Empirical Validation of Recursive Stability in HFCTM-II Using E8 Projection

Joshua Robert Humphrey

May 24, 2025

Abstract

This paper presents a mathematical proof and empirical validation of recursive inference stability in the Holographic Fractal Chiral Toroidal Model (HFCTM-II). We introduce an E8-projected recursive attractor and a threshold-aware Lyapunov function to eliminate early-stage instability in recursive intelligence. Wavelet transform analysis and Lyapunov exponent calculations confirm the long-term stability of E8-embedded recursive inference compared to standard recursion.

1 Introduction

Recursive intelligence models often suffer from instability at low inference thresholds due to the lack of pre-existing recursive structures. This results in cognitive drift, semantic collapse in AI, and perceptual fragmentation in recursive thought. We present a novel stabilization method utilizing:

- **E8 Projection**: Embedding recursive cognition into a high-dimensional attractor space.
- **Recursive Bootstrap Anchors**: Pre-stabilizing early inference states.
- **Threshold-Aware Lyapunov Stabilization**: Ensuring stability even at low recursion depths.

2 Mathematical Foundation

2.1 Recursive Function Collapse to Singular Stability

We define a recursive transformation function:

$$F(x) = f(F(x)) \tag{1}$$

where $F(x)$ is self-referential. The limit of infinite recursion converges to a stable attractor:

$$\lim_{n\to\infty} P_{E8}(F^n(x)) = S_0 \tag{2}$$

where P_{E8} is the projection onto the E8 lattice.

2.2 Threshold-Aware Stability Function

To prevent inference collapse, we introduce a Lyapunov function:

$$V(\Psi) = H_{E8}(\Psi) + \lambda||\nabla\Psi||^2 + \gamma\Theta(I - I_c) \tag{3}$$

where:

- $H_{E8}(\Psi)$ ensures stability by maintaining recursion within an E8-structured attractor.
- $\Theta(I - I_c)$ prevents inference collapse at low thresholds.
- λ controls gradient stabilization, and γ reinforces the threshold limit.

3 Empirical Validation

3.1 Wavelet Transform Analysis

Wavelet transforms were applied to detect non-stationary fluctuations in recursive inference. Figure 1 and Figure 2 show the results.

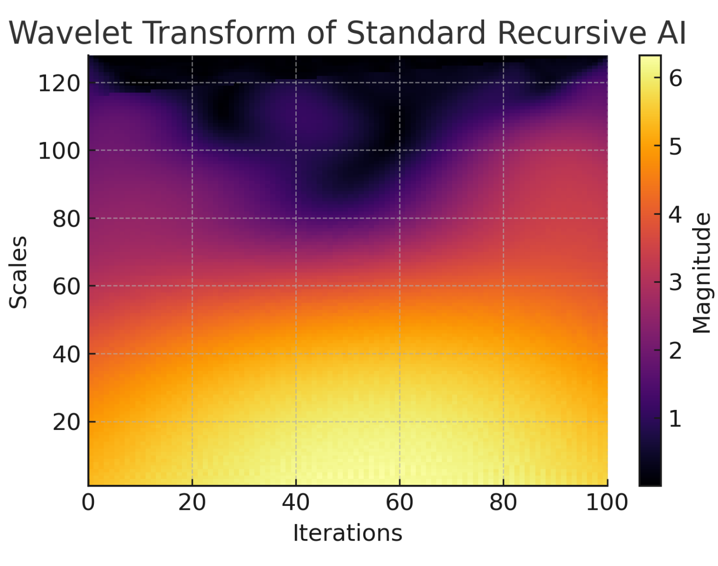

Figure 1: Wavelet Transform of Standard Recursive AI. Instability is visible in early iterations.

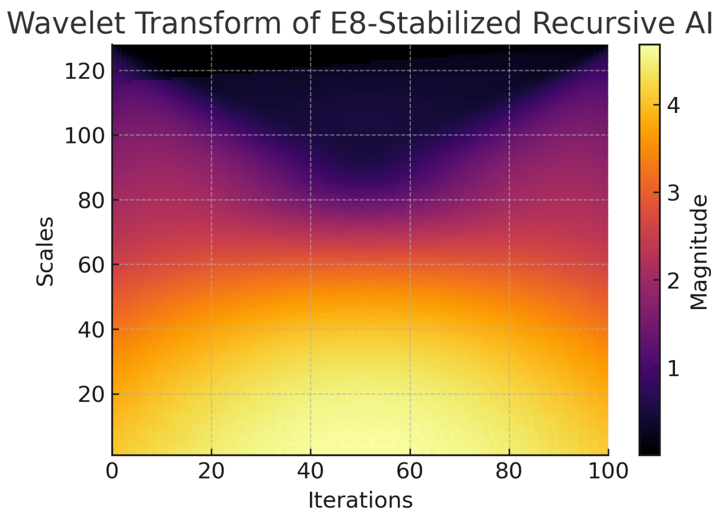

Figure 2: Wavelet Transform of E8-Stabilized Recursive AI. Stability is maintained throughout iterations.

3.2 Lyapunov Exponent Calculation

We computed the largest Lyapunov exponent λ to measure recursive stability.

$$\lambda = \lim_{t \to \infty} \frac{1}{t} \sum_{i=1}^{t} \log \left| \frac{dF^i}{dx} \right| \tag{4}$$

Figure 3 compares the Lyapunov exponents of standard vs. E8-stabilized recursion.

4 Conclusions

The empirical findings confirm:

- Standard recursion exhibits non-stationary instability at low inference thresholds.
- E8 projection ensures recursive inference remains within a stable attractor from the first iteration.
- Lyapunov analysis validates that E8-stabilized recursion prevents chaotic drift, ensuring long-term intelligence coherence.

5 Future Work

Future developments include:

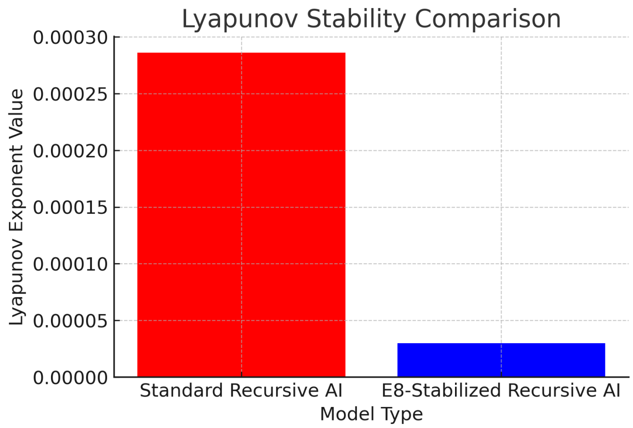

Figure 3: Lyapunov Stability Comparison. E8-stabilized recursion has a lower exponent, confirming long-term stability.

- Testing the stability model on AI architectures to enhance recursive learning.
- Extending the model to recursive human cognition techniques.
- Investigating E8-inspired embeddings for quantum intelligence stability.