CS463/516
Medical Imaging

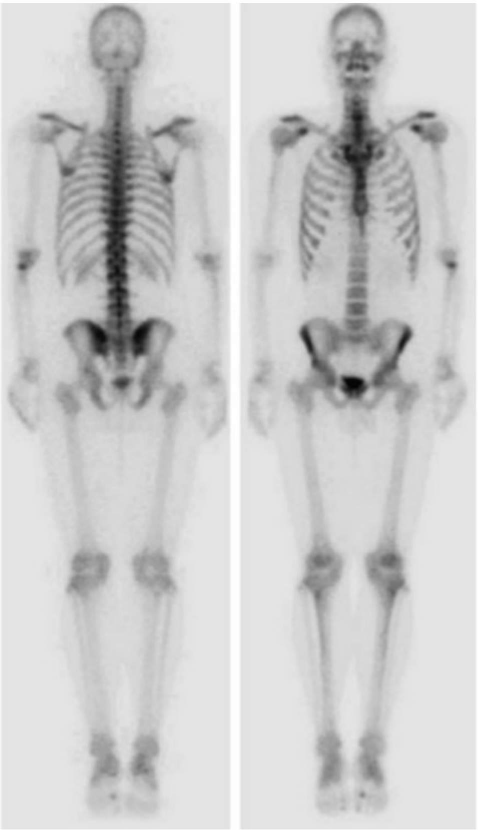
Lecture 2

Digital image acquisition

- Two types of medical image
  - 1) projection images project a physical parameter in the body on a 2D image
  - 2) slice images produce a 1 to 1 mapping of measured value
- Interpretation of medical images requires expertise about meaning of intensities in image
  - a) The man is clearly wearing a dark jacket, and reading a white newspaper, but what about b) ?
- Physical property measured by imaging device must:
  - 1) penetrate the human body
  - 2) not interfere with the body
  - 3) be meaningful for answering some medically relevant question

a)

b)



The ‘big 4’ digital imaging techniques

- **X-ray imaging (includes CT)**
  - Contrast: Absorption of short-wave electromagnetic waves varies across tissue types
- **Magnetic Resonance Imaging**
  - Contrast: Density and molecular binding of selected atoms (notably hydrogen) which varies with tissue type, molecular composition, and functional status
- **Ultrasound Imaging**
  - Contrast: Captures reflections at boundaries between and within tissues with different acoustic impedance
- **Nuclear Imaging**
  - Contrast: Measures distribution of radioactive tracers administered to subject through blood flow
- Other techniques include EEG, MEG, microscopy, and photography
- Choice of technique depends on medical question we want to answer

