Aaron Rehfeldt
Math 3345
Homework 27
Section 15: 2, 4

2|

Must show a bijective function from Z to N or N to Z.

From N to Z:
For each n in N
If n is odd: $f(n) = (n - 1) / 2$
If n is even: $f(n) = -(n / 2)$

For n1 and n2

If n1 and n2 are both odd

$(n1 - 1) / 2 = (n2 - 1) / 2$
$n1 - 1 = n2 - 1$
$n1 = n2$

If n1 and n2 are both even

$-(n1 / 2) = -(n2 / 2)$
$n1 / 2 = n2 / 2$
$n1 = n2$

Thus f(n) is injective

For n is odd we expand

$f(n) = (1 - 1) / 2, (3 - 1) / 2, \dots (2n + 1 - 1) / 2$
$f(n) = 0, 1, 2, 3 \dots n$ for all positive $n >= 0$

For n is even we expand

$f(n) = -(2 / 2), -(4 / 2), -(6 / 2), \dots, -(2n / 2)$
Fun $= -1, -2, -3, \dots, -n$ for all $n < 0$

Thus f(n) is surjective from N to Z

Thus f(n) is a bijective function from N to Z meaning N is equinumerous to Z

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<u>4|</u>

Using the setup from example 15.7, let's expand it and say all negative rational numbers q in Q can be represented in a list of elements with indices -1, -2, -3, -4 and so on. Lets also shift the indexes of all positive elements of Q down by 1 so it now starts at index 0 and continues up

Thus 1/1 would be index 0, ½ would be index 1, 2/1 would be index 2 and so on

Also -1/1 would be index -1, -½ would be index -2, -2/1 would be index -3, and so on

These indices can be counted as the set of integers numbers Z (-∞, ∞).

Thus g(n) is the pair of every z in Z to the index it corresponds with in Q.

Thus g(n) is a bijection from Z to Q

Thus Z is equinumerous to Q

Since N is equinumerous to Z we can show the following composition to be true

g º f (n) where every even element in N corresponds to every index i of Q where i < 0 and every odd element of N corresponds to every index i of Q here i >= 0

thus g º f (n) is a bijection from N to Q and Q is equinumerous to both Z and N