Multivariate Regression Model for Explaining House Price

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Data Import & Variables Checking

```
library(readxl)
dat <- read_excel("Data/Real estate valuation data set.xlsx")
dat=dat[-1]
colnames(dat)=c("x1","x2","x3","x4","x5","x6","y")
anyNA(dat) # check if any missing value
```

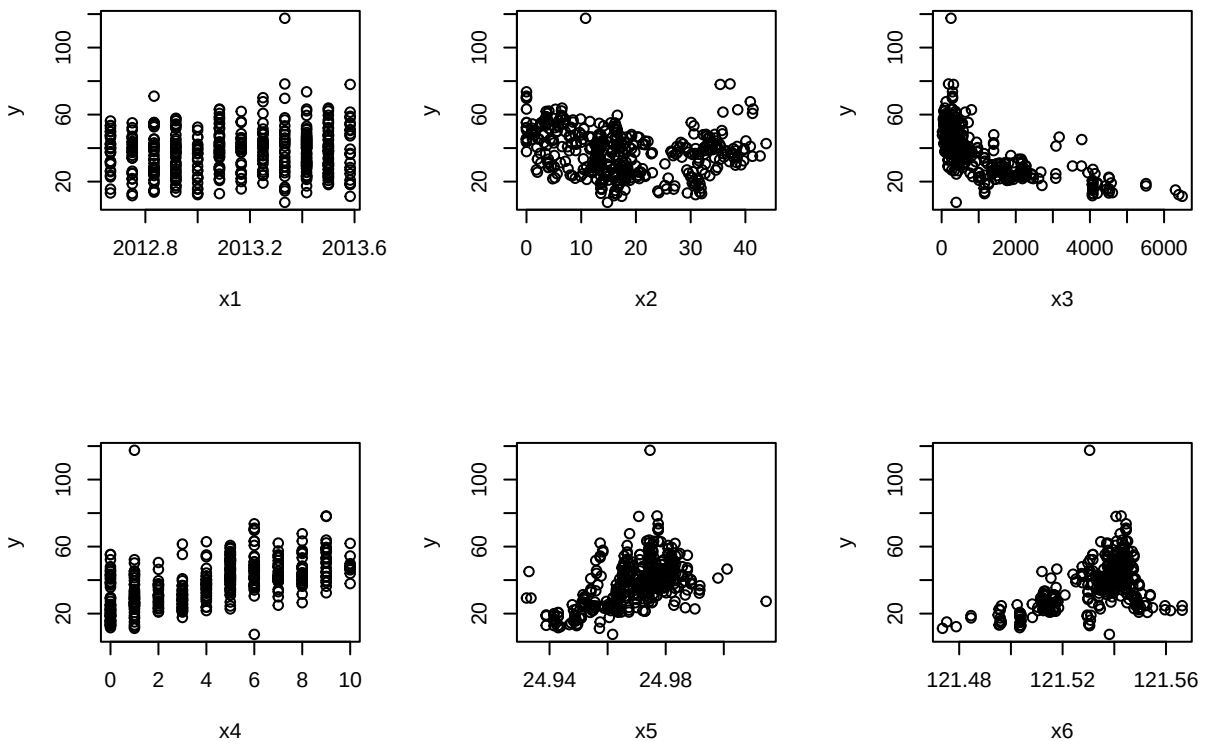
```
## [1] FALSE
```

```
summary(dat,digits=6)
```

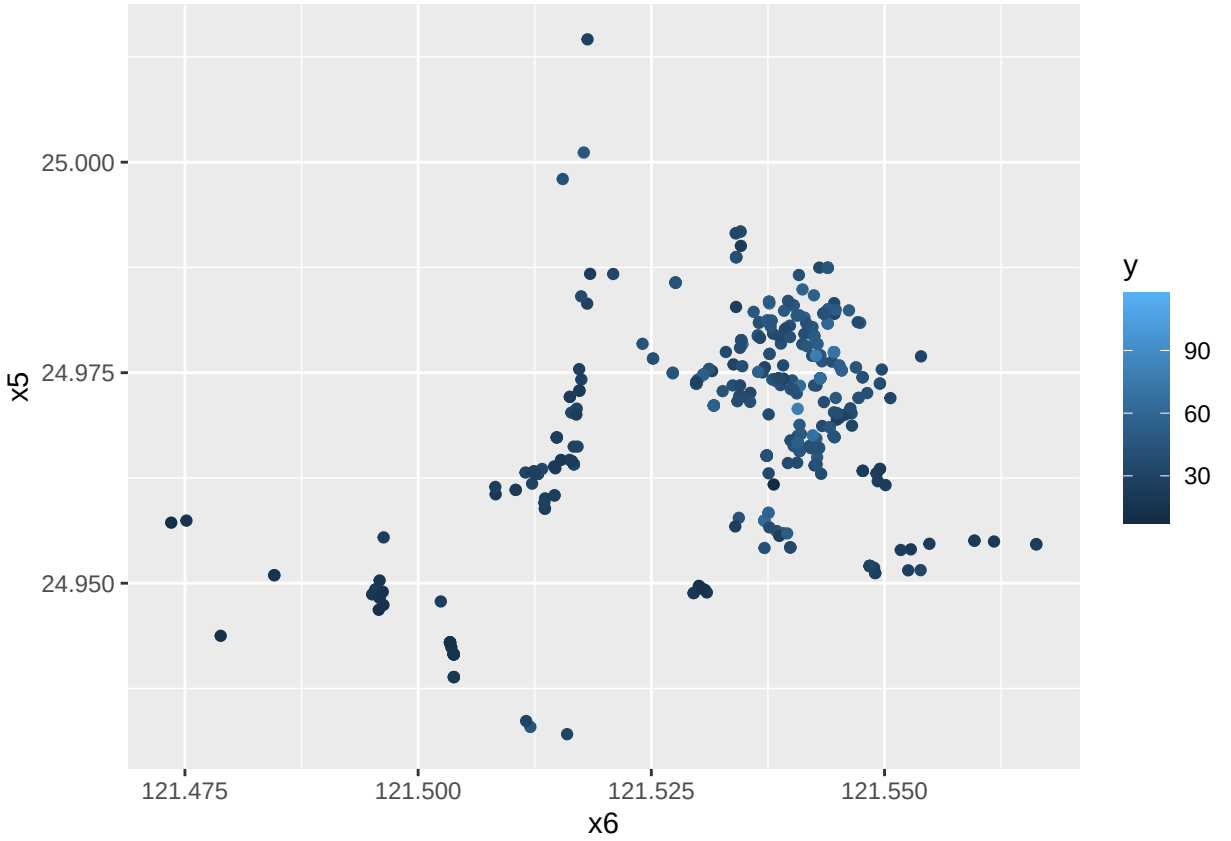
```
##        x1               x2               x3               x4
##  Min.   :2012.67   Min.   : 0.0000   Min.   :  23.383   Min.   : 0.0000
##  1st Qu.:2012.92   1st Qu.: 9.0250   1st Qu.: 289.325   1st Qu.: 1.0000
##  Median :2013.17   Median :16.1000   Median : 492.231   Median : 4.0000
##  Mean   :2013.15   Mean   :17.7126   Mean   :1083.886   Mean   : 4.0942
##  3rd Qu.:2013.42   3rd Qu.:28.1500   3rd Qu.:1454.279   3rd Qu.: 6.0000
##  Max.   :2013.58   Max.   :43.8000   Max.   :6488.021   Max.   :10.0000
##        x5               x6               y
##  Min.   :24.9321   Min.   :121.474   Min.   :  7.6000
##  1st Qu.:24.9630   1st Qu.:121.528   1st Qu.: 27.7000
##  Median :24.9711   Median :121.539   Median : 38.4500
##  Mean   :24.9690   Mean   :121.533   Mean   : 37.9802
##  3rd Qu.:24.9775   3rd Qu.:121.543   3rd Qu.: 46.6000
##  Max.   :25.0146   Max.   :121.566   Max.   :117.5000
```

