Question 13 (13 marks)

Ellie has retired from her job and plans to live off her acquired wealth. She has a superannuation balance of $1 205 788 and a housing loan of $205 749. Ellie sets up an annuity after paying off her housing loan. The annuity will have monthly investment returns and payments, and is modelled by the recursive rule

$$T_{n+1} = 1.005T_n - 6000, \quad T_0 = Z$$

(a) State how much Ellie receives as a monthly payment. (1 mark)

(b) Determine the value of Z in the recursive rule. (1 mark)

(c) Calculate what annual interest rate the annuity pays. (1 mark)

The annuity has a start date of 1 March 2024. At the end of the month when the balance of the annuity first drops below $700 000, Ellie plans to convert the annuity to a perpetuity, paid at the end of each month, so that she will leave behind some family inheritance.

(d) (i) Determine the year and month when the annuity will end. (3 marks)

(ii) Determine the final balance at the end of the annuity. (1 mark)

Ellie receives advice that a perpetuity could now be arranged, receiving 5.95% per annum compounded daily.

(e) Determine the monthly perpetuity payment. (3 marks)

(f) Ellie is curious as to whether the annuity or the perpetuity pays the better annual rate of interest. Determine which pays the higher interest rate. (3 marks)