From Skeletal Motion to Conformal Geometric Algebra: A Novel Approach to Human Motion Modeling

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Introduction

Modeling human motion is critical in robotics, biomechanics, and sports analytics, but traditional tools like quaternions and rotation matrices face limitations:

- **Quaternions:** Compact but lack support for translations without added complexity.
- **Rotation Matrices**: Interpretable but computationally expensive for high- dimensional data.

Conformal Geometric Algebra (CGA) offers a unified framework to encode rotations, translations, and dilations, preserving geometric relationships efficiently.

Figure 1. Overlaid skeleton visualization using OpenPose, showing a hockey player's slapshot motion.

This work introduces CGA-specific autoencoder that compactly encodes and reconstructs motion with high fidelity, paving the way for applications in sports analytics, robotics, and biomechanics.

Data Description

The dataset originates from a 6-second video of a hockey slapshot, processed using OpenPose to extract 3D skeletal data. The data was transformed into Conformal Geometric Algebra (CGA) representations for efficient motion modeling.

Data Elements:

Points: 3D joint positions projected to 5D conformal space:

$$P = xe_1 + ye_2 + ze_3 + \tfrac{1}{2}(x^2 + y^2 + z^2)e_\infty + e_0$$

Translations: Captured with CGA translators:

$$T = 1 + \tfrac{1}{2}(\Delta x e_1 + \Delta y e_2 + \Delta z e_3)e_\infty$$

Rotations: Represented as CGA rotors:

$$R = \cos\frac{\theta}{2} + \sin\frac{\theta}{2}\,(r_x e_1 + r_y e_2 + r_z e_3)$$

Multivector Representation:

$$M = T \cdot R \cdot P \cdot R^\dagger \cdot T^\dagger$$

where P is the CGA point, R is the rotor, T is the translator, and $R^\dagger, T^\dagger$ are their reverses, ensuring proper transformations.

This encoding preserves geometric relationships, enabling precise and compact motion modeling.

Autoencoder Structure and Novelty

To process CGA motion data, a custom autoencoder was designed to encode and reconstruct motion while preserving geometric properties.

Architecture Overview:

Input: 7D vectors for each CGA component (rotors, translators, points).
Encoder: Independent branches for rotors, translators, and points, merged into a shared latent space:

Latent Vector: Compact representation capturing key motion features in the latent space.

Decoder: Reconstructs components with minimal loss using tailored loss functions.

Key Innovations:

Multivector Loss Functions:

$$L_{\text{rotor}} = \frac{1}{N}\sum_{i=1}^{N} \|R_i - \hat{R}_i\|^2 + \lambda \|\|\hat{R}_i\| - 1\|$$

Designed to preserve CGA rotor normalization and minimize reconstruction error.

Geometric Fidelity: Ensures the latent space respects CGA's geometric relationships.

Interpretability: Latent space clusters frames by motion intensity (validated with PCA/t-SNE).

This autoencoder is the first to leverage CGA representations, achieving low reconstruction loss (0.00059) and maintaining high fidelity for rotors (MSE 0.1373), translators (MSE 0.1460), and points (MSE 0.3377).

Reconstruction Validation and Visualization

After encoding and reconstructing motion data using the CGA-based autoencoder, the recon- structed CGA representations were projected back into 3D space for validation.

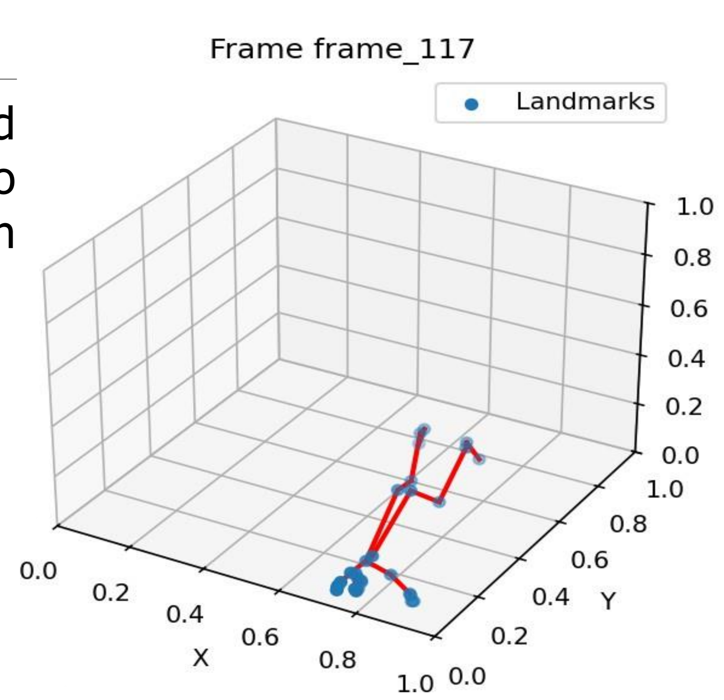

Figure 2. Example Frame 117 Reconstruction

To validate reconstruction fidelity, reconstructed 3D points were compared against the original dataset using a predefined error threshold. A frame-by-frame analysis revealed that deviations were **consistently below the threshold**, highlighting the autoencoder's precision in preserving motion dynamics. A visual animation of the reconstructed motion confirmed that both rotational and translational components aligned closely with the original skeletal motion, highlighting the autoencoder's precision in preserving motion dynamics.

Key metrics included:

Mean Point Error: Errors in reconstructed 3D joint positions averaged below 0.02 units.

Temporal Coherence: The motion trajectory remained smooth and continuous across frames.

Threshold Compliance: No reconstructed point deviated beyond the preset threshold, affirming geometric consistency.

This level of precision validates the autoencoder's ability to encode and re- construct complex human motion while retaining essential geometric relation- ships, making it highly suitable for applications in sports analysis, robotics, and biomechanics.

