Question 11 **(10 marks)**

A protest group looking to abolish the circulation of physical money (notes and coins) claims that 74% of Perth residents have not used physical money in the past 12 months. To investigate this claim, a data scientist surveys a random sample of Perth residents, with 70% claiming they had not used physical money in the past 12 months.

Let p denote the proportion of Perth residents who have not used physical money in the past 12 months.

(a) Given that the width of the 95% confidence interval for p is 0.096

(i) determine the 95% confidence interval for p. (2 marks)

(ii) determine the number of people surveyed. (2 marks)

(b) What does the data scientist's confidence interval suggest about the protest group's claim? (2 marks)

The data scientist is interested in whether the proportion is similar to that in other Australian capital cities, and conducts surveys in Brisbane, Sydney and Hobart. The following information was recorded based on the survey results:

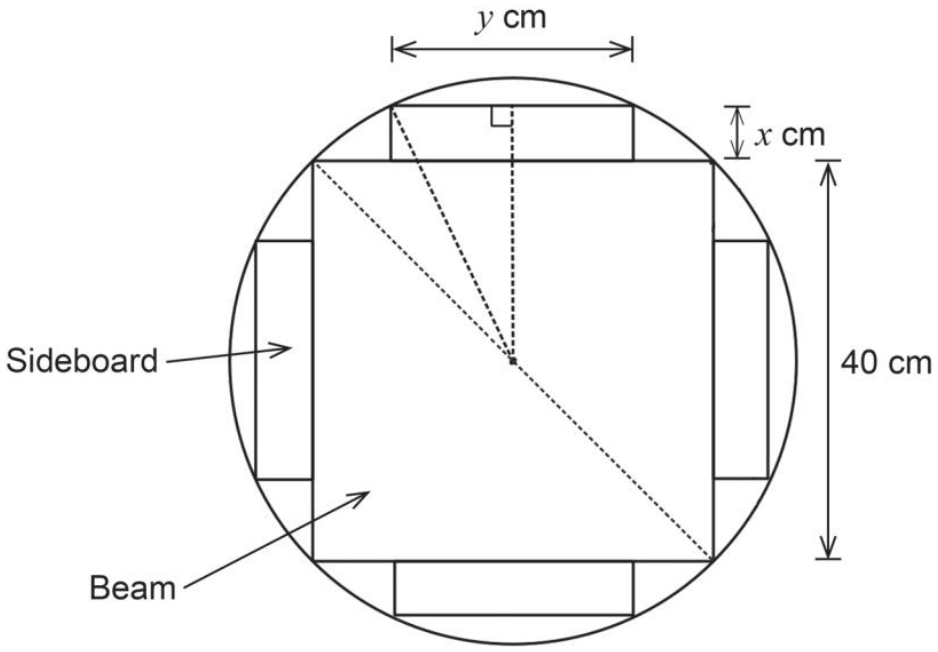
City	Sample size	Sample proportion
Brisbane	N	0.65
Sydney	$2N$	0.65
Hobart	N	0.75

(c) For each of the following city pairs, identify which had the widest 95% confidence interval. Justify your answer.

(i) Brisbane and Sydney (2 marks)

(ii) Brisbane and Hobart (2 marks)

A log with a circular cross-section is being prepared for milling. The log is to be cut into five usable pieces, one large beam with a square cross-section and four sideboards, as shown in the diagram below.



(a) Determine the exact radius of the log. (2 marks)

(b) Using the variables defined in the diagram, show that the cross-sectional area, in cm^2, of a single sideboard is $A(x) = 2x\sqrt{400 - 40x - x^2}$. (3 marks)

(c) Use calculus techniques to determine the dimensions x and y that maximise the cross-sectional area of one sideboard. (4 marks)