Techniques for Mesh Movement and Curved Mesh Generation

for Computational Fluid Dynamics

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Why mesh movement?

Mesh movement is useful in several areas of CFD

- aeroelasticity, and fluid-structure interaction in general
- shape optimization
- generation of curved meshes for spatially high-order discretizations
- others

Fluid-Structure Interaction

- For a body-fitted grid, robust mesh-movement is required to maintain validity and quality of the mesh after imposing motion of the structural domain[1].
- Immersed boundary methods can also be used; both have advantages and disadvantages[2].

[1] C. Farhat and M. Lesoinne. "Two efficient staggered algorithms for the serial and parallel solution of three-dimensional nonlinear transient aeroelastic problems". In: *Comput. Methods Appl. Mech. Engrg.* 182 (2000), pp. 499–515.

[2] G. Hou, J. Wang, and A. Layton. "Numerical methods for fluid-structure interaction - a review". In: *Commun. Comput. Phys.* 12.2 (2012), pp. 337–377.

Shape optimization

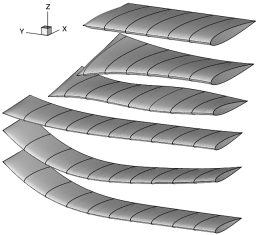

Figure : Convergence history of lift-constrained drag minimization

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[3] J.E. Hicken and D.W. Zingg. “Aerodynamic optimization algorithm with integrated geometry parameterization and mesh movement”. In: *AIAA Journal* 48.2 (2010), pp. 400–413.

High-order methods

- According to Wang, Fidkowski *et. al.*[4], spatially high-order methods perform better than prevailing second order methods for some kinds of simulations considering CPU time taken to achieve a given error level.
- One area of challenge they mention is generation of high-order meshes.

[4] Z.J. Wang et al. "High-order CFD methods: current status and perspective". In: *Intl. J. Numer. Meth. Fluids* 72 (2013), pp. 811–845.

The need for curved meshes

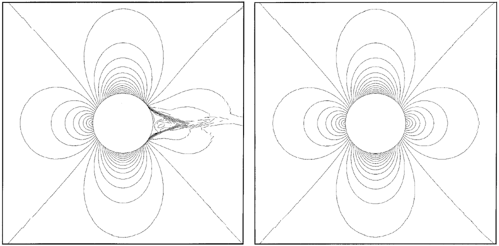

Figure : Inviscid subsonic flow over a cylinder; left: DGP1 solution with regular linear mesh, right: DGP1 solution with quadratic (‘Q2’) mesh[5]

[5] F. Bassi and S. Rebay. “High-order accurate discontinuous finite element solution of the 2D Euler equations”. In: *J. Comput. Phys.* 128 (1997), pp. 251–285.

The need for curved meshes

Even in less extreme cases, curved meshes are required to obtain design (p+1) order of accuracy for high-order methods such as discontinuous Galerkin methods[6].

[6] X. Luo, M.S. Shephard, and J.-F. Remacle. “The influence of geometric approximation on the accuracy of high order methods”. In: *Rensselaer SCOREC report* 1 (2001).

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Introduction

The problem is to move the interior nodes of a mesh when a given displacement is imposed on the boundary.

- At the very least: mesh elements should not get invalidated.
- Mesh elements should not suffer much deterioration in quality.
- The technique should be computationally inexpensive.

Many mesh-movement methods for unstructured meshes can be found in literature. They can be broadly classified into two types.

- Elasticity-based methods
- Interpolation methods

Combination of these with each other and with other techniques such as topological (connectivity) smoothing are also used[7].

[7] F. Alauzet. "A changing-topology moving mesh technique for large displacaments". In: *Engrg. Comput.* 30 (2014), pp. 175–200.

Lineal spring analogy

Every mesh edge is treated as a linear spring in each coordinate direction[8].

$$\sum_j k_{ij}(\Delta \mathbf{r}_i - \Delta \mathbf{r}_j) = \mathbf{0} \quad \forall i \tag{1}$$

where i ranges over all nodes, j ranges over points surrounding node i and $\Delta \mathbf{r}_i$ is the displacement of node i. k_{ij} is the stiffness of the spring between nodes i and j, which can be taken as

$$k_{ij} = \frac{1}{||\mathbf{r}_i - \mathbf{r}_j||}. \tag{2}$$

[8] J.T. Batina. "Unsteady Euler Algorithm with Unstructured Dynamic Mesh for Complex-Aircraft Aerodynamic Analysis". In: *AIAA Journal* 29.3 (1991), pp. 327–333.