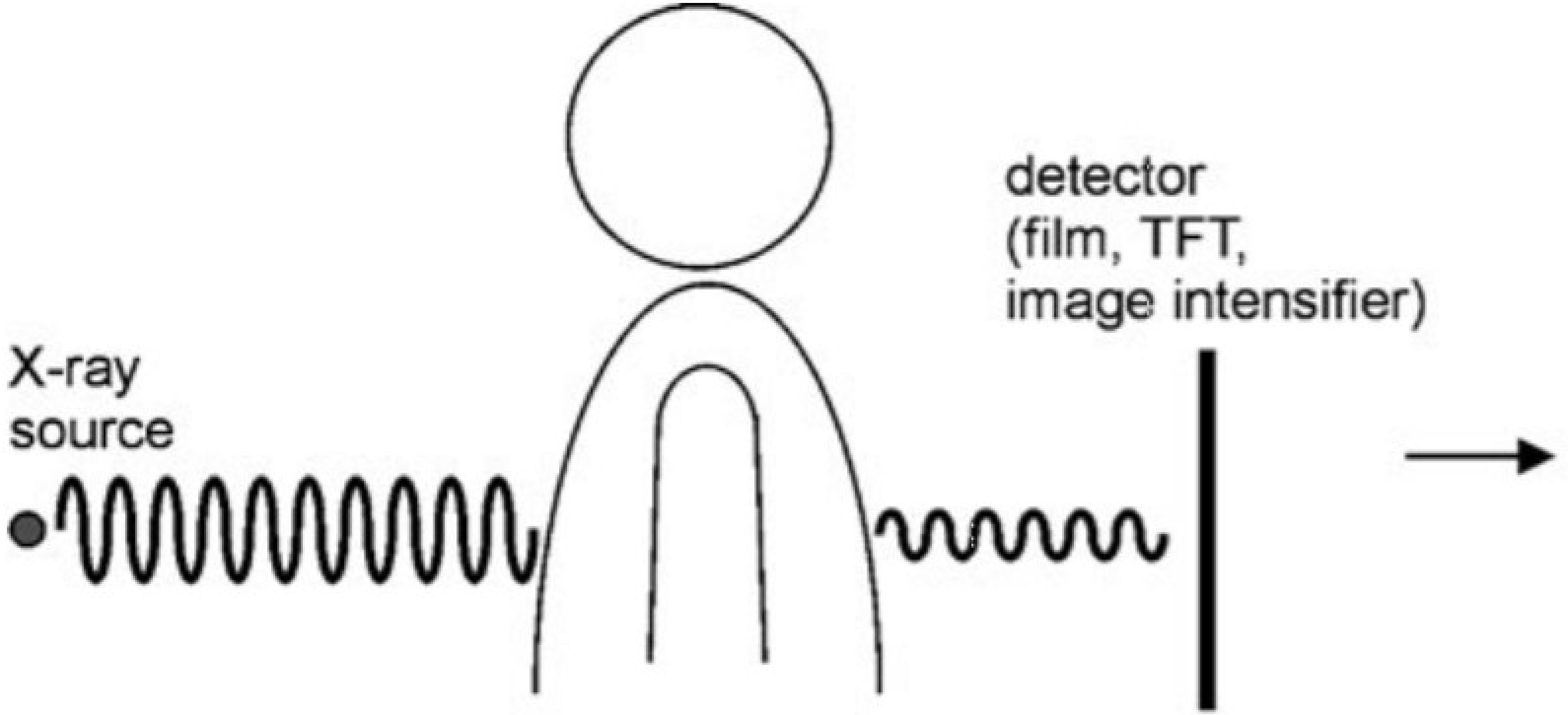
First medical image ever taken→

- 1896
- Wilhelm Röntgen takes X-ray of wife’s hand
  - Bones of hand and wedding ring visible →

