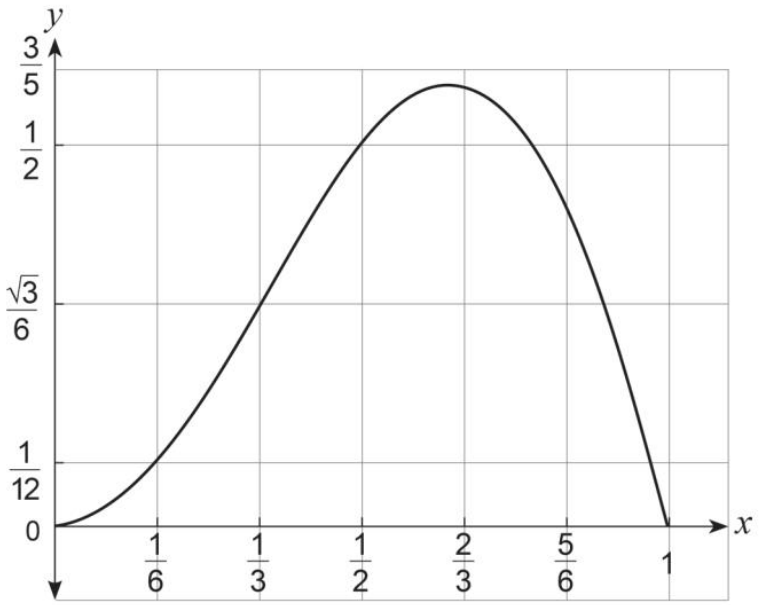
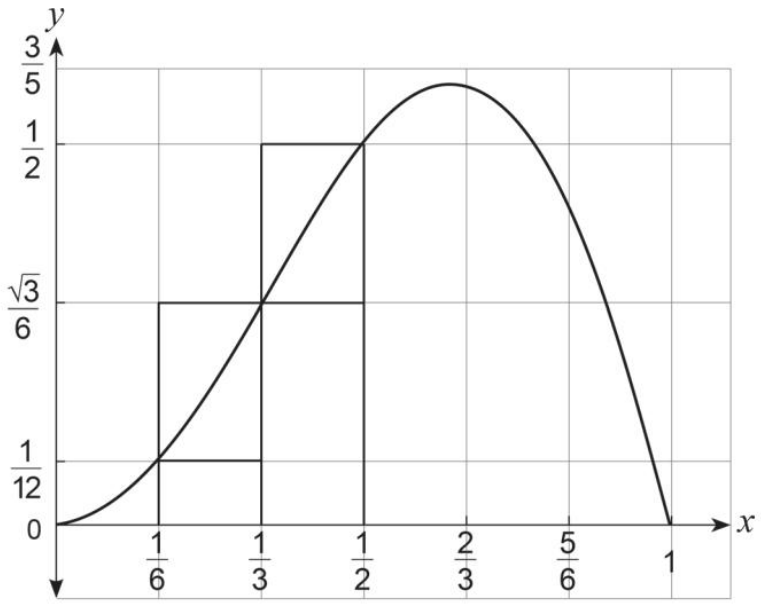
Question 6 **(5 marks)**

The graph of $y = x \sin(\pi x)$ for $0 \leq x \leq 1$ is shown below.



(a) On the diagram above, shade a region whose area is equal to $\int_{\frac{1}{6}}^{\frac{1}{2}} x \sin(\pi x)\, dx$. (1 mark)

A spare diagram is provided at the end of this Question/Answer booklet. If you need to use it, cross out this attempt and indicate that you have redrawn it on the spare diagram.



(b) (i) By considering the areas of the rectangles shown in the graph of $y = x \sin(\pi x)$ above, demonstrate and explain why

$$\frac{1+2\sqrt{3}}{72} < \int_{\frac{1}{6}}^{\frac{1}{2}} x \sin(\pi x)\, dx < \frac{3+\sqrt{3}}{36}.$$

(3 marks)

(ii) State **one** suggestion as to how the approximation from part (b)(i) could be improved. (1 mark)