University of Patras

Department of Engineering. Computer & Information Technology

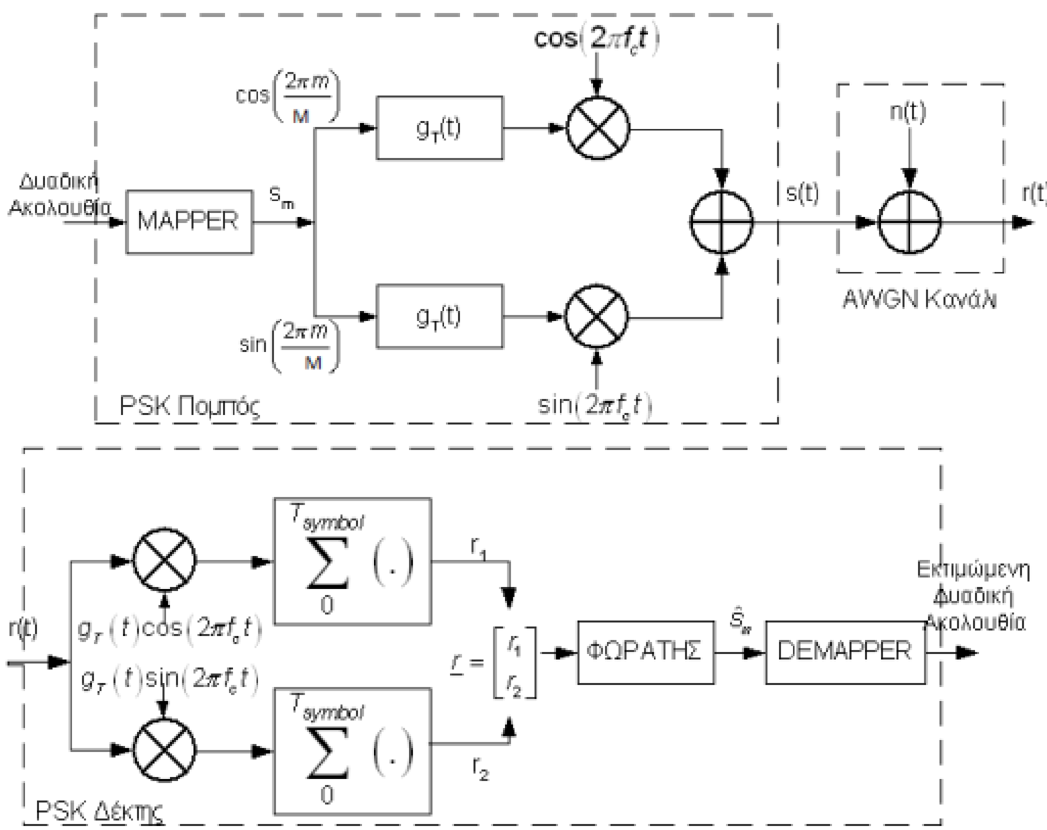
DIGITAL COMMUNICATIONS

Laboratory Exercises 2023-2024 : 2nd set (optional)

Study of Coherent Bandpass M-PSK Systems

In this exercise, you are asked to study the performance of M-PSK modulation for M = 8, 16. This comparison will be based on bit error rate (BER) measurements, which will be carried out in a coherent bandpass system with a rectangular base pulse.

A. Coherent M-PSK



As shown in the figure, the transmitter of the M-PSK system receives a binary sequence as input, converts it into symbols, multiplies it by the rectangular pulse, and then the signal is transferred to the transmission band via the modulator. AWGN noise is added to the transmitted signal, and it reaches the system's receiver. There, the signal is demodulated, resulting in a two-dimensional vector, which is fed into the detector where a decision is made regarding which symbol was sent. Finally, the demapper performs the reverse mapping from symbols to bits. These systems are described in the following sections.