Lineal spring analogy

This scheme requires the solution of a linear system of size N_n (the number of mesh nodes), n_{dim} (the spatial dimension of the problem) times.

Torsional springs

The lineal spring analogy fails for large deformations or stretched elements. Farhat *et. al.* came up with a more robust scheme, which is also a spring analogy[9]. They introduce two

improvements over Batina's model.

- The model is closer to a structural analogy in that the displacements in each coordinate direction are coupled.
- 'Torsional springs' are introduced at each node in each element. These are designed to prevent edges collapsing into each other due to rotational motion.

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[9] C. Farhat et al. "Torsional springs for two-dimensional dynamic unstructured fluid meshes". In: *Computer methods in applied mechanics and engineering* 163 (1998), pp. 231–245.

Torsional springs

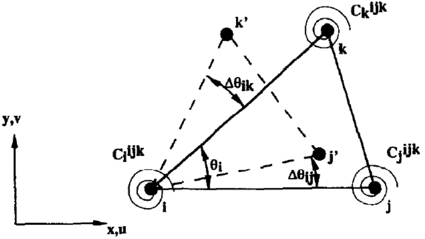

Figure : Movement of an element in torsion spring analogy

A coupled system of $n_{dim} N_n$ equations must be solved.

Linear elasticity

The mesh is assumed to model a deformable solid body, which is then deformed according to the equations of solid mechanics, that is, linear or non-linear elasticity.
The simplest approach is linear elasticity.

$$\nabla \cdot \sigma = \mathbf{0} \quad \text{in}\, \Omega \tag{3}$$

$$\boldsymbol{\sigma} = 2\mu\boldsymbol{\epsilon} + \lambda(\mathrm{tr}\boldsymbol{\epsilon})\boldsymbol{I} \tag{4}$$

$$\boldsymbol{\epsilon} = \frac{1}{2}(\nabla\boldsymbol{u} + \nabla\boldsymbol{u}^T) \tag{5}$$

$$\boldsymbol{u} = \boldsymbol{u}_b \quad \text{on}\, \partial\Omega \tag{6}$$

Stiffened Linear elasticity

- The linear elasticity scheme is often modified by ‘stiffening’ the mesh appropriately.
- We attain some control over the propagation of deformation into the interior of the mesh, as done, for instance, by Stein *et. al.*[10].
- The material is stiffened based on the determinant of the local Jacobian matrix of the reference-to-physical mapping; i.e., smaller elements are stiffer than larger ones.

[10] K. Stein, T. Tezduyar, and R. Benney. “Mesh moving techniques for fluid-structure interactions with large displacements”. In: *J. Appl. Mech.* 70 (2003), pp. 58–63.

Nonlinear elasticity

Claimed to be a highly robust method for mesh movement by Persson and Peraire[11].
The constitutive equation (4) and strain-displacement relation (5) are replaced by the 'neo-Hookean' constitutive model

$$\boldsymbol{\sigma} = \mu((\boldsymbol{F}^T\boldsymbol{F})\boldsymbol{F}^{-T} - \boldsymbol{F}^{-T}) + \lambda(\ln\det\boldsymbol{F})\boldsymbol{F}^{-T} \tag{7}$$

Here, $\boldsymbol{F} = \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{\xi}}$, $\boldsymbol{x}$ is the physical position vector of a point with coordinate ξ in the reference configuration.
The system is solved using Newton-GMRES iterations.

[11] P.-O. Persson and J. Peraire. "Curved mesh generation and mesh refinement using lagrangian solid mechanics". In: *47th AIAA Aerospace Sciences Meeting*. 2009.

Elasticity-based methods

- Advantages: stiffened linear elasticity is found to be robust, and nonlinear elasticity is claimed to be very robust.
- Disadvantage: expensive!
- Also, implementation is dependent on element type and spatial dimension.

Delaunay graph mapping

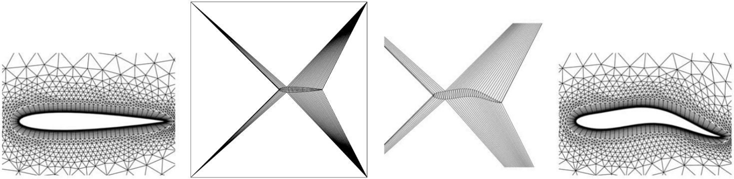

Figure : The DGM process (from left to right): original mesh, Delaunay graph, deformed Delaunay graph, deformed mesh (ref:[12])

[12] X. Liu, N. Qin, and H. Xia. "Fast dynamic grid deformation based on Delaunay graph mapping". In: *Journal of Computational Physics* 211 (2006), pp. 405–423.

Mesh quality metrics

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