Graphics and Computational Programming Assignment 1

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# Part 1

## Brute-Force Ray-Triangle Intersection

### Theory

The brute-force approach for testing a ray-triangle intersection is to test each ray against all other triangles or vice-versa, so called due to the fact that there is no other preliminary testing with computationally cheaper methods to see whether there is any reasonable chance of collision before the more-expensive ray-triangle intersection test is done.

The actual ray-triangle intersection test is done through first testing whether the ray will collide with the supporting plane of the triangle, then gathering the intersection point with that plane and testing whether it is within the bound of the triangle. This is outlined in more detail through a series of equations below.

(1)

A point on a ray is defined using the as using the parametric equation (1), where defines the ray’s origin, the direction and a parameter which is a scalar value defining how far along the point lies.

(2)

(3)

The supporting plane is defined by (2), but can be expressed as a product of its normal by (3). If any point on this ray, as defined by (1) is to exist on the plane then it must satisfy this equation. Therefore if is substituted by, then the equation can be rearranged into the form below.

(4)

Logically, this means that if or then the ray does not intersect with that plane due to either being parallel in the former case or the plane existing in the inverse direction of the ray, as shown in Figure 1.

Once the intersection with the plane has been validated the intersection point can be found by using (1) to give the intersection point. A test must then be done for whether this point is within the bounds of the triangle, defined by three vertices and. Using the defining vertex points of the triangle, the three edges of the triangle can be defined as (5).

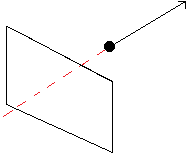


Figure 1 - Simple diagram of a plane (shown with a rectangular sub-section) existing behind a ray with an arrowed direction. As shown, the section of the ray that would exist for a line (shown in red) would collide with plane, but does not for the ray.

(5)

By calculating the triple product of , where and are the vertex points that make up each edge of the triangle, a scalar value is given that will be positive for if the contact point is within the triangle bounds of that edge. Therefore, if this is tested positively for each bounding edge of the triangle then the point is within the triangle. This is shown in (6).

(6)

### Implementation

The implementation of

At a higher level than the actual test for a ray-triangle intersection, ordering of operations can be done to reduce the number of overall operations. The ordering of these calculations have been done in a specific way as shown in the pseudocode below.

|  |
| --- |
| Brute-Force Pseudocode |
| for each triangle  n := CalculateNormal()    for n from 0 to 255    for m from 0 to 255  #Defining origin and direction of ray  p := (0,m,n)  d := (1,0,0)    #Testing collision  if collision(p,d,triangle)  Draw(m,n)  end if    end for    end for    end for |

In this pseudocode it can be seen that the triangles are iterating over the rays, this is so that the normal for each triangle is being calculated only one as this is a computationally expensive operation and should be done the minimal amount of times.

## Bounding Volumes

### General Theory

A bounding volume (BV) is a representation of an object in world-space using some primitive shape, such as a cube or a sphere. This allows for more complicated geometry to be represented in a simplistic, if not completely accurate, form. A popular implementation of BVs is the Axis-Aligned Bounding Box (AABB), this encompasses an object in a box that is relative to the X, Y and Z axis as opposed to being oriented to the object (although this implementation exists as the Oriented Bounding Box). The purpose of this in the context of this assignment is that it allows for each ray to be tested against the bounding volume before being tested against triangle, this is a computationally cheaper test and has been explained further below.

As can be seen in Figure 2, the arbitrary object in this example is taking a relatively small portion of the screen. This means if each ray, which in the context of this example is equivalent to a pixel on the screen, was to test against each triangle on the object then there would be a lot of expensive calculation being done needlessly as there it is clear there is no way in which those rays will be colliding with the object. Therefore, a check must be done to ensure that the ray’s origin is within the BV in the relevant axis, for this instance the only relevant axis are Y and Z as the ray’s direction is always constant in the X axis as 1.

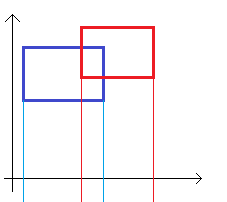


Figure 3 - Diagram showing the projection of the minimum and maximum values of two AABBs onto an arbitrary axis for collision testing.

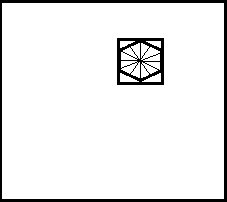


Figure 2 - Simple diagram of an arbitrary triangulated object, surrounded with a tight-fitting AABB, displayed in a on a screen.

