Question 14 **(15 marks)**

Patients are anaesthetised before surgery. The time, in minutes, it takes for a patient to return to consciousness after surgery, called the 'recovery time', is an important measure of the procedure.

Let μ be the population mean recovery time. From historical data, the population standard deviation is $\sigma = 25$ minutes.

A random sample of 100 recovery times is taken of patients who have undergone a tonsillectomy. Let $\overline{X}$ denote the sample mean recovery time.

(a) State the approximate distribution for $\overline{X}$. (3 marks)

(b) Determine the probability that the sample mean is greater than the population mean by more than 3 minutes. (2 marks)

(c) If it is required that the sample mean is to have a 50% chance of being within m minutes of the population mean, determine the value of m correct to 0.1 minutes. (2 marks)

A particular sample of size 100 produces $\overline{x} = 15$ minutes.

(d) Calculate a 99% confidence interval, I_1, for the population mean recovery time. (2 marks)

A new procedure for tonsillectomy using an assisting robot was trialled. A random sample of 400 recovery times was taken for patients who underwent the new procedure. The observed sample mean was 14.5 minutes.

The 99% confidence interval, I_2 , for the population mean recovery time for the new procedure, was found to be $11.28 < \mu\,(new) < 17.72$ minutes.

Two junior doctors made the following statements.

Anja: "The new procedure is superior as its sample mean of 14.5 minutes from 400 patients is lower than the sample mean of 15 minutes from 100 patients and by using a larger sample size we can be more confident."

Sanjeet: "Since the interval I_2 lies completely within interval I_1 , then it can be inferred that the population recovery time for the new procedure is the same as that for the old procedure."

(e) (i) State whether Anja's statement is true or false. Justify your answer. (2 marks)

(ii) State whether Sanjeet's statement is true or false. Justify your answer. (2 marks)

(f) Calculate the minimum sample size required to estimate the mean recovery time for the new procedure with an interval width of at most 4 minutes using a 95% confidence level. (2 marks)