Visualization

```
par(mfrow=c(2,3))
plot(dat$y~dat$x1,xlab='x1',ylab='y')
plot(dat$y~dat$x2,xlab='x2',ylab='y')
plot(dat$y~dat$x3,xlab='x3',ylab='y')
plot(dat$y~dat$x4,xlab='x4',ylab='y')
plot(dat$y~dat$x5,xlab='x5',ylab='y')
plot(dat$y~dat$x6,xlab='x6',ylab='y')
```




```
# patterns between inputs and response
par(mfrow=c(1,1))
library(ggplot2)
ggplot()+geom_point(data=dat,aes(x=x6,y=x5,col=y)) #geographical plots
```



Variable Selection/Model Construction

```
lm0=lm(y~(x1+x2+x3+x4+x5+x6)^2,data=dat)
summary(lm0)
```

```
##
## Call:
## lm(formula = y ~ (x1 + x2 + x3 + x4 + x5 + x6)^2, data = dat)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -36.474  -4.329  -0.738   3.293  70.522
##
## Coefficients: (3 not defined because of singularities)
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -5.040e+04  1.724e+04  -2.923  0.00366 **
## x1           6.782e+00  4.415e+00   1.536  0.12527
## x2           3.325e+02  7.527e+02   0.442  0.65898
## x3           8.767e+00  4.172e+00   2.101  0.03625 *
## x4           5.422e+03  4.136e+03   1.311  0.19061
## x5           9.939e+02  1.631e+02   6.092 2.65e-09 ***
## x6           9.854e+01  1.178e+02   0.837  0.40329
## x1:x2        1.043e-01  1.259e-01   0.828  0.40825
```

```
## x1:x3       -8.601e-04  1.352e-03  -0.636  0.52490
## x1:x4       -3.887e-01  6.074e-01  -0.640  0.52257
## x1:x5               NA         NA      NA       NA
## x1:x6               NA         NA      NA       NA
## x2:x3        8.616e-06  7.877e-05   0.109  0.91295
## x2:x4        9.448e-03  1.384e-02   0.682  0.49534
## x2:x5       -2.897e+00  4.374e+00  -0.662  0.50816
## x2:x6       -3.870e+00  5.761e+00  -0.672  0.50219
## x3:x4       -1.687e-03  3.852e-04  -4.378 1.53e-05 ***
## x3:x5       -2.229e-01  4.396e-02  -5.072 6.08e-07 ***
## x3:x6       -1.212e-02  2.702e-02  -0.449  0.65394
## x4:x5       -1.071e+02  2.150e+01  -4.981 9.48e-07 ***
## x4:x6       -1.616e+01  3.118e+01  -0.518  0.60460
## x5:x6               NA         NA      NA       NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.903 on 395 degrees of freedom
## Multiple R-squared:  0.6773, Adjusted R-squared:  0.6626
## F-statistic: 46.06 on 18 and 395 DF,  p-value: < 2.2e-16
```

```
# do step-wise variable selection
step(lm0,direction="both",trace=FALSE)
```

```
##
## Call:
## lm(formula = y ~ x1 + x2 + x3 + x4 + x5 + x6 + x2:x6 + x3:x4 +
##     x3:x5 + x4:x5, data = dat)
##
## Coefficients:
## (Intercept)           x1           x2           x3           x4           x5
##  -4.299e+04    5.946e+00    5.645e+02    5.619e+00    2.695e+03    9.495e+02
##          x6        x2:x6        x3:x4        x3:x5        x4:x5
##   6.049e+01   -4.647e+00   -1.574e-03   -2.252e-01   -1.079e+02
```

```
lm1=lm(y~x1+x2+x3+x4+x5+x6+x2:x6+x3:x4+x3:x5+x4:x5,data=dat)
summary(lm1)
```

```
##
## Call:
## lm(formula = y ~ x1 + x2 + x3 + x4 + x5 + x6 + x2:x6 + x3:x4 +
##     x3:x5 + x4:x5, data = dat)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -36.470  -4.295  -0.544   3.206  71.136
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -4.299e+04  9.226e+03  -4.659 4.32e-06 ***
## x1           5.946e+00  1.383e+00   4.301 2.14e-05 ***
## x2           5.645e+02  3.768e+02   1.498    0.135
## x3           5.619e+00  9.233e-01   6.086 2.70e-09 ***
```

```
## x4           2.695e+03  4.773e+02   5.646 3.11e-08 ***
## x5           9.495e+02  1.115e+02   8.518 3.28e-16 ***
## x6           6.049e+01  6.536e+01   0.925    0.355
## x2:x6       -4.647e+00  3.101e+00  -1.499    0.135
## x3:x4       -1.574e-03  2.507e-04  -6.279 8.80e-10 ***
## x3:x5       -2.252e-01  3.698e-02  -6.090 2.64e-09 ***
## x4:x5       -1.079e+02  1.911e+01  -5.643 3.16e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.845 on 403 degrees of freedom
## Multiple R-squared:  0.6756, Adjusted R-squared:  0.6676
## F-statistic: 83.93 on 10 and 403 DF,  p-value: < 2.2e-16
```

```
# drop x6 and x2:x6
lm2=lm(y~x1+x2+x3+x4+x5+x3:x4+x3:x5+x4:x5,data=dat)
summary(lm2)
```

```
##
## Call:
## lm(formula = y ~ x1 + x2 + x3 + x4 + x5 + x3:x4 + x3:x5 + x4:x5,
##     data = dat)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -36.414  -4.333  -0.746   3.328  71.191
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -3.640e+04  3.837e+03  -9.486  < 2e-16 ***
## x1           5.982e+00  1.383e+00   4.327 1.91e-05 ***
## x2          -2.618e-01  3.436e-02  -7.620 1.82e-13 ***
## x3           5.824e+00  8.661e-01   6.724 6.05e-11 ***
## x4           2.797e+03  4.464e+02   6.266 9.48e-10 ***
## x5           9.771e+02  1.032e+02   9.470  < 2e-16 ***
## x3:x4       -1.590e-03  2.385e-04  -6.667 8.56e-11 ***
## x3:x5       -2.334e-01  3.470e-02  -6.726 5.96e-11 ***
## x4:x5       -1.120e+02  1.788e+01  -6.263 9.65e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.848 on 405 degrees of freedom
## Multiple R-squared:  0.6738, Adjusted R-squared:  0.6673
## F-statistic: 104.6 on 8 and 405 DF,  p-value: < 2.2e-16
```

```
library(faraway)
# check multicollinearity
vif(lm2)
```

```
##           x1           x2           x3           x4           x5        x3:x4
## 1.019343e+00 1.027448e+00 8.013781e+06 1.159611e+07 1.099518e+01 1.520145e+00
##        x3:x5        x4:x5
## 8.007367e+06 1.159943e+07
```

