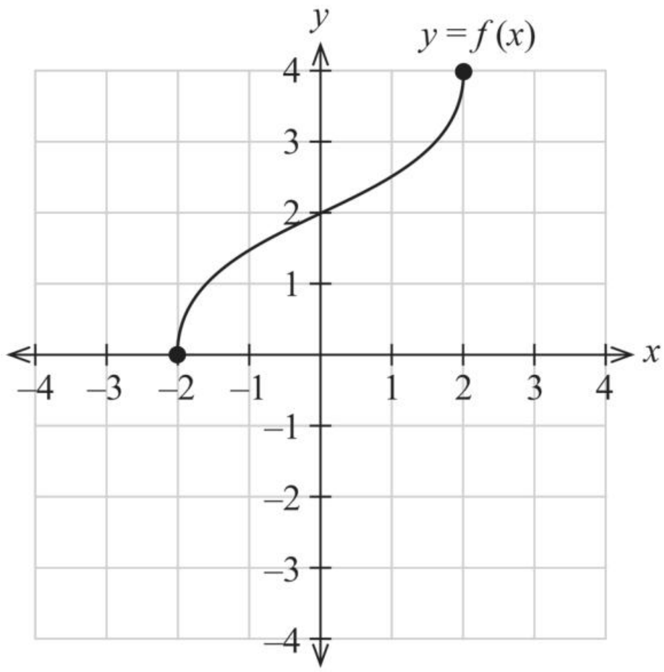
Question 8 **(13 marks)**

The statement $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$ can be written as $\frac{\pi}{3} = \cos^{-1}\left(\frac{1}{2}\right)$, where $\cos^{-1}$ represents the inverse cosine function.

The graph of $y = f(x) = \frac{4}{\pi}\cos^{-1}\left(-\frac{x}{2}\right)$ for $-2 \le x \le 2$ is shown below.



(a) Explain why $y = f^{-1}(x)$ exists. (1 mark)

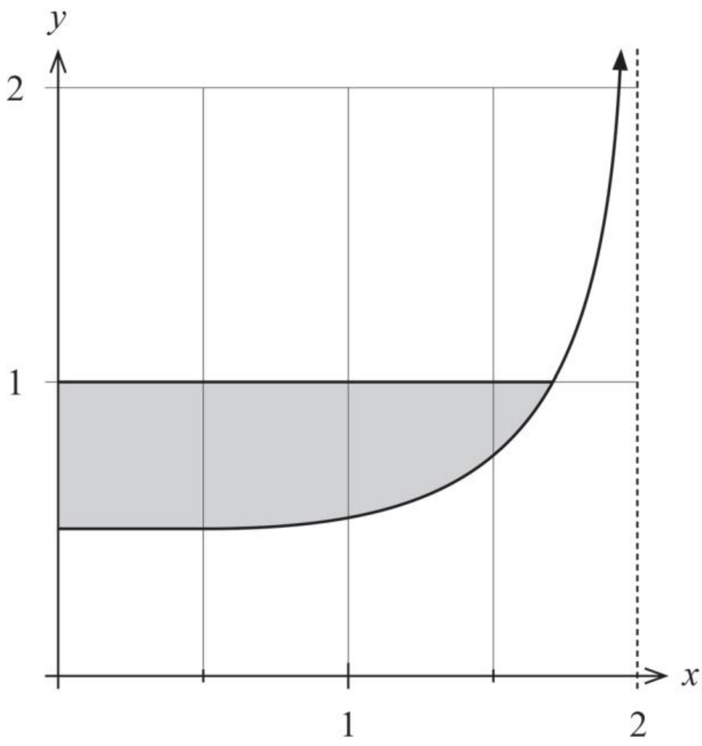
(b) Determine the defining rule for $y = f^{-1}(x)$. (2 marks)

(c) Sketch the graph of $y = f^{-1}(x)$ on the axes on page 12. (2 marks)

A spare grid is provided at the end of this Question/Answer booklet. If you need to use it, cross out this attempt and indicate that you have redrawn it on the spare grid.

(d) If $y = \frac{4}{\pi}\cos^{-1}\left(-\frac{x}{2}\right)$, using implicit differentiation, show that $\frac{dy}{dx} = \frac{4}{\pi\sqrt{4-x^2}}$. (5 marks)

The graph of $y = \frac{1}{\sqrt{4-x^2}}$ is shown below for $0 \le x < 2$.



The shaded region is bounded by the curve $y = \frac{1}{\sqrt{4-x^2}}$, the line $y = 1$ and the y axis.

(e) Determine the exact area of the shaded region. (3 marks)