Question 19 **(18 marks)**

de Broglie and Bohr

The 1913 Bohr model of the hydrogen atom was replaced by Schrödinger's wave mechanical model in 1926. However, Bohr's model is still taught today because of its conceptual and mathematical simplicity, and because it introduced a number of key quantum mechanical ideas such as the quantum number.

In 1924, de Broglie postulated wave-particle duality for the electron, which removed some of the arbitrary nature of Bohr's model. For example, an electron possessing wave properties is subject to constructive and destructive interference. If we assumed circular electron orbits around the nucleus, this meant that only certain whole numbers of a particular wavelength could 'fit' into the circumference of the orbit.

$$n\lambda = 2\pi r \qquad \text{Equation 1}$$

This introduces the quantum number n, which can have values of 1,2,3, ...

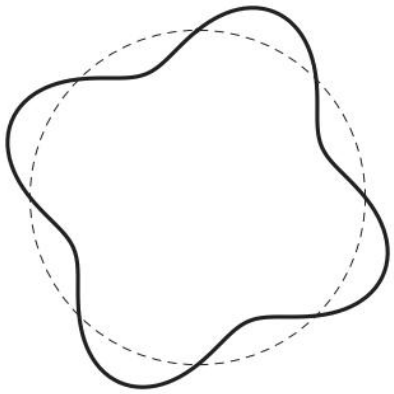

Figure 3: The $n = 4$ state of an electron

If we combine Equation 1 with the de Broglie equation, $\lambda = \frac{h}{p} = \frac{h}{m_e v}$, we get

$$m_e v = \frac{nh}{2\pi r} \qquad \text{Equation 2}$$

where m_e is the mass of an electron.

This equation holds for non-relativistic speeds. Relativistic momentum must be considered with speeds above 0.50 c.

If we then combine Equation 2 with the equation for kinetic energy, $KE = \frac{m_e v^2}{2}$, we get

$$KE = \frac{n^2 h^2}{8 m_e \pi^2 r^2} \qquad \text{Equation 3}$$

This shows that the kinetic energy of electrons bound to nuclei is quantised and the total energy of these electrons is quantised. This explains why electrons can be found in different orbits but not in-between. Therefore, when electrons de-excite and jump down from one orbit to another, photons with specific energies are emitted. This is supported by observing emission spectra of elements in which only certain wavelengths appear.

(a) Explain why de Broglie's theory that electrons can be treated as waves led to the introduction of quantum numbers. (3 marks)

(b) Derive Equation 3 from Equation 2 and the equation for kinetic energy. (5 marks)

(c) With the use of a diagram, describe the experimental evidence stated in the text on page 32 that supports the quantisation of electron energy. (5 marks)

(d) Calculate the wavelength of an electron travelling at 0.750 c. (5 marks)

Answer: _____ m