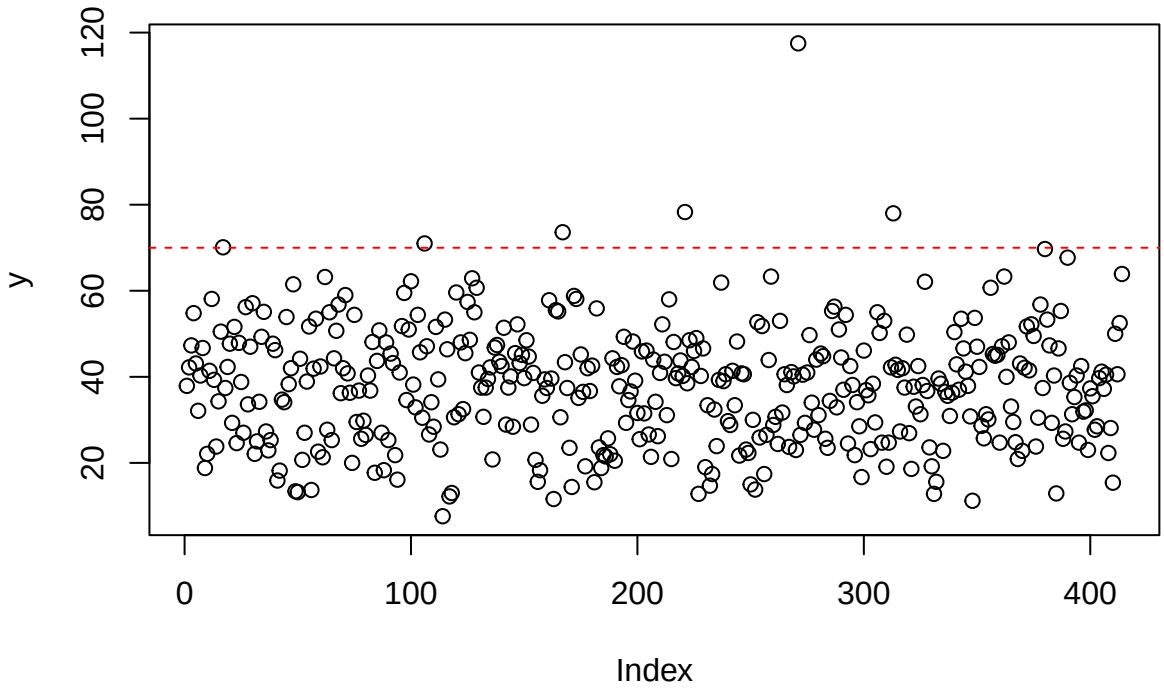
```r
# there exists multicollinearity
# drop x3:x5 and x4:x5
lm_1=lm(log(y)~x1+x2+x3+x4+x5+x3:x4,data=dat)
summary(lm_1) # predictors are significant
```

```
##
## Call:
## lm(formula = log(y) ~ x1 + x2 + x3 + x4 + x5 + x3:x4, data = dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.66445 -0.11075  0.00606  0.10768  1.02761
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -5.119e+02  7.824e+01  -6.544 1.81e-10 ***
## x1           1.411e-01  3.744e-02   3.768 0.000189 ***
## x2          -7.100e-03  9.264e-04  -7.664 1.34e-13 ***
## x3          -1.153e-04  1.318e-05  -8.749  < 2e-16 ***
## x4           3.822e-02  4.870e-03   7.849 3.75e-14 ***
## x5           9.280e+00  1.090e+00   8.510 3.37e-16 ***
## x3:x4       -3.474e-05  5.988e-06  -5.802 1.32e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.213 on 407 degrees of freedom
## Multiple R-squared:  0.7097, Adjusted R-squared:  0.7054
## F-statistic: 165.8 on 6 and 407 DF,  p-value: < 2.2e-16
```

```r
vif(lm_1) # no multicollinearity
```

```
##       x1       x2       x3       x4       x5    x3:x4
## 1.014400 1.013775 2.518950 1.872938 1.666907 1.300180
```

```r
plot(dat$y,ylab='y')
abline(h=70,col='red',lty=2)
```




```
sum(dat$y>70)
```

```
## [1] 6
```

```
center_x=mean(dat[which(dat$y>70),]$x6)
center_y=mean(dat[which(dat$y>70),]$x5)
c(center_x,center_y)
```

```
## [1] 121.54126  24.97578
```

```
dat$r=sqrt((dat$x6-center_x)^2+(dat$x5-center_y)^2)
dat$theta=atan((dat$x6-center_x)/(dat$x5-center_y))


lm4=lm(log(y)~(x1+x2+x3+x4+r+theta)^2,data=dat)
summary(lm4)
```

```
##
## Call:
## lm(formula = log(y) ~ (x1 + x2 + x3 + x4 + r + theta)^2, data = dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.71744 -0.10860  0.00278  0.10429  0.93667
```

```
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -4.052e+02  2.792e+02  -1.451  0.14751
## x1           2.033e-01  1.387e-01   1.466  0.14352
## x2          -4.581e+00  6.835e+00  -0.670  0.50313
## x3          -1.571e-02  2.021e-01  -0.078  0.93808
## x4           4.753e+01  3.381e+01   1.406  0.16054
## r            3.769e+01  1.842e+04   0.002  0.99837
## theta       -4.320e+01  1.045e+02  -0.413  0.67955
## x1:x2        2.270e-03  3.395e-03   0.669  0.50415
## x1:x3        7.793e-06  1.004e-04   0.078  0.93817
## x1:x4       -2.361e-02  1.679e-02  -1.406  0.16058
## x1:r        -3.228e-02  9.149e+00  -0.004  0.99719
## x1:theta     2.150e-02  5.192e-02   0.414  0.67903
## x2:x3        6.082e-06  2.580e-06   2.358  0.01887 *
## x2:x4        6.053e-04  4.019e-04   1.506  0.13280
## x2:r        -3.664e-01  2.116e-01  -1.732  0.08415 .
## x2:theta     3.419e-04  1.250e-03   0.273  0.78464
## x3:x4       -4.505e-05  1.373e-05  -3.280  0.00113 **
## x3:r         2.384e-03  9.581e-04   2.488  0.01327 *
## x3:theta    -1.016e-04  5.390e-05  -1.885  0.06018 .
## x4:r         3.153e+00  9.956e-01   3.167  0.00166 **
## x4:theta    -2.093e-03  6.874e-03  -0.304  0.76098
## r:theta      3.903e+00  4.426e+00   0.882  0.37842
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2058 on 392 degrees of freedom
## Multiple R-squared:  0.7389, Adjusted R-squared:  0.7249
## F-statistic: 52.83 on 21 and 392 DF,  p-value: < 2.2e-16
```

```
# do step-wise variable selection
step(lm4,direction="both",trace=FALSE)
```

```
##
## Call:
## lm(formula = log(y) ~ x1 + x2 + x3 + x4 + r + theta + x1:x4 +
##     x2:x3 + x2:x4 + x2:r + x3:x4 + x3:r + x3:theta + x4:r, data = dat)
##
## Coefficients:
## (Intercept)           x1           x2           x3           x4            r
##  -5.426e+02    2.715e-01   -1.109e-02   -4.661e-05    5.577e+01   -2.492e+01
##       theta        x1:x4        x2:x3        x2:x4         x2:r        x3:x4
##   8.269e-02   -2.770e-02    6.413e-06    6.287e-04   -3.801e-01   -4.189e-05
##        x3:r     x3:theta         x4:r
##   2.235e-03   -5.327e-05    2.852e+00
```

```
lm5=lm(log(y)~x1+x2+x3+x4+r+theta+x1:x4+x2:x3+x2:r+x3:x4+x3:r+x3:theta+x4:r,data=dat)
summary(lm5)
```

```
##
## Call:
```

```
## lm(formula = log(y) ~ x1 + x2 + x3 + x4 + r + theta + x1:x4 +
##     x2:x3 + x2:r + x3:x4 + x3:r + x3:theta + x4:r, data = dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.71979 -0.10347  0.00277  0.10384  0.97194
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -5.291e+02  1.252e+02  -4.226 2.94e-05 ***
## x1           2.648e-01  6.219e-02   4.258 2.57e-05 ***
## x2          -7.011e-03  1.416e-03  -4.951 1.09e-06 ***
## x3          -7.148e-05  8.307e-05  -0.860 0.390069
## x4           5.385e+01  2.475e+01   2.176 0.030124 *
## r           -2.200e+01  6.519e+00  -3.375 0.000811 ***
## theta        8.732e-02  2.244e-02   3.891 0.000117 ***
## x1:x4       -2.674e-02  1.229e-02  -2.176 0.030177 *
## x2:x3        6.187e-06  2.467e-06   2.508 0.012531 *
## x2:r        -4.562e-01  1.994e-01  -2.288 0.022641 *
## x3:x4       -3.594e-05  1.283e-05  -2.800 0.005351 **
## x3:r         2.590e-03  9.058e-04   2.859 0.004469 **
## x3:theta    -6.142e-05  2.055e-05  -2.989 0.002970 **
## x4:r         2.505e+00  9.119e-01   2.747 0.006287 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2051 on 400 degrees of freedom
## Multiple R-squared:  0.7355, Adjusted R-squared:  0.7269
## F-statistic: 85.54 on 13 and 400 DF,  p-value: < 2.2e-16
```

```
# drop insignificant predictors
lm6=lm(log(y)~x1+x2+x3+x4+r+theta+x2:x3+x2:r+x3:r+x3:theta,data=dat)
summary(lm6)
```

```
##
## Call:
## lm(formula = log(y) ~ x1 + x2 + x3 + x4 + r + theta + x2:x3 +
##     x2:r + x3:r + x3:theta, data = dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.70506 -0.11044  0.00293  0.10933  0.96676
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -2.907e+02  7.381e+01  -3.938 9.69e-05 ***
## x1           1.463e-01  3.667e-02   3.990 7.84e-05 ***
## x2          -6.633e-03  1.428e-03  -4.644 4.63e-06 ***
## x3          -2.545e-04  5.583e-05  -4.559 6.84e-06 ***
## x4           2.801e-02  5.509e-03   5.085 5.64e-07 ***
## r           -8.174e+00  4.214e+00  -1.939 0.053149 .
## theta        9.231e-02  2.258e-02   4.088 5.25e-05 ***
## x2:x3        7.868e-06  2.355e-06   3.341 0.000914 ***
## x2:r        -5.946e-01  1.910e-01  -3.113 0.001985 **
```

```
## x3:r         3.362e-03  6.768e-04   4.967 1.01e-06 ***
## x3:theta    -7.889e-05  1.868e-05  -4.222 2.99e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2082 on 403 degrees of freedom
## Multiple R-squared:  0.7253, Adjusted R-squared:  0.7185
## F-statistic: 106.4 on 10 and 403 DF,  p-value: < 2.2e-16
```

```
# try to drop some interactions
lm7=lm(log(y)~x1+x2+x3+x4+x2:x3+x2:r+x3:r+theta+x3:theta,data=dat)
summary(lm7)
```

```
##
## Call:
## lm(formula = log(y) ~ x1 + x2 + x3 + x4 + x2:x3 + x2:r + x3:r +
##     theta + x3:theta, data = dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.72025 -0.11073  0.00565  0.10720  0.96665
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -2.981e+02  7.397e+01  -4.030 6.66e-05 ***
## x1           1.500e-01  3.674e-02   4.082 5.38e-05 ***
## x2          -5.136e-03  1.206e-03  -4.260 2.55e-05 ***
## x3          -3.381e-04  3.564e-05  -9.485  < 2e-16 ***
## x4           2.872e-02  5.515e-03   5.207 3.07e-07 ***
## theta        8.669e-02  2.247e-02   3.858 0.000133 ***
## x2:x3        1.121e-05  1.610e-06   6.962 1.36e-11 ***
## x2:r        -8.964e-01  1.111e-01  -8.067 8.29e-15 ***
## x3:r         3.296e-03  6.782e-04   4.860 1.69e-06 ***
## x3:theta    -7.947e-05  1.875e-05  -4.239 2.78e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2089 on 404 degrees of freedom
## Multiple R-squared:  0.7227, Adjusted R-squared:  0.7166
## F-statistic:   117 on 9 and 404 DF,  p-value: < 2.2e-16
```

```
library(lmtest)
```

```
## Loading required package: zoo
```

```
##
## Attaching package: 'zoo'
```

```
## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
```

```
lrtest(lm7,lm6) # compare lm6 and lm7 --- lm6 is better
```

```
## Likelihood ratio test
##
## Model 1: log(y) ~ x1 + x2 + x3 + x4 + x2:x3 + x2:r + x3:r + theta + x3:theta
## Model 2: log(y) ~ x1 + x2 + x3 + x4 + r + theta + x2:x3 + x2:r + x3:r +
##     x3:theta
##   #Df LogLik Df Chisq Pr(>Chisq)
## 1  11 65.814
## 2  12 67.737  1 3.846    0.04986 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
vif(lm6) # multicollinearity exists
```

```
##        x1        x2        x3        x4         r     theta     x2:x3      x2:r
##  1.018331  2.521451 47.293269  2.507638 36.495953  3.367712 41.332762 39.128120
##      x3:r  x3:theta
## 21.963585  8.056467
```

```
lm8=lm(log(y)~x1+x2+log(x3)+x4+r,data=dat)
summary(lm8)
```

```
##
## Call:
## lm(formula = log(y) ~ x1 + x2 + log(x3) + x4 + r, data = dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.68927 -0.10562 -0.00026  0.10647  0.99431
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -3.387e+02  7.548e+01  -4.487 9.39e-06 ***
## x1           1.705e-01  3.750e-02   4.546 7.21e-06 ***
## x2          -6.888e-03  9.385e-04  -7.339 1.18e-12 ***
## log(x3)     -1.042e-01  1.781e-02  -5.849 1.01e-08 ***
## x4           1.304e-02  4.990e-03   2.613  0.00931 **
## r           -1.305e+01  1.245e+00 -10.484  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2122 on 408 degrees of freedom
## Multiple R-squared:  0.7111, Adjusted R-squared:  0.7076
## F-statistic: 200.9 on 5 and 408 DF,  p-value: < 2.2e-16
```

```
vif(lm8) # no multicollinearity
```