PCA Analysis of Latent Space

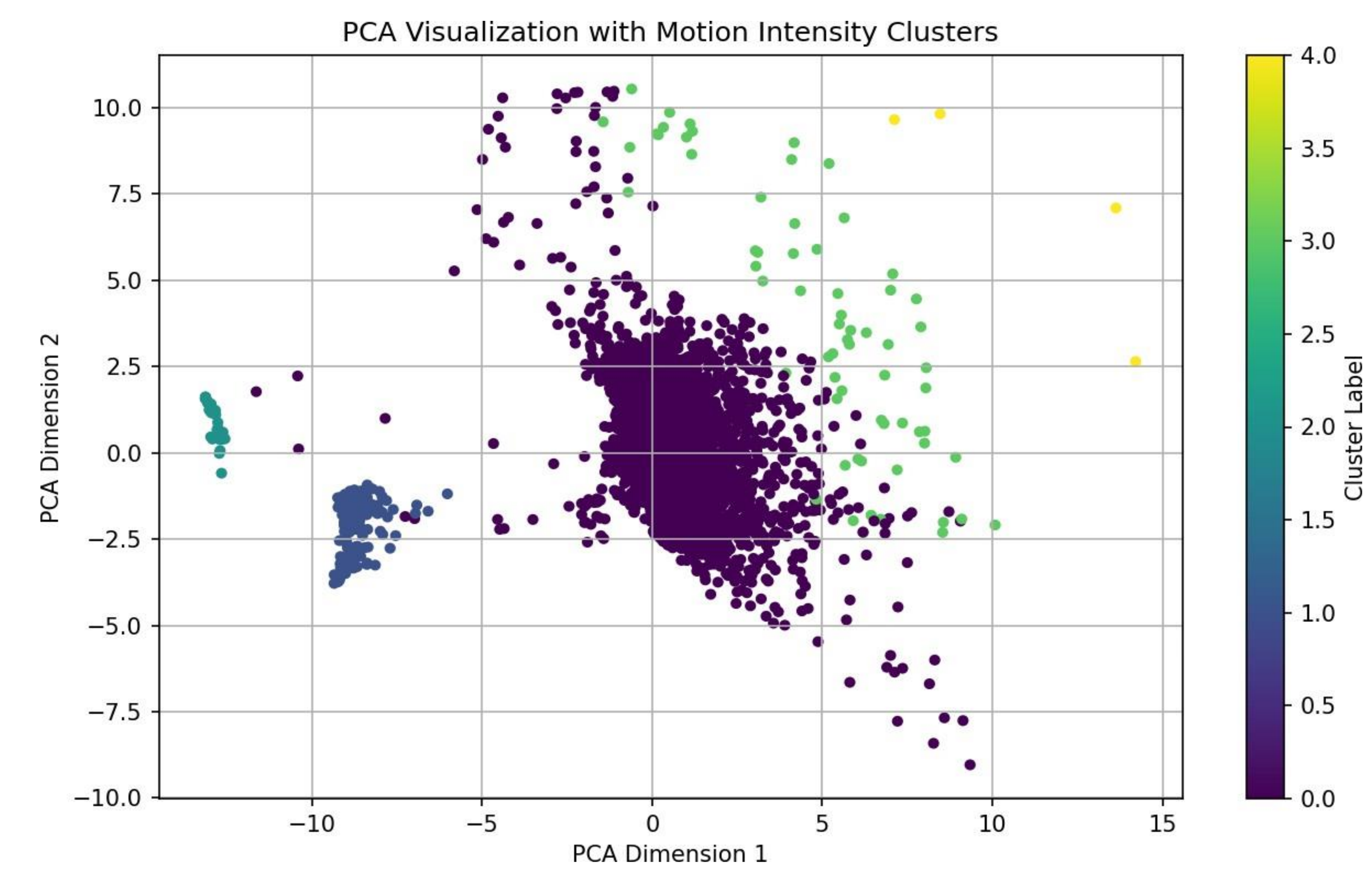

Figure 3. PCA visualization of latent space showing clusters of varying motion intensity.

The PCA analysis revealed distinct clusters within the latent space, corresponding to different types of motion:

- **Low-Intensity Motion:** Static or repetitive frames form tight clusters.
- **Moderate Motion:** Transitional movements are grouped distinctly.
- **High-Intensity Motion:** Dynamic phases like the slapshot execution are more dispersed.

This clustering demonstrates the autoencoder's ability to organize motion data in a compact, interpretable latent space. The success of this analysis underscores the transformative potential of CGA for encoding and understanding high-dimensional motion data, marking a significant advancement in CGA-based motion modeling.

Conclusion and Future Work

This project represents a significant step forward in leveraging Conformal Geometric Algebra (CGA) for human motion modeling. The development and successful implementation of a CGA-specific autoencoder capable of compactly encoding and reconstructing motion with high fidelity demonstrate the transformative potential of CGA. By organizing motion into an interpretable latent space and validating reconstructed motions against original data with minimal deviations, this work highlights CGA's robustness and suitability for applications in sports analytics, robotics, and biomechanics.

Future Work: To build on these results, the next steps involve:

- **Expanding the Dataset:** Incorporating a more diverse range of motion types, including different sports, daily activities, and dynamic environments, to test the model's scalability and generalizability.
- **Testing on New Data:** Evaluating the model on unseen motion datasets to assess its robustness across various contexts.
- **Ablation Studies:** Comparing the CGA autoencoder against quaternion-based and other traditional models to quantify the advantages and limitations of CGA.

References

[1] Zhe Cao, Tomas Simon, Shih-En Wei, and Yaser Sheikh. Openpose: Real-time multi-person 2d pose estimation, 2021.

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