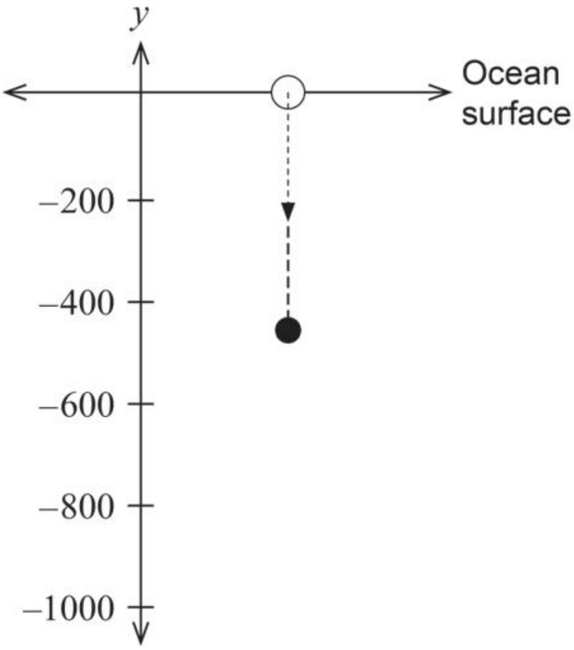
Question 17 (9 marks)

A pressure sensitive device measures depth as it sinks toward the seabed. The device is released from rest at the ocean surface, and as it sinks downward, the water exerts a resistance force to oppose its motion.



Let $t =$ the time (in seconds) elapsed from release.

$y(t) =$ the displacement of the device relative to the surface (metres).

$v(t) =$ the velocity of the device (metres per second).

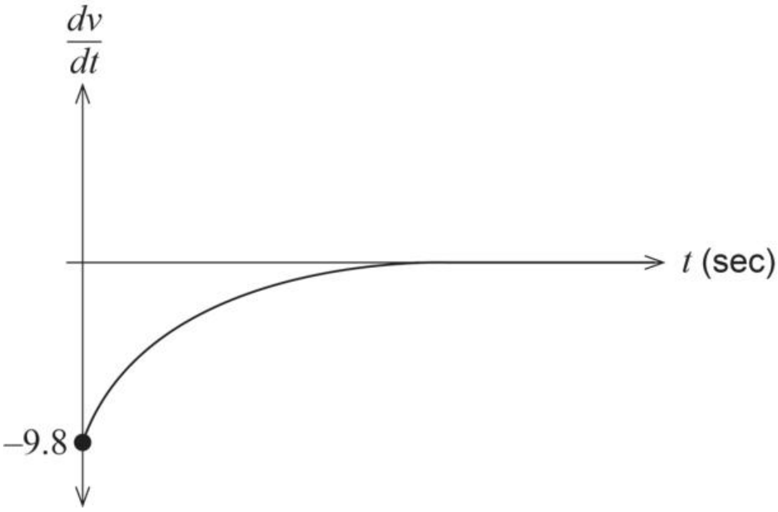
$a(t) =$ the acceleration of the device (metres/second^2).

The diagram shows that after 95 seconds, the device is 463.05 metres below the surface i.e. $y(95) = -463.05$.

The acceleration of the device, at any point in time, is given by $\dfrac{dv}{dt} = -9.8 - 2v$.

(a) Calculate the acceleration of the device, when the device is falling at a rate of 3 metres per second. (2 marks)

The graph of the acceleration $\dfrac{dv}{dt}$ is shown below.



(b) Explain **two** features of the graph of the acceleration $a(t)$ on page 16, referring to the differential equation $\dfrac{dv}{dt} = -9.8 - 2v$ or to the resistance force. (2 marks)

(c) Show, using the separation of variables technique, that $v(t) = 4.9(e^{-2t} - 1)$. (3 marks)

At a particular location, the device is released from rest at the surface of the ocean and falls until it strikes the seabed.

(d) If the device takes exactly 2 minutes 30 seconds to hit the seabed, calculate the depth of the seabed at this location, correct to the nearest metre. (2 marks)