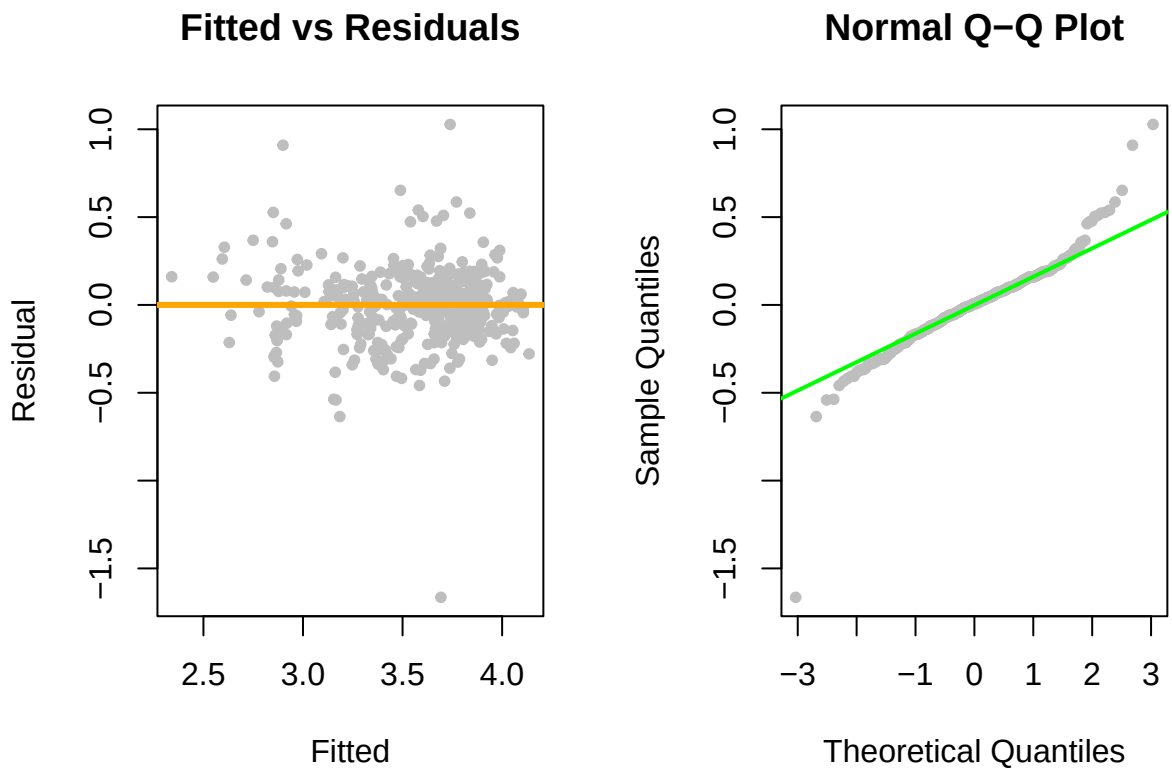
```
##       x1       x2  log(x3)       x4        r
## 1.025534 1.048311 3.646632 1.980900 3.065354
```

```
lm_2=lm8
```

