07.03

1. $$\int \frac{dx}{1+3\cos^2 x} = \int \frac{\sec^2(x)}{tg^2(x)+4}\,dx = \left| u = tg(x) \right| =$$

$$= \int \frac{1}{u^2+4}\,du = \left| v = u/2 \right| = \int \frac{2}{4v^2+4}\,dv =$$

$$= \frac{arctg(v)}{2} = \frac{arctg\left(\frac{tg(x)}{2}\right)}{2} + C$$

2. $$\int \frac{dx}{\sin^2(x) - 5\sin x \cos x} = \int \frac{\sec^2(x)}{tg^2(x) - 5tg(x)}\,dx =$$

$$= \left| u = tg(x) \right| = \int \frac{du}{u^2 - 5u} = \int \frac{du}{u^2(1-\frac{5}{u})} = \left| v = 1 - \frac{5}{u} \right| =$$

$$= \frac{1}{5}\int \frac{dv}{v} = \frac{\ln\left(1 - \frac{5}{tg(x)}\right)}{5} + C$$

3. $$\int \frac{dx}{8 - 4\sin(x) + 7\cos x} = -\int \frac{dx}{4\sin(x) - 7\cos(x) - 8} =$$

$$-\int \frac{dx}{\frac{8tg(\frac{x}{2})}{tg(\frac{x}{2})+1} - \frac{7(1-tg^2(\frac{x}{2}))}{tg^2(\frac{x}{2})+1} - 8} = \left| u = tg(\tfrac{x}{2}) \right| = 2\int \frac{du}{u^2 - 8u + 15} =$$

$$= 2\int \frac{du}{(u-5)(u-3)} = 2\int \left(\frac{1}{2(u-5)} - \frac{1}{2(u-3)} \right) du =$$

$$\frac{A}{u-5} + \frac{B}{u-3} = \frac{1}{(u-5)(u-3)}$$

$$Au - 3A + Bu - 5B = 1$$

$$(A+B)u + (-3A - 5B) = 1$$

$$\begin{cases} A + B = 0 \\ -3A - 5B = 1 \end{cases} \quad \begin{cases} A = -B = \frac{1}{2} \\ B = -\frac{1}{2} \end{cases}$$

$$= \int \frac{du}{u-5} - \int \frac{du}{u-3} =$$

$$= \ln(u-5) - \ln(u-3) =$$

$$= \ln(|tg(\tfrac{x}{2}) - 5|) - \ln(|tg(\tfrac{x}{2}) - 3|)$$

$$+ C$$

4. $\int \sqrt{4-x^2}\, dx = \left| u = \arcsin\left(\frac{x}{2}\right) \right| =$

$= \int 2\cos(u)\sqrt{4-4\sin^2(u)}\, du = 4\int \cos^2(u)\, du =$

$= 2\cos(u)\sin(u) + \frac{u}{2} = x\sqrt{1-\frac{x^2}{4}} + 2\arcsin\left(\frac{x}{2}\right) + \mathbb{C}$

5. $\int \frac{x^2-4}{x}\, dx = \int x\, dx - \int \frac{4dx}{x} =$

$= \frac{x^2}{2} - 4\ln|x| + \mathbb{C}$