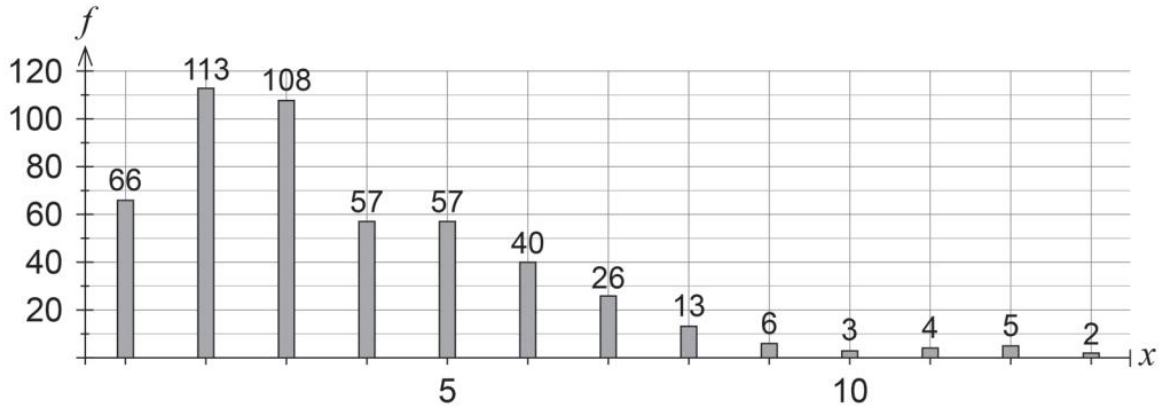
Question 14 **(14 marks)**

A mathematics teacher, Mr Ulam, devises a new game that uses four standard dice. A player rolls all the dice. Any dice with a 1 facing up are winning dice and are removed. The remaining dice are rolled again. The game is finished when a player has two or more winning dice.

Mr Ulam plays the game 500 times and records how many rolls it takes to win. Let X be a random variable denoting the number of rolls needed to win a game. The frequency results, f, are displayed in the graph below.



(a) Using the experimental data above, estimate the probability of

(i) winning in exactly two rolls. (1 mark)

(ii) not winning in two or less rolls. (2 marks)

(b) State **two** reasons why the game cannot be modelled using a binomial distribution.

(2 marks)

Mr Ulam uses a computer to simulate 10 000 games to better estimate the probability of each outcome. The results of the simulations are summarised in the table below.

x	1	2	3	4	5	6	7	8
$\mathrm{P}(X=x)$	0.134	0.215	0.208	0.153	0.106	0.067	0.047	0.030

x	9	10	11	12	13	14	15	16
$\mathrm{P}(X=x)$	0.016	0.012	0.005	0.003	0.002	0.001	0.001	0.000

To fundraise for a local charity, Mr Ulam charges \$1 for each game, and pays out winnings as follows:

- if the game is won in one or two rolls the player wins \$2
- if the game is won in three rolls the player wins \$1 (i.e. their money back)
- if the game is won in four or more rolls the player wins nothing.

Let Y be a random variable denoting the **profit** for a player who plays a \$1 game.

(c) Using the data above, complete the probability distribution table for Y. (3 marks)

y			
$\mathrm{P}(Y=y)$			

(d) Calculate the

(i) expected value of Y. (2 marks)

(ii) variance of Y. (2 marks)

(e) In the long run, do you expect that the game will be profitable for the charity? Justify your answer. (2 marks)