Model Diagnostics

#Model(1)

```
library(zoo)
par(mfrow=c(1,2))
plot(fitted(lm_1),resid(lm_1),xlab='Fitted',ylab='Residual',
     main='Fitted vs Residuals',
     col = "grey",
     pch = 20)
abline(h=0, col = "orange", lwd = 3)
qqnorm(resid(lm_1),col="grey",pch=20)
qqline(resid(lm_1),col="green",lwd=2)
```




```
library(lmtest)
bptest(lm_1) # equal variance holds
```

```
##
##  studentized Breusch-Pagan test
##
## data:  lm_1
## BP = 8.7722, df = 6, p-value = 0.1868
```

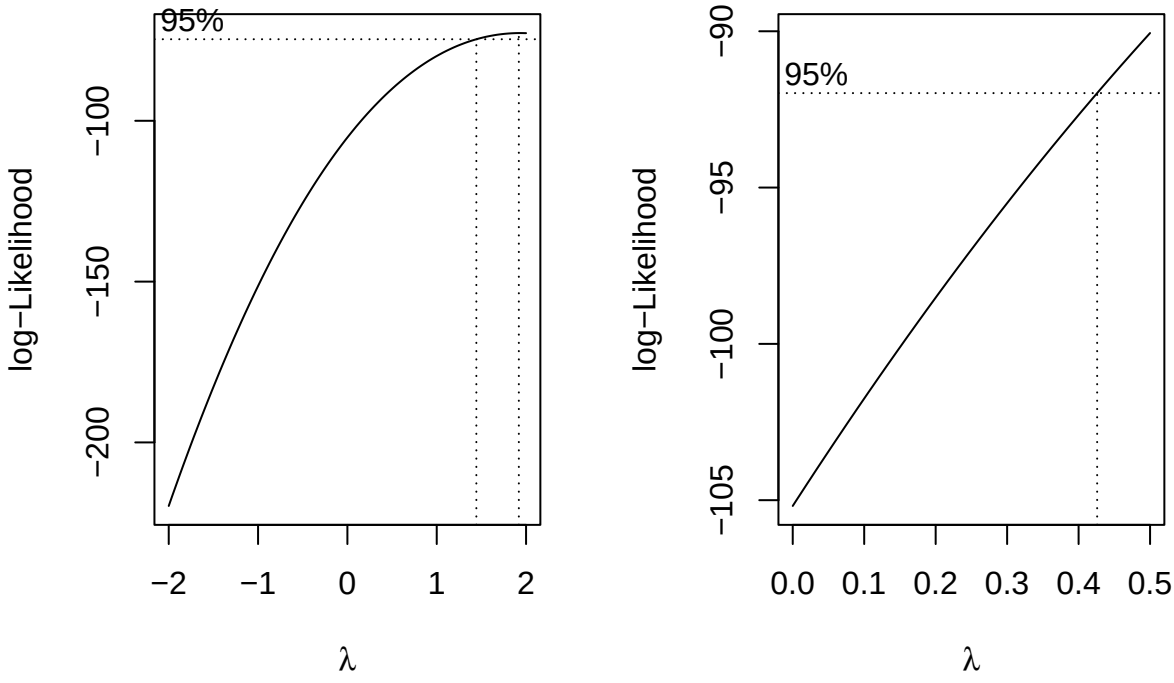
```
shapiro.test(resid(lm_1)) # normality is violated
```

```
##
##  Shapiro-Wilk normality test
##
## data:  resid(lm_1)
## W = 0.90754, p-value = 3.413e-15
```

```
dwtest(lm_1) # no autocorrelation
```

```
##
##  Durbin-Watson test
##
## data:  lm_1
## DW = 2.1482, p-value = 0.9351
## alternative hypothesis: true autocorrelation is greater than 0
```