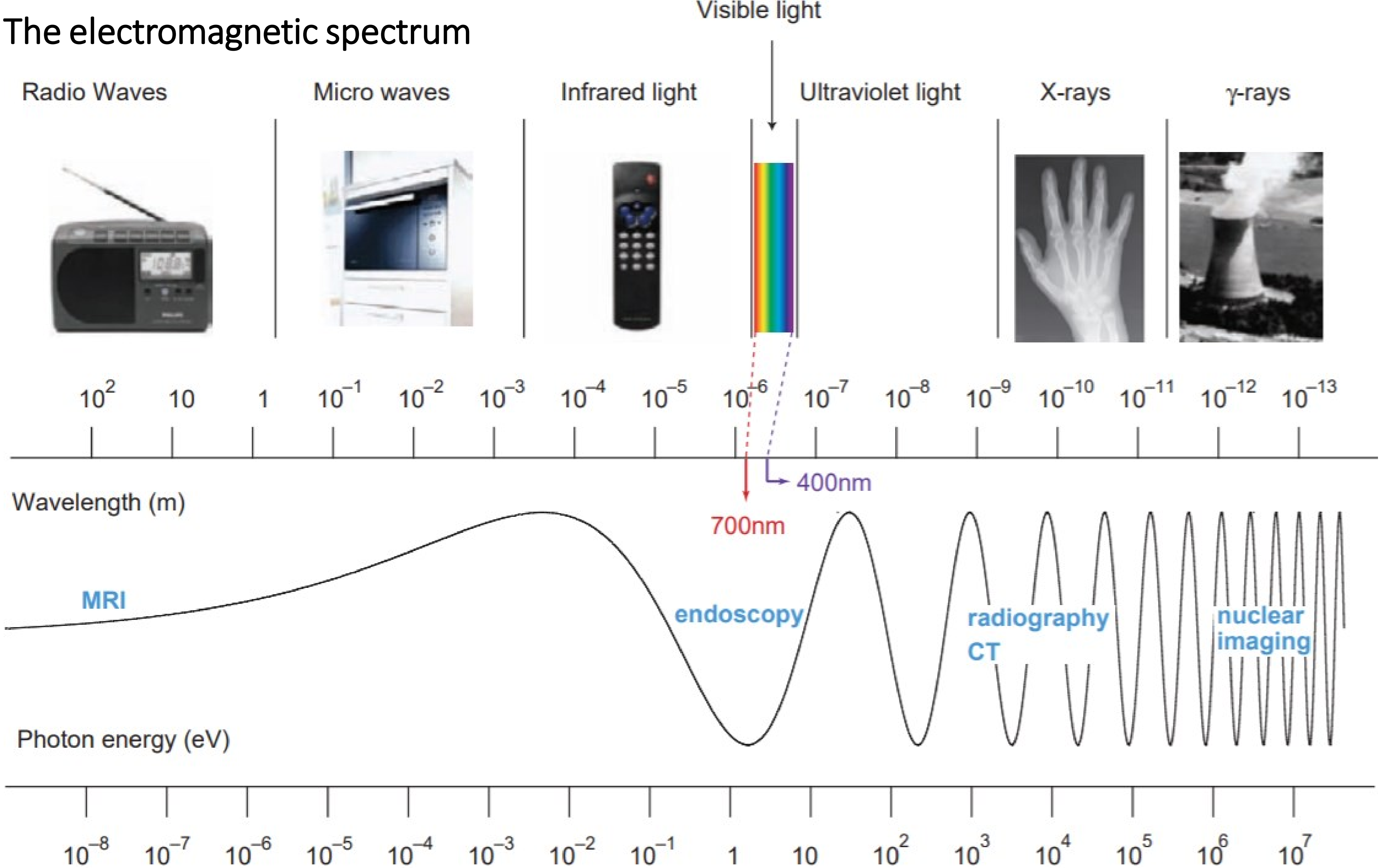
Wilhelm Röntgen

X-ray imaging

- X-rays penetrate body and produce image showing integral of tissue-specific absorption along path from X-ray source to detector
- Discovered in 1895 by Wilhelm Röntgen, harmful effects not known in early years
- <u>First technique to allow for non-invasive insight into human body</u>

