Question 19 (14 marks)

Using the correct technique, Olympic ski jumpers can slow down their descent, by creating lift to counteract gravity. These jumpers must land successfully to have their distance recorded and land on sloped ground to prevent serious injury.

For copyright reasons this image cannot be reproduced in the online version of this document.

A skier begins his descent at point B accelerating down the ramp. At the end of the ramp the skier is travelling horizontally at point E at 32 metres per second (115.2 kilometres per hour).

Let t = the number of seconds in flight after point E (100, 120).
$h(t)$ = the height of the skier above the horizontal ground $y = 0$ (metres)
$x(t)$ = the horizontal position of the skier (metres)

The sloped ground for landing is given by $y = 170 - 0.5x$ where $100 \leq x \leq 340$.

It is found that the expression for the position vector for the skier during the flight is given by:

$$\underset{\sim}{r}(t) = \begin{pmatrix} 740 - 640e^{-0.05t} \\ 120 - 2.5t^2 \end{pmatrix}$$

(b) Calculate the height of the skier above the sloped ground after 3 seconds of flight, correct to the nearest 0.01 metre.   (3 marks)
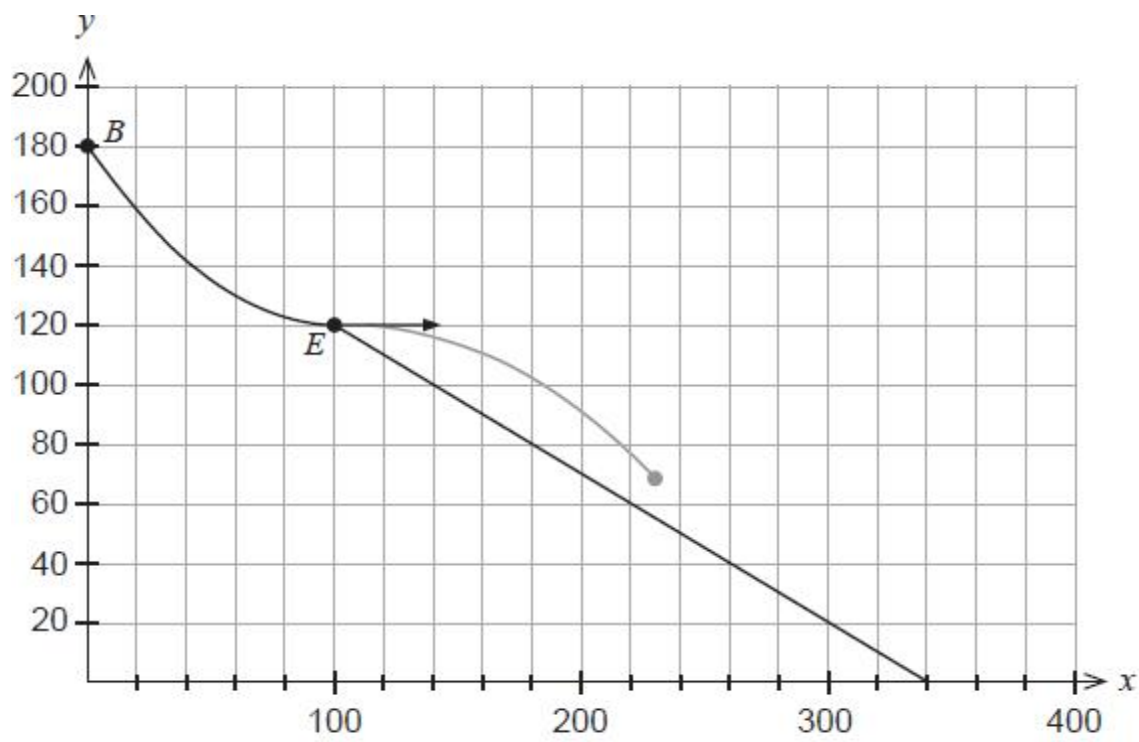
The ski jumper's suit and skis decrease the horizontal velocity $x'(t)$ so that $x'(t) = 32e^{-0.05t}$.

(a)   Show that $x(t) = 740 - 640e^{-0.05t}$.   (2 marks)

(e)   Calculate the angle at which the skier impacts the sloped ground, correct to the nearest
        0.1 degree.   (3 marks)

It can be shown that the Cartesian equation for the skier's flight is given by:

$$y = 120 - 1000 \left(\ln \left(\frac{740 - x}{640} \right) \right)^2$$

(d)   Calculate the time taken for the skier to land on the sloped ground, correct to the nearest 0.01 second.   (3 marks)

(c)   Determine the vertical lift s (m/s²) provided by the skier's suit and equipment in the descent if $\dfrac{d^2h}{dt^2} = s - 9.8$, where s is a constant.   (3 marks)