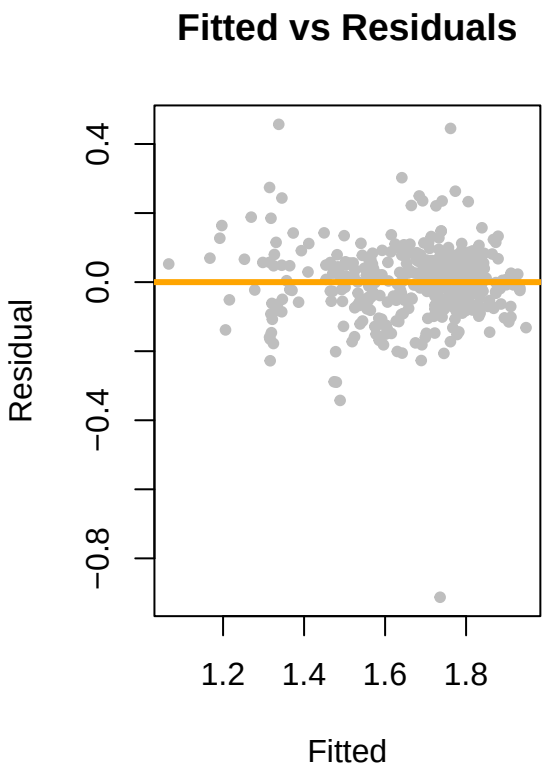
```
# try box-cox transformation
library(MASS)
boxcox(lm_1)
boxcox(lm_1, lambda = seq(0, 0.5, by = 0.05))
```

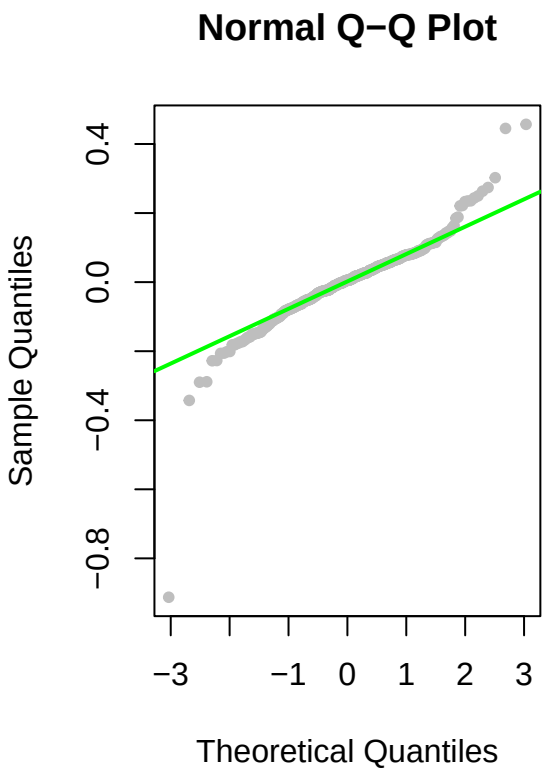


```r
# Let's transform Y using lambda = 0.42
lambda = 0.42
dat_lm_transf=lm(((log(y)^(lambda)-1)/(lambda))~x1+x2+x3+x4+x5+x3:x4,data=dat)
summary(dat_lm_transf)
```

```
##
## Call:
## lm(formula = ((log(y)^(lambda) - 1)/(lambda)) ~ x1 + x2 + x3 +
##     x4 + x5 + x3:x4, data = dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.91253 -0.05151  0.00515  0.05545  0.45686
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -2.500e+02  3.869e+01  -6.462 2.95e-10 ***
## x1           6.829e-02  1.851e-02   3.689 0.000256 ***
## x2          -3.391e-03  4.581e-04  -7.402 7.77e-13 ***
## x3          -6.013e-05  6.519e-06  -9.225  < 2e-16 ***
## x4           1.751e-02  2.408e-03   7.269 1.87e-12 ***
## x5           4.578e+00  5.393e-01   8.489 3.94e-16 ***
## x3:x4       -1.533e-05  2.961e-06  -5.176 3.57e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1053 on 407 degrees of freedom
## Multiple R-squared:  0.7082, Adjusted R-squared:  0.7039
## F-statistic: 164.6 on 6 and 407 DF,  p-value: < 2.2e-16
```

```r
par(mfrow=c(1,2))
plot(fitted(dat_lm_transf),resid(dat_lm_transf),
     xlab='Fitted',ylab='Residual',
     main='Fitted vs Residuals',
     col = "grey",
     pch = 20)
abline(h=0, col = "orange", lwd = 3)
qqnorm(resid(dat_lm_transf),col="grey",pch=20)
qqline(resid(dat_lm_transf),col="green",lwd=2)
```






```
bptest(dat_lm_transf)
```

```
##
##  studentized Breusch-Pagan test
##
## data:  dat_lm_transf
## BP = 8.47, df = 6, p-value = 0.2056
```

```
shapiro.test(resid(dat_lm_transf))
```

```
##
##  Shapiro-Wilk normality test
##
## data:  resid(dat_lm_transf)
## W = 0.88545, p-value < 2.2e-16
```

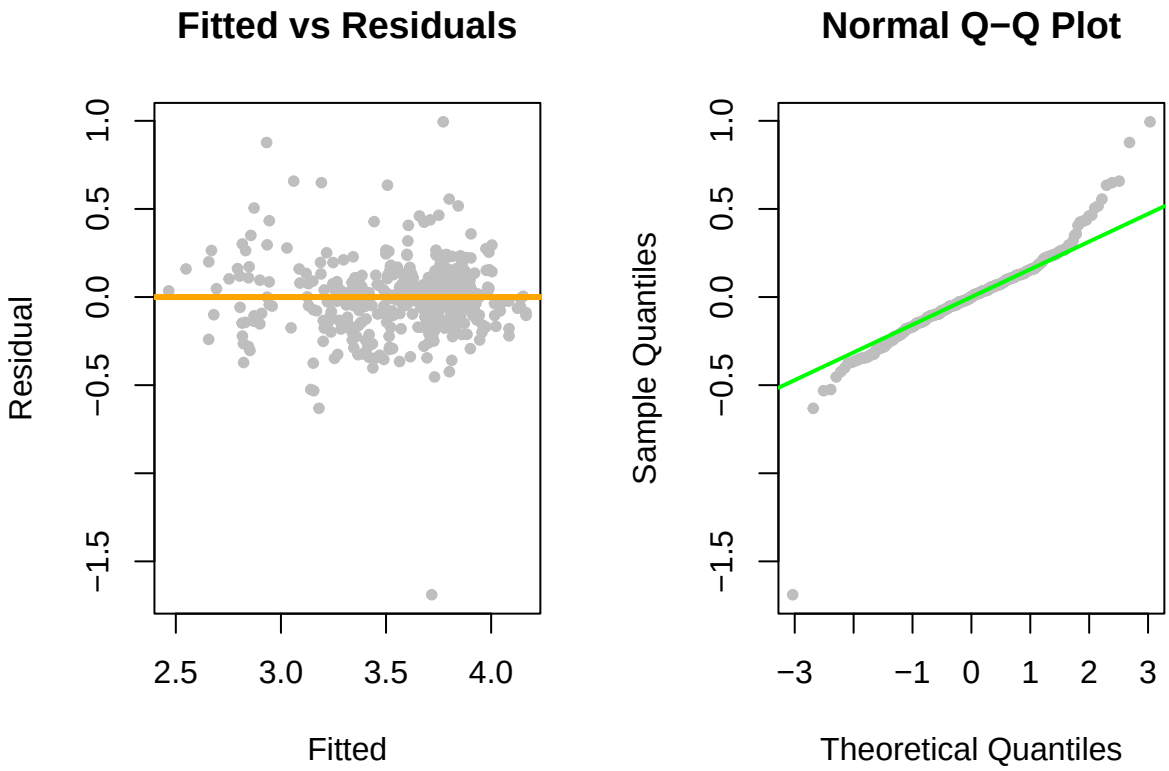
```
dwtest(dat_lm_transf)
```

```
##
##  Durbin-Watson test
##
## data:  dat_lm_transf
## DW = 2.135, p-value = 0.9164
## alternative hypothesis: true autocorrelation is greater than 0
```

```
# The Box-Cox transformation doesn't help to correct normality
```