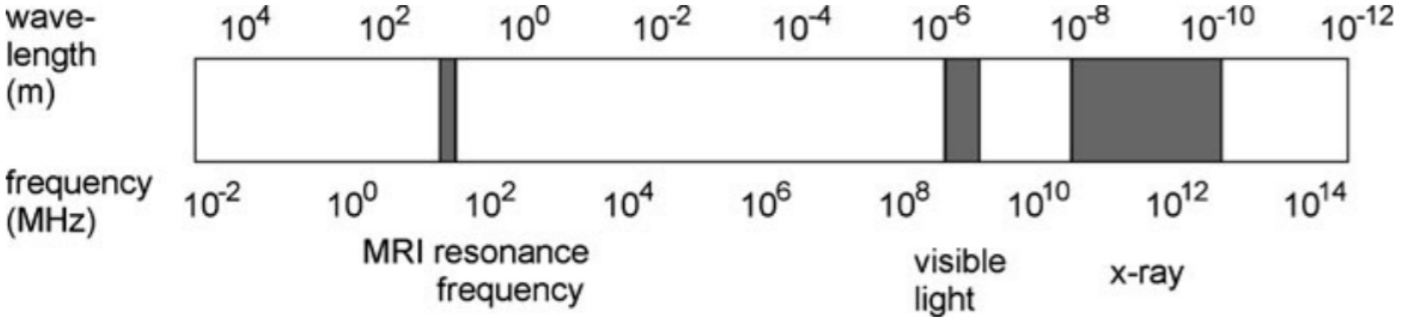


The electromagnetic spectrum

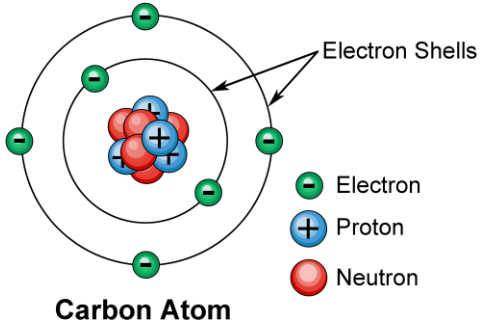


Generation, attenuation, and detection of X-rays

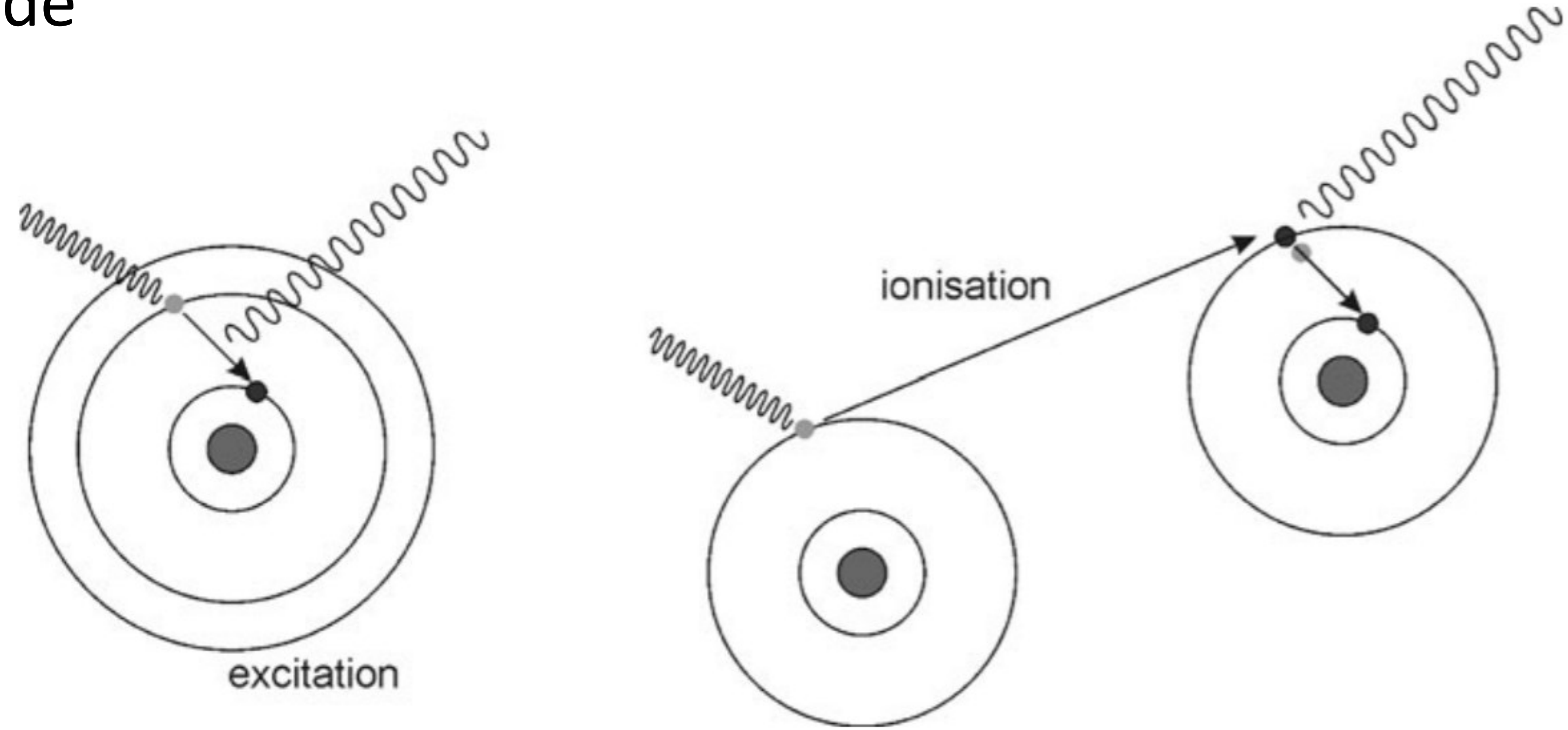
- X-rays are electromagnetic waves with wavelength above visible spectrum
- Electromagnetic waves travel at speed of light c with wavelength λ, frequency f
- $c = \lambda f$
- Energy of a photon in electron volts (eV) is the energy acquired by single electron when moving through potential of 1 V
- Energy of photon characterized by its wavelength, $e = 1.24/\lambda$
  - If unit of measurement is kilo electron volt (keV), wavelength measured in nanometers nm
  - In order of increasing energy: radio wave, visible light, x-ray (gamma ray)
  - Gamma rays created in nucleus of atom while x-rays are not
- X-rays characterized by their *exposure*, or amount of charge per unit volume of air, units of röntgen



