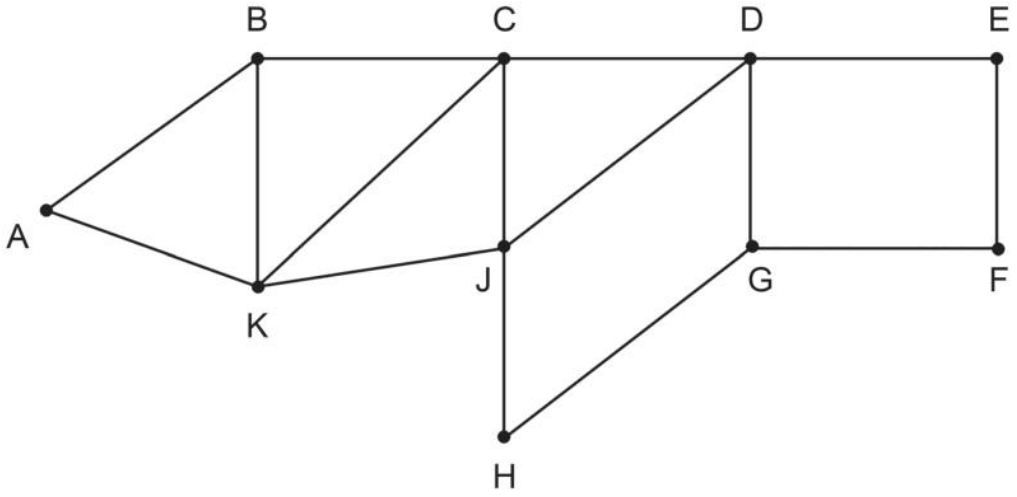
Question 5 **(10 marks)**

A volunteer tour guide has identified several historical places of interest around the Perth central business district. The network shown below has a place of interest at each vertex and the best possible connections between them.



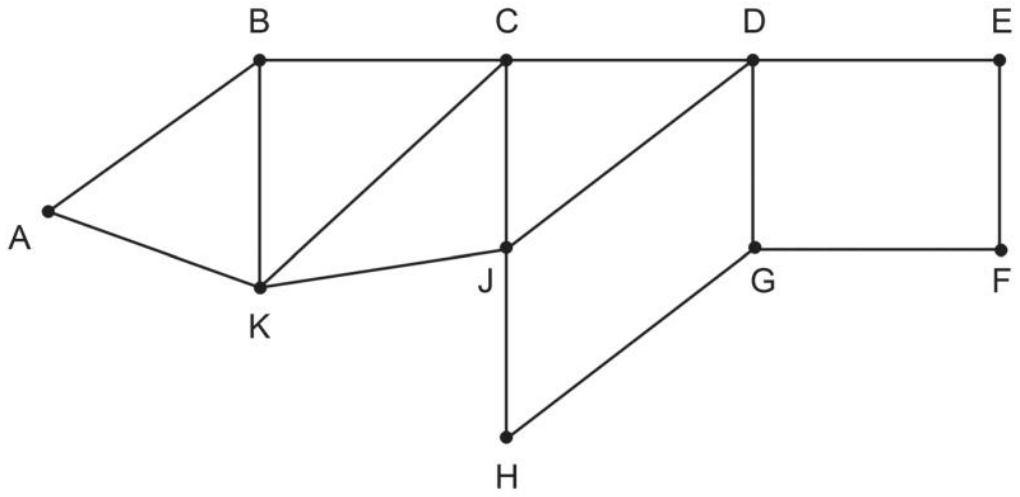
(a) (i) Verify Euler's formula holds true for the above graph. (2 marks)

(ii) Given that part (a)(i) shows the above graph is planar, state what else this means about the graph. (1 mark)

The tour guide wants to offer two types of tours: one that just focuses on visiting each place of interest and the other that visits each place of interest and travels along every connection in order to show more of what Perth has to offer.

(b) Determine whether the tour is able to visit each place of interest and travel along every connection with an Eulerian or semi-Eulerian trail. Justify your response. (2 marks)

(c) Show that the graph below is Hamiltonian by highlighting a possible tour that visits every place of interest. (2 marks)



(d) Determine the number of subgraphs with at most three edges, that contain all of the vertices A, B and C. Each subgraph must be a continuous path. Show a neat sketch of each subgraph. (3 marks)