Binary Input Sequence

The input to both systems is a sequence of bits, where the values 0 and 1 appear with equal probability. Such a sequence can be generated using one of the functions randsrc, rand, or randn appropriately. The number of bits to be sent should be on the order of O(100K).

Bit-to-Symbol Mapping

The mapper essentially serves as a converter from bits to symbols. Given that the system we will implement is M-ary, each symbol corresponds to a specific group of log2(M) bits. Therefore, the mapper should, for every log2(M) bits, output one of the M symbols of the modulation scheme. Similarly, the demapper receives as input the symbol detected by the decision device of the receiver and extracts the corresponding log2(M) bits.

In the case of PSK, PAM, and QAM modulations, an important aspect of this mapping is Gray coding. According to this scheme, if two symbols are adjacent in the two-dimensional signal space, they are assigned bit patterns that differ by only one bit.

Rectangular Pulse

The M-PSK system you are asked to simulate uses a rectangular pulse for symbol transmission.

The rectangular pulse is defined as:

$$g_T\ (t) = \begin{cases} \sqrt{\dfrac{2E_s}{T_{symbol}}} = \sqrt{\dfrac{2}{T_{symbol}}}, & 0 \le t \le T_{symbol} \\ 0\ , \alpha\lambda\lambda o\acute{\upsilon} & \end{cases}$$

where Es is the energy per symbol, which we normalize as Es = 1, and $Tsymbol$ s the symbol period.

Modulation M-PSK

Each symbol in M-PSK modulation is defined by two components:

$$s_m = \begin{bmatrix} \sqrt{E_s}\cos\left(\frac{2\pi m}{M}\right) \\ \sqrt{E_s}\sin\left(\frac{2\pi m}{M}\right) \end{bmatrix}, \qquad m = 0,1,\ldots,M-1$$

where in our case Es = 1 and for this reason, the energy is not indicated in the figure. Each component, after being multiplied by the rectangular pulse, is modulated by the carrier frequency, resulting in the bandpass signal:

$$s_m(t) = \cos\left(\frac{2\pi m}{M}\right) g_T(t)\cos(2\pi f_c t) + \sin\left(\frac{2\pi m}{M}\right) g_T(t)\sin(2\pi f_c t), 0 \leq t \leq T_{symbol}$$

Time values of the simulation

The systems we want to simulate transmit symbols at a rate of $Rsymbol$ = 250 $Kbps$, so the symbol period is $Tsymbol$ = 4 μsec. The transmission band uses a carrier frequency fc = 2,5 MHz, so the carrier period is Tc = 0,4 μsec. For the simulation, to achieve a satisfactory representation of the bandpass signals, sampling is performed at twice the Nyquist rate, meaning we take 4 samples per carrier period, resulting in a sampling period of $Tsample$ = Tc 4 = 0,1 μsec. Since the two systems are simulated at the sampling rate, each value of the vectors corresponds to a time $Tsample$ = 0.1 μsec, which we can normalize to $Tsample$ = 1. Thus, we obtain the following normalized times: $Tsample$ = 1, Tc = 4, $Tsymbol$ = 40

which means that in each carrier period, we keep 4 samples, and each symbol period contains 10 carrier cycles or 40 samples.

AWGN Channel

The bandpass signal transmitted by the system's transmitter passes through an ideal additive noise channel. The noise is white and follows a Gaussian distribution with zero mean and variance σ2=N0/2\sigma^2 = N_0 / 2σ2=N0/2. The noise can be generated using the randn function as follows:

$$noise = \sqrt{\sigma^2} * randn\left(\left(\frac{L_b}{\log_2(M)}\right) * 40{,}1\right)$$

The noise variance is determined each time by the SNR/bit that we want to achieve at the system's receiver. It is reminded that, due to the normalizations we have applied, the energy per symbol in both systems is Es = 1, so the energy per bit is Eb = 1/ log2(M) . Therefore, if we want to have

$$SNR = 10 * \log_{10}\left(\frac{E_b}{N_0}\right) = 10, \text{θα πρέπει } N_0 = 0{,}05 \text{ και } \sigma^2 = 0{,}025.$$

M-PSK Demodulator

The demodulator of the M-PSK system correlates, meaning it multiplies and integrates (sums) the received signal with the carrier and the rectangular pulse. The correlation is performed within the time frame of a symbol period. In the simulation, we assume that the M-PSK is coherent. This means that the receiver knows the carrier phase and the timing of each symbol, i.e., it is fully synchronized with the transmitter.

