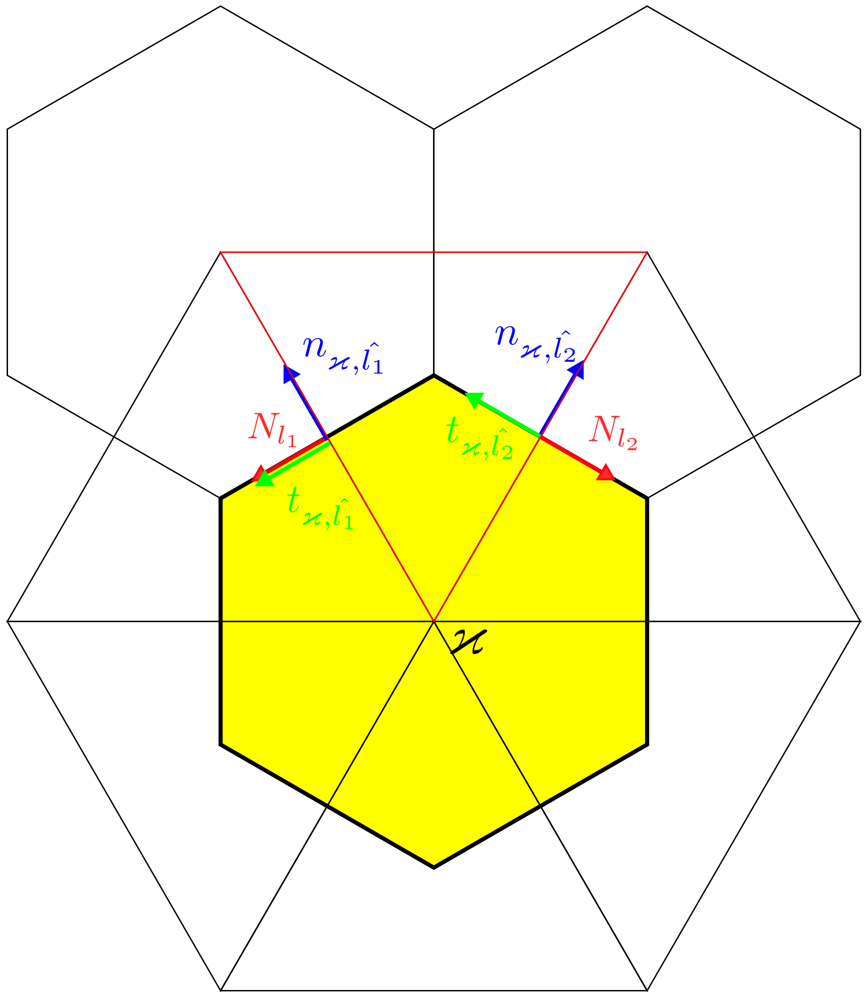

Here, $\boldsymbol{N}_{l_1}$ and $\boldsymbol{N}_{l_2}$ are of such directions because we define the flux on downward triangle (the red one) to be outward. On the yellow dual cell denoted by vertex $\varkappa$, the ourward unit vectors are $\boldsymbol{n}_{\varkappa,\hat{l_1}}$ and $\boldsymbol{n}_{\varkappa,\hat{l_2}}$. Since we need $\boldsymbol{n}_{\varkappa,\hat{l}} \times \boldsymbol{t}_{\varkappa,\hat{l}} = \boldsymbol{k}_l$, where $\boldsymbol{k}_l$ denotes the unit vector in the upward direction of the local coordinate at the midpoint of edge l, the green $\boldsymbol{t}_{\varkappa,\hat{l_1}}$ and $\boldsymbol{t}_{\varkappa,\hat{l_2}}$ are as shown in the figure.

The curl operator is

$$\mathrm{curl}(\boldsymbol{v})_{\varkappa} = \frac{1}{A_{\varkappa}} \sum_{\hat{l}} v_{n_l} (\boldsymbol{N_l} \cdot \boldsymbol{t}_{\varkappa,\hat{l}}) \hat{l}$$

Note the part $\boldsymbol{N_l} \cdot \boldsymbol{t}_{\varkappa,\hat{l}}$, we see that this sign is different on edges $\hat{l}$ for a dual cell $\varkappa$ (1 for $\hat{l}_1$ and -1 for $\hat{l}_2$). Therefore the previous approach we used for a divergence operator, which is to assume a uniform sign on all edges of a cell, cannot be applied here. A separate formula based on the color of an edge is not valid either, because for $\hat{l}_1$ and the bottom-right dual edge, they are of different signs.

There are some solutions to this problem, but I cannot see a simple one without modifying the GridTools. For example:

- a field on both vertexes and edges
- a different `on_edges()` traversal for a vertex. For example, `on_odd_edges()` and `on_even_edges()`.

It remains to be seen whether the second proposal would be general enough. I still need to read the literature to find more operators. I think for a Laplacian there will be no such problem of multiple values on one place, or nonuniform formula on fields. But I need to find out if there are more operators involving traversing surrounding edges and treating each of them differently. Well I have to read the papers now because I am stuck here anyway :P