

APPLIED MATH 1930
Randomized Algorithms for Counting, Integration and Optimization
Syllabus—Fall 2021

1. Overview of the course.
2. Probability review (Chapter 1 and other reference books):
Random variables, distributions, independence, expected values, normal (Gaussian) distributions, limit theorems, Poisson and Markov processes, large time properties of Markov processes and stationary distributions, measures of information, optimization.
3. Simulation of random numbers, random variables, and random processes (Chapter 2):
How to generate samples from (i.e., simulate) random numbers, random variables, random vectors, random processes.
4. Reducing variance (Chapter 5, 7):
Formulation and analysis of standard methods that have been developed for reducing variance in Monte Carlo simulations, including importance sampling.
5. Markov chain Monte Carlo (Chapter 6):
Introduction to methods for approximating the stationary distribution of a Markov process, and various applications. Qualitative discussion on why rare events slow convergence.
6. The splitting method (Chapter 9):
Schemes based on branching processes for generating samples from sets with low probability, and their use in problems from different areas, including counting in combinatorial problems and rare event estimation.
7. Advanced topics selected from importance sampling, stochastic enumeration, parallel tempering. Some of these topics may developed as part of the end-of-semester projects.