

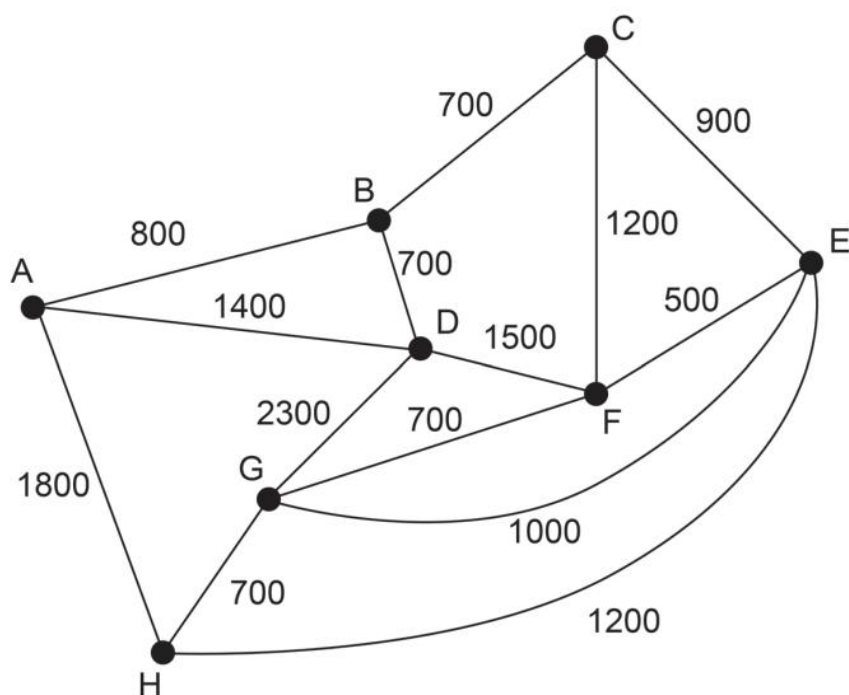
#### Question 4

(13 marks)

A council in the South West region of Western Australia has erected a number of water stations for hikers travelling through the forest on a walking trail. Over the years, the council has increased the number of water stations, with direct connecting paths to some other water stations, where the ground makes this feasible.

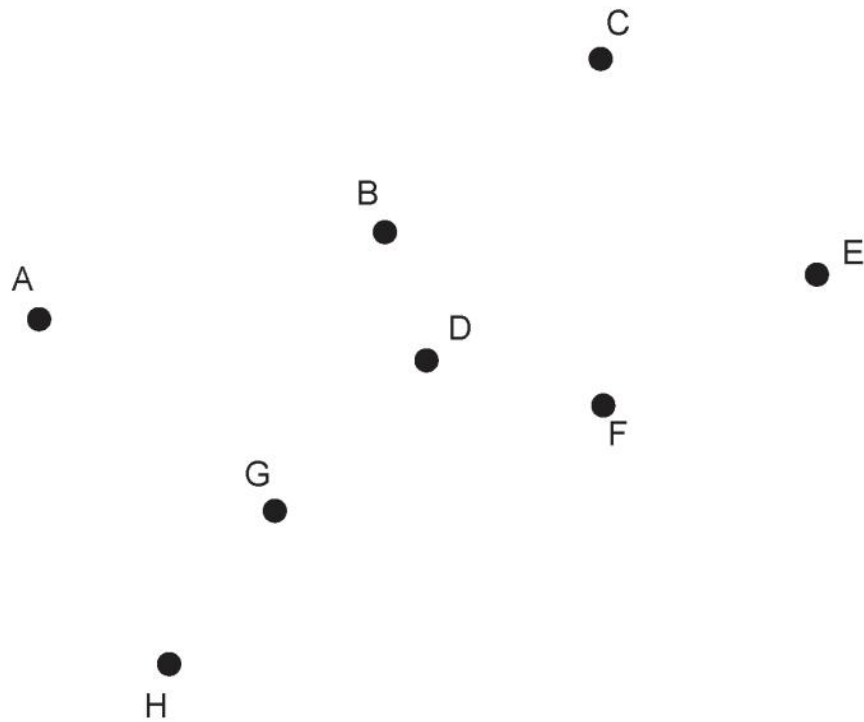
The table and graph below show the direct distances (in metres), between water stations of the paths that have been constructed.

	A	B	C	D	E	F	G	H
A		800		1400				1800
B	800		700	700				
C		700			900	1200		
D	1400	700				1500	2300	
E			900			500	1000	1200
F			1200	1500	500		700	
G				2300	1000	700		700
H	1800				1200		700	



- (a) The council has decided to upgrade some of the paths between water stations. It aims to create the shortest distance connecting all water stations, using the upgraded path(s) for travel. Using Prim's algorithm, determine which paths should be upgraded to ensure the distance is minimised. (3 marks)
- (b) Upgrading the paths is costed at \$30 per metre. If the budget is \$180 000, will this be sufficient to upgrade the appropriate paths? Justify your answer. (2 marks)

- (c) (i) Draw the network identified by Prim's algorithm. (1 mark)



- (ii) Show this network is bipartite by drawing a diagram. (2 marks)

- (d) Each day, a park ranger must check every path to ensure there are no obstacles interfering with a hiker's track. Can this be done using a semi-Eulerian or Eulerian trail? Justify your answer. (2 marks)

- (e) A park ranger is stopped at watering station G and receives a message to travel to watering station B to assist an injured hiker. Calculate the shortest distance the ranger must travel, and the pathway, to get from G to B. Working must be shown. (3 marks)