In order to do this, the minimum and maximum values of the BV must be projected onto the axis, as is shown in Figure 3 for an arbitrary axis. In this figure the minimum and maximum values have been displayed against the axis using lines. When AABBs are used for collision detection between objects, each object has a BV in 3D and is said to pass the collision test if the minimum value of one is less than the maximum value of another (or vice-versa) on all axis, however in this context the ray does not need to have a bounding volume per-say as it does not exist as a physical object. Instead the Y and Z bounding values for the object’s AABB are projected and used to test against the origin of the ray. The equations for this simple test are shown below.

(7)

Where is the origin of the ray, and and define the minimum and maximum bounds of the AABB of an object.

(8)

As can be seen in (5), the ray’s origin is defined in the Y and Z axis in this example and as it’s direction is constant in the X axis, the only axis of the object that need to be tested of the object are the Y and Z axis as is shown in (6).

If this test has not passed then this ray will not test against any triangles at all whereas the ray would have tested against every single triangle using brute-force, wasting processing time. If the test has passed, then it can be seen that there is a chance for this ray to collide with a triangle within the object (although it is important to note that this is not guaranteed, due to the inaccuracy in AABB and it not exactly representing the object) therefore a brute-force test will continue for this ray.

### Per-Triangle

#### Differences

While an AABB can be calculated to encompass the entire object to initially test for a possible collision and then carrying on with a brute-force pass if this passes, it is also possible to create an AABB for each triangle in order to reduce the overhead in this brute-force testing pass.

#### Memory Overhead vs. Performance

### Bounding Volume Hierarchy

* What it is, the benefits and why I’ve not done it (laziness in truth, but easy marks in the report)

## Algorithm Analysis

### Brute-Force

In order to evaluate the brute-force algorithm accurately, a high number of performance benchmarks have been taken on a single machine using a variety of models containing different numbers of triangles. This ranges from a simple, publicly available, model of a Gourd that uses 648 triangles to a triangulated 3D scan of man that uses 79,996 triangles. 256 full brute-force tests were run for each model, apart from the highest-poly mesh due to the length of time each test was taking. Each test was timed using ‘steady\_clock’ from C++’s standard library, this implementation of a timer contains some steadying to account for process scheduling done by the OS, that would be otherwise unaccounted for if ‘high\_resolution\_timer’ was used, as the only time that should be measured when comparing them is the time that is actually spent computing the process due to potential differences in process scheduling between tests. Despite this there was still a relatively large variance in the time taken between tests which increased as the time taken for the algorithm finished, ranging from a standard error of 76.2ms for the Gourd and 1416.0ms for the 3D scan. Graphs of all of this data is available in appendices 1.a-1.f; the trend-lines have been plotted with the gradients shown, the gradients of which are a good indicator of the variance between tests.