The demodulator correlates the received signal with the two components of the carrier, resulting in two values, forming a vector rrr that corresponds to the position of the received signal on the M-PSK constellation plane.

M-PSK Detector

The detector receives the vector rrr as input and decides which symbol (as defined in the vector form above) is closest to it. The vector sms_msm that has the smallest distance from rrr corresponds to the symbol that was transmitted.

B. BER Measurements

To measure the BER (Bit Error Rate), that is, the probability of bit error occurrence, you need to compare the received bit value with the one you transmitted. To obtain reliable BER measurements, they should come from a sufficiently large number of data points. A rough rule of thumb is that to measure a BER value on the order of 10−210^{-2}10−2, you need 10,000 bits of data, for a BER on the order of 10−310^{-3}10−3, you need 100,000 bits of data, and so on. This rule doesn't mean that you need to simulate more than 100,000 bits!

BER curves are typically plotted on a logarithmic scale on the y-axis, representing the error probability. The theoretical error probability for 4-PSK is given by the following formula:

$$P_4 \approx 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\left[1-\frac{1}{2}Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\right].$$

The theoretical error probability for M-PSK, where M>4M > 4M>4, is given by the following formula:

$$P_M \approx 2Q\left(\sqrt{\frac{2\log_2(M)E_b}{N_0}}\sin\left(\frac{\pi}{M}\right)\right).$$

C. Power Spectrum

In this exercise, we are also interested in observing the bandwidth occupied by the signals from both systems. To determine the bandwidth, we need to compute the power spectrum of the corresponding transmitted signal (without added noise).

To calculate the power spectrum of a signal, we must take successive segments of the signal (let's say 2048 samples), compute their Fourier transform, retain the square of its magnitude, and finally, average these values over all the 2048-sample segments.

Questions - Requirements

1. Based on the above guidelines, implement the M-PSK system and describe its key points.
2. Draw the constellation of symbols for the M-PSK modulations in the exercise (M=8,16). Repeat this for the case of Gray coding.
3. [Theoretical] Briefly explain why it makes sense to use Gray coding in an M-PSK system.
4. For each of the two systems (M=8,16 with Gray coding), measure the error probability and plot the BER curves for SNR values in the range [−4:2:20] dB. The BER curves should be plotted on the same graph. On the same graph, also plot the theoretical BER for each M-PSK system you implemented. Discuss the results. Which system performs better in terms of error probability for the same SNR? How much more SNR is required for the worse system to achieve the same error probability as the better one?
5. For each of the two systems, calculate the power spectrum of the transmitted signal. Plot the two spectra on the same graph in logarithmic scale. What do you observe regarding the bandwidth required by each system? Comment on the bandwidth and the amplitude of the main and secondary lobes for each modulation scheme.

Notes

- The report is submitted only electronically via e-class (under the "Assignments" section). At the end of the report, include the code you implemented. The report file must be in PDF format and named after your student ID. For example, if the exercise is done by the student with ID 5678, the file should be named 5678.pdf.
- To upload an assignment, you must first be enrolled in the course. If you are not enrolled in the course, the system will not allow you to upload the assignment. Enrollment can be done through the options available on e-class.
- Make sure that your assignment has been submitted correctly on e-class. Late submissions will not be accepted with the excuse that you submitted it, but for some reason, the assignment is not on e-class.
- The assignment is individual and OPTIONAL.
- The deadline for submission is February 18, 2024.
- Any questions regarding the assignment will be addressed through the course forum on e-class under "Discussions" and during the office hours announced online.