#Model(2)

```
par(mfrow=c(1,2))
plot(fitted(lm_2),resid(lm_2),xlab='Fitted',ylab='Residual',
     main='Fitted vs Residuals',
     col = "grey",
     pch = 20)
abline(h=0, col = "orange", lwd = 3)
qqnorm(resid(lm_2),col="grey",pch=20)
qqline(resid(lm_2),col="green",lwd=2)
```




```
library(lmtest)
bptest(lm_2) # equal variance holds
```

```
##
##  studentized Breusch-Pagan test
##
## data:  lm_2
## BP = 6.4616, df = 5, p-value = 0.2639
```

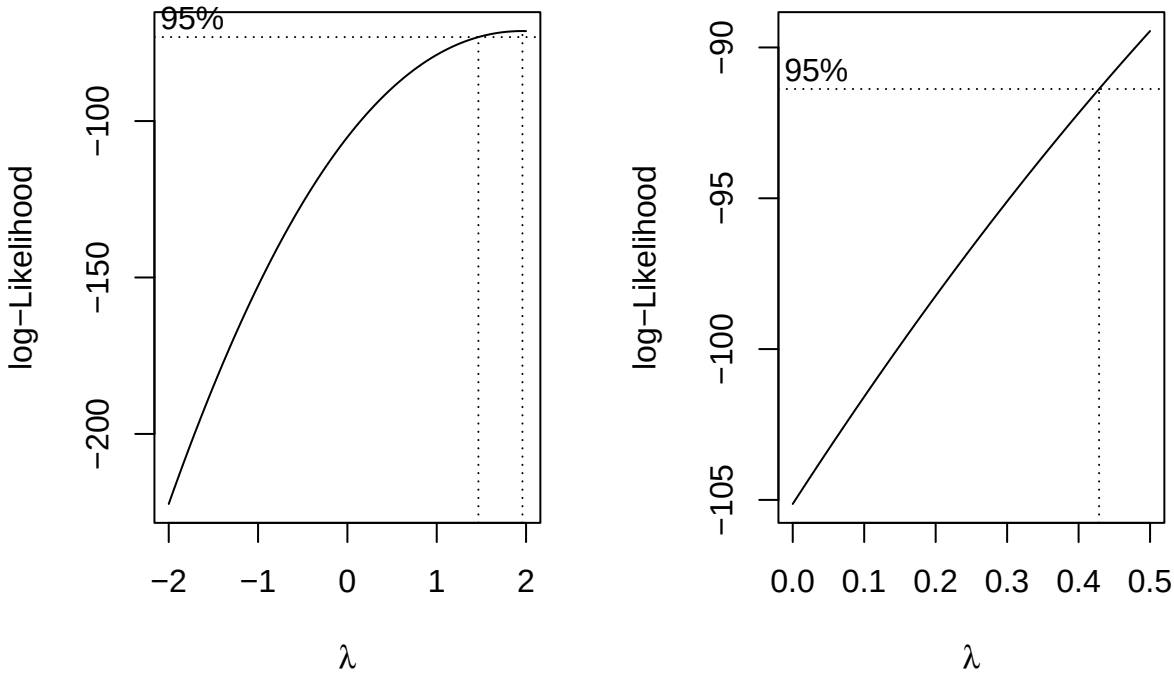
```
shapiro.test(resid(lm_2)) # normality is violated
```

```
##
##  Shapiro-Wilk normality test
##
## data:  resid(lm_2)
## W = 0.9037, p-value = 1.577e-15
```

```
dwtest(lm_2) # no autocorrelation
```

```
##
##  Durbin-Watson test
##
## data:  lm_2
## DW = 2.1192, p-value = 0.89
## alternative hypothesis: true autocorrelation is greater than 0
```

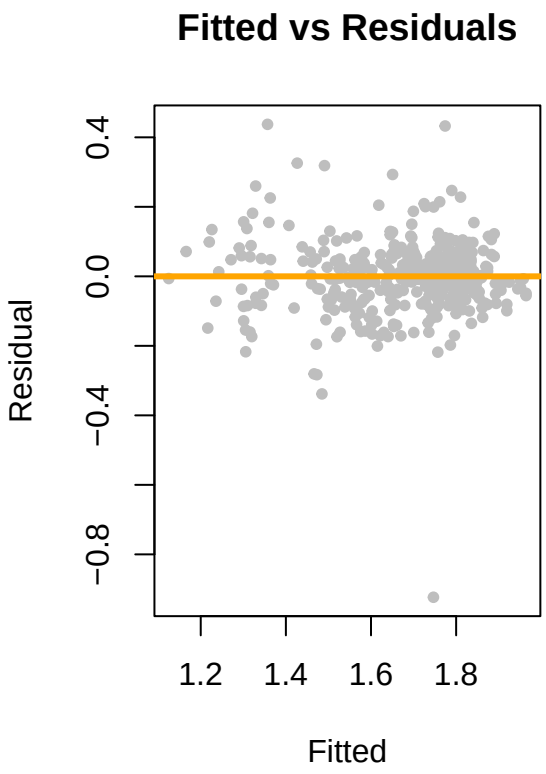
```
# try box-cox transformation
library(MASS)
boxcox(lm_2)
boxcox(lm_2, lambda = seq(0, 0.5, by = 0.05))
```