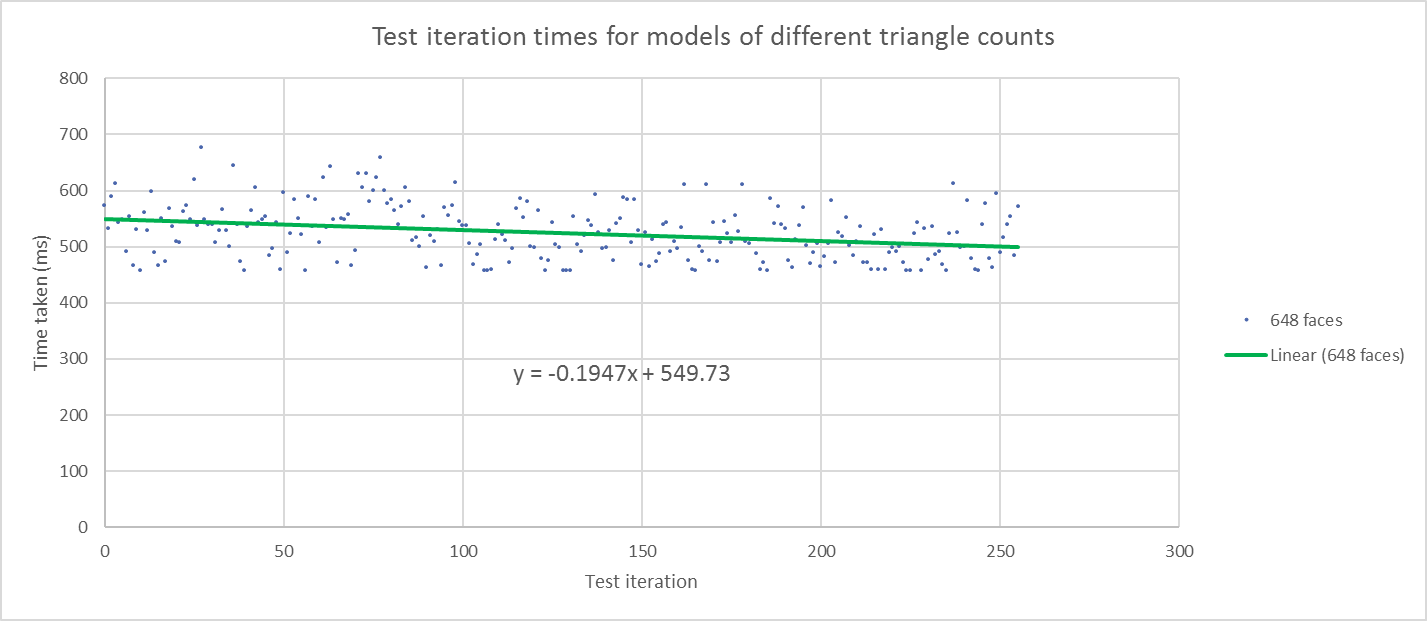
### Per-Object AABB

### Per-Triangle AABB

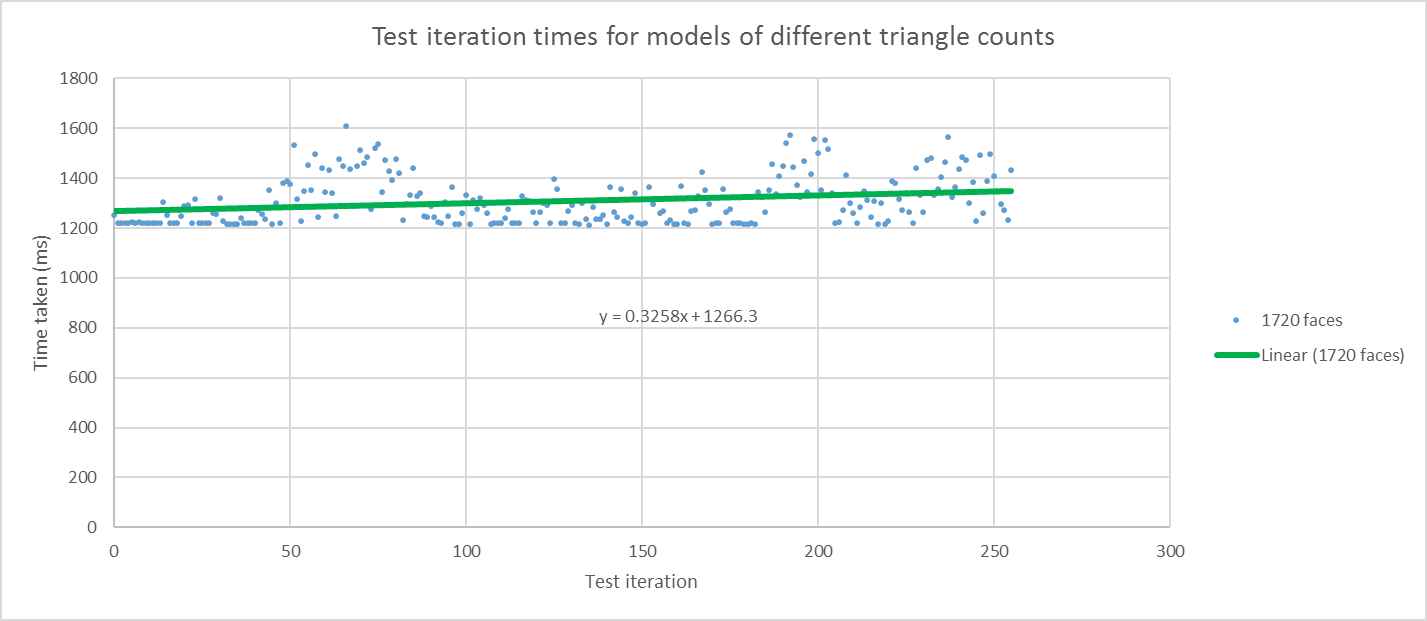
# Part 2

# Appendices

