Question 7 (12 marks)

A new entertainment streaming service is concerned because it is receiving cancellations and not gaining any new subscriptions. The number of subscriptions (in thousands) at the end of each week have been recorded in the table below.

Week (n)	1	2	3	4	5
Subscriptions (in thousands)		97	94.09	91.267	

(a) Show that the number of subscriptions is decreasing by 3% each week. (2 marks)

(b) Complete the table above. (1 mark)

(c) Deduce a rule for the nth term for the number of subscriptions at the end of each week. (2 marks)

(d) During which week does the number of subscriptions first fall below 70 000? (2 marks)

In an attempt to increase the total number of subscriptions to the streaming service, two possible loyalty programs are being developed, with a plan for one of these to be launched when the number of subscriptions falls to 50 000.

(e)   It is hoped that the first loyalty program will change the percentage decrease of subscriptions per week from 3% to 1.4% and also increase the number of new subscriptions by 100 000 each week.

The total number of subscriptions can be represented by the recursive rule

$$T_{n+1} = aT_n + b, \quad T_0 = 50.$$

Determine the value of a and b. (2 marks)

(f)   The total number of subscriptions from the second loyalty program can be represented by the recursive rule, $T_{n+1} = 0.99T_n + 95, \quad T_0 = 50.$

(i)   Interpret the number 0.99 in the context of this question. (1 mark)

(ii)   The streaming service will need to maintain 8.5 million total subscriptions if it is to be considered sustainable. Determine whether this is achievable with the second loyalty program. (2 marks)