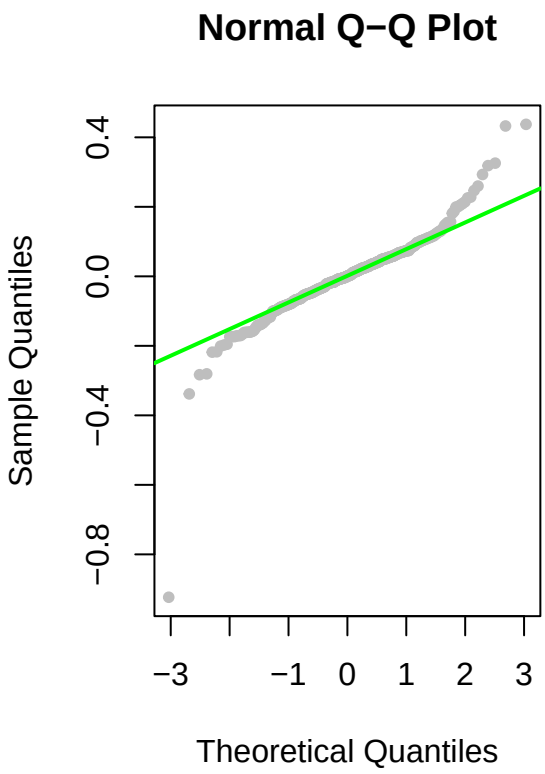


```r
# Let's transform Y using lambda = 0.42
lambda = 0.42
dat_lm_transf2=lm(((log(y)^(lambda)-1)/(lambda))~x1+x2+log(x3)+x4+r,data=dat)
summary(dat_lm_transf2)
```

```
##
## Call:
## lm(formula = ((log(y)^(lambda) - 1)/(lambda)) ~ x1 + x2 + log(x3) +
##     x4 + r, data = dat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.92337 -0.05012  0.00047  0.05334  0.43733
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.624e+02  3.737e+01  -4.345 1.76e-05 ***
## x1           8.172e-02  1.857e-02   4.401 1.38e-05 ***
## x2          -3.349e-03  4.647e-04  -7.206 2.81e-12 ***
## log(x3)     -4.717e-02  8.819e-03  -5.349 1.48e-07 ***
## x4           6.177e-03  2.471e-03   2.500   0.0128 *
## r           -6.784e+00  6.164e-01 -11.007  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1051 on 408 degrees of freedom
## Multiple R-squared:  0.7089, Adjusted R-squared:  0.7053
## F-statistic: 198.7 on 5 and 408 DF,  p-value: < 2.2e-16
```

```r
par(mfrow=c(1,2))
plot(fitted(dat_lm_transf2),resid(dat_lm_transf2),
     xlab='Fitted',ylab='Residual',
     main='Fitted vs Residuals',
     col = "grey",
     pch = 20)
abline(h=0, col = "orange", lwd = 3)
qqnorm(resid(dat_lm_transf2),col="grey",pch=20)
qqline(resid(dat_lm_transf2),col="green",lwd=2)
```






```
library(lmtest)
bptest(dat_lm_transf2)
```

```
##
##  studentized Breusch-Pagan test
##
## data:  dat_lm_transf2
## BP = 5.5304, df = 5, p-value = 0.3546
```

```
shapiro.test(resid(dat_lm_transf2))
```

```
##
##  Shapiro-Wilk normality test
##
## data:  resid(dat_lm_transf2)
## W = 0.88297, p-value < 2.2e-16
```

```
dwtest(dat_lm_transf2)
```

```
##
##  Durbin-Watson test
##
## data:  dat_lm_transf2
## DW = 2.1081, p-value = 0.8671
## alternative hypothesis: true autocorrelation is greater than 0
```

```
# The Box-Cox transformation doesn't help
```

Outliers,Influential Points

```
sum(abs(rstandard(lm_1)) > 2)
```

```
## [1] 18
```

```
sum(cooks.distance(lm_1) > 4 / length(cooks.distance(lm_1)))
```

```
## [1] 28
```

```
sum(abs(rstandard(lm_2)) > 2)
```

```
## [1] 20
```

```
sum(cooks.distance(lm_2) > 4 / length(cooks.distance(lm_2)))
```

```
## [1] 24
```

Evaluation

```
library(lmtest)
lrtest(lm_2,lm_1)
```

```
## Likelihood ratio test
##
## Model 1: log(y) ~ x1 + x2 + log(x3) + x4 + r
## Model 2: log(y) ~ x1 + x2 + x3 + x4 + x5 + x3:x4
##   #Df LogLik Df  Chisq Pr(>Chisq)
## 1   7 57.319
## 2   8 56.291  1 2.0545     0.1518
```

```
# p-value=0.1518 is large, we have no evidence to say lm_1 is better
AIC(lm_1,lm_2)
```

```
##      df        AIC
## lm_1  8  -96.58275
## lm_2  7 -100.63721
```

```
BIC(lm_1,lm_2)
```

```
##      df       BIC
## lm_1  8 -64.37583
## lm_2  7 -72.45615
```

```
summary(lm_1)$adj.r.square
```

```
## [1] 0.7054138
```

```
summary(lm_2)$adj.r.square
```

```
## [1] 0.7075905
```

```
coef(lm_2)
```

```
##   (Intercept)            x1            x2       log(x3)            x4
## -3.387203e+02  1.704993e-01 -6.887733e-03 -1.041800e-01  1.303711e-02
##             r
## -1.305119e+01
```

```
exp(coef(lm_2))
```

```
##   (Intercept)            x1            x2       log(x3)            x4
## 7.863642e-148  1.185897e+00  9.931359e-01  9.010631e-01  1.013122e+00
##             r
##  2.147540e-06
```

Furthur discussion

```
# try to use different thresholds for choosing origin of model(2)
center_x=mean(dat[which(dat$y>0),]$x6)
center_y=mean(dat[which(dat$y>0),]$x5)
dat$r=sqrt((dat$x6-center_x)^2+(dat$x5-center_y)^2)
dat$theta=atan((dat$x6-center_x)/(dat$x5-center_y))
lm_2_1=lm(log(y)~x1+x2+log(x3)+x4+r,data=dat)
lrtest(lm_2_1,lm_1)
```

```
## Likelihood ratio test
##
## Model 1: log(y) ~ x1 + x2 + log(x3) + x4 + r
## Model 2: log(y) ~ x1 + x2 + x3 + x4 + x5 + x3:x4
##   #Df LogLik Df  Chisq Pr(>Chisq)
## 1   7 21.995
## 2   8 56.291  1 68.592  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# lm_1 is better, p-value is very small

center_x=mean(dat[which(dat$y>50),]$x6)
center_y=mean(dat[which(dat$y>50),]$x5)
dat$r=sqrt((dat$x6-center_x)^2+(dat$x5-center_y)^2)
dat$theta=atan((dat$x6-center_x)/(dat$x5-center_y))
lm_2_2=lm(log(y)~x1+x2+log(x3)+x4+r,data=dat)
lrtest(lm_2_2,lm_1)
```

```
## Likelihood ratio test
##
## Model 1: log(y) ~ x1 + x2 + log(x3) + x4 + r
## Model 2: log(y) ~ x1 + x2 + x3 + x4 + x5 + x3:x4
##   #Df LogLik Df  Chisq Pr(>Chisq)
## 1   7 52.093
## 2   8 56.291  1 8.3968   0.003759 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# lm_1 is better, but p-value is larger

lrtest(lm_2,lm_1)
```

```
## Likelihood ratio test
##
## Model 1: log(y) ~ x1 + x2 + log(x3) + x4 + r
## Model 2: log(y) ~ x1 + x2 + x3 + x4 + x5 + x3:x4
##   #Df LogLik Df  Chisq Pr(>Chisq)
## 1   7 57.319
## 2   8 56.291  1 2.0545     0.1518
```

```
# p-value is large, lm_2 is better

center_x=mean(dat[which(dat$y>80),]$x6)
center_y=mean(dat[which(dat$y>80),]$x5)
dat$r=sqrt((dat$x6-center_x)^2+(dat$x5-center_y)^2)
dat$theta=atan((dat$x6-center_x)/(dat$x5-center_y))
lm_2_3=lm(log(y)~x1+x2+log(x3)+x4+r,data=dat)
lrtest(lm_2_3,lm_1)
```

```
## Likelihood ratio test
##
## Model 1: log(y) ~ x1 + x2 + log(x3) + x4 + r
## Model 2: log(y) ~ x1 + x2 + x3 + x4 + x5 + x3:x4
##   #Df LogLik Df  Chisq Pr(>Chisq)
## 1   7 39.448
## 2   8 56.291  1 33.687  6.472e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# lm_1 is better, p-value is very small
```