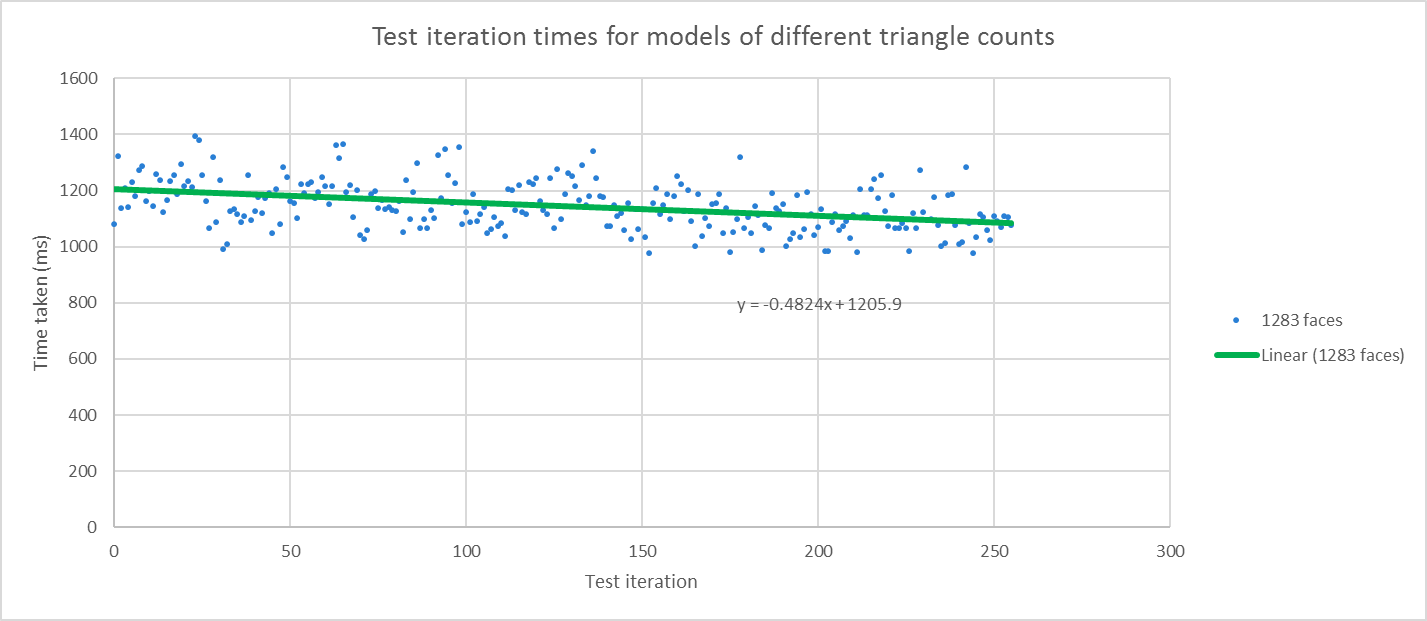
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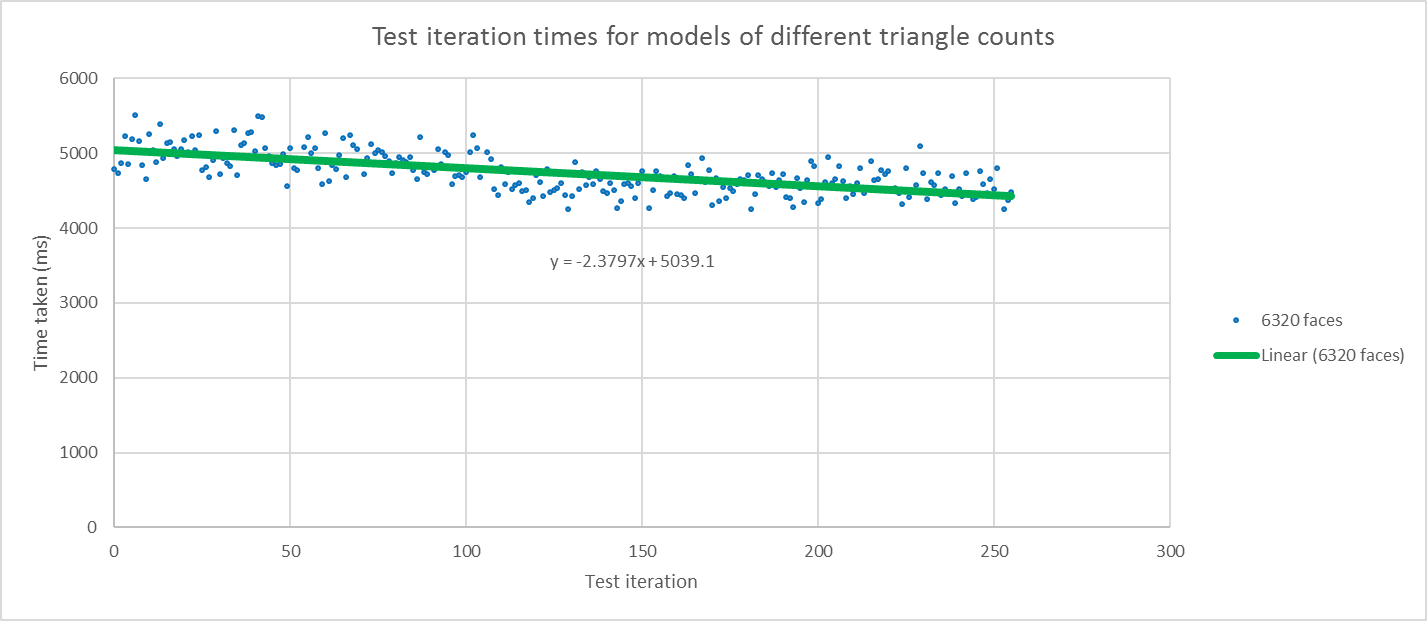
Appendix 1.0.a



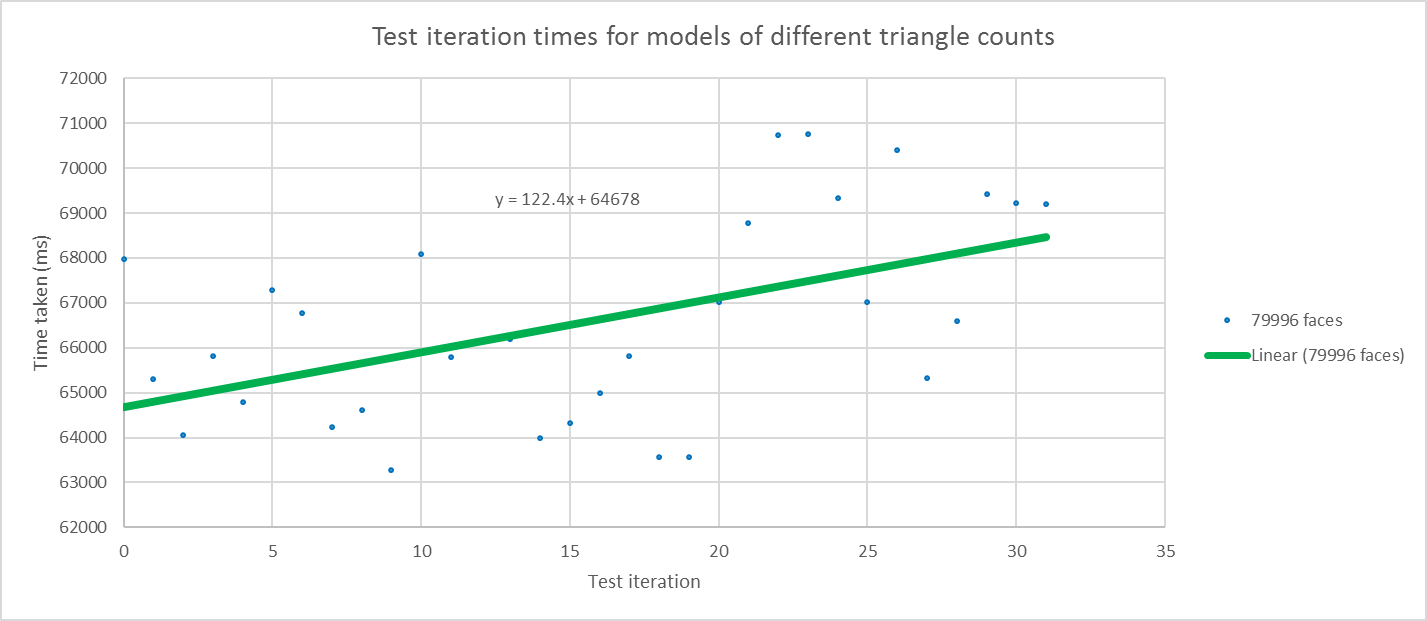
Appendix 1.0.b



Appendix 1.0.c



Appendix 1.0.d



Appendix 1.0.e

Appendix 1.f