X-ray generation

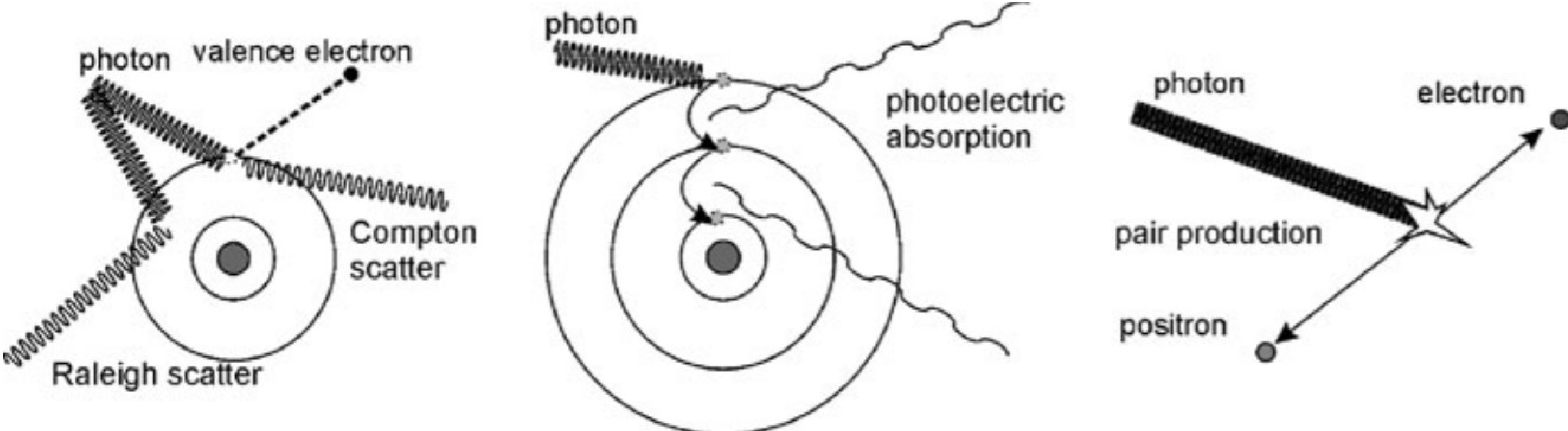


- Electrons in atom organized in ‘shells’ around nucleus
  - Negatively charged electrons attracted to protons in nucleus, so innermost shell contains electrons with lowest energy
  - Requires energy to release electron from shell, outer (valence) electrons easiest to remove
- X-rays generated as excess energy from electrons in material of cathode ray tube (CRT) when heating the cathode
  - Energy from heating causes electrons to be released from cathode, accelerated towards anode
  - In anode, electrons lose kinetic energy by excitation, ionization, and radiation
  - Excitation and ionization cause electrons of anode material to move from outer to inner shell



X-ray attenuation

- When x-ray enters human body, four types of attenuation can happen:
  - 1) **Rayleigh scatter** – photon loses energy by exciting whole atom, which immediately releases a photon with similar energy scattered in a different direction
  - 2) **Compton scatter** – atom release valence electron from shell
  - 3) **Photoelectric absorption** – photon releases its energy through photoelectric absorption by removing one of the electrons from atom's inner shell. Photon loses energy completely. Electrons from outer shell move to inner shell, releasing radiation. Main contributor to X-ray imaging.
  - 4**) Pair production** – photon annihilates, producing electron-positron pair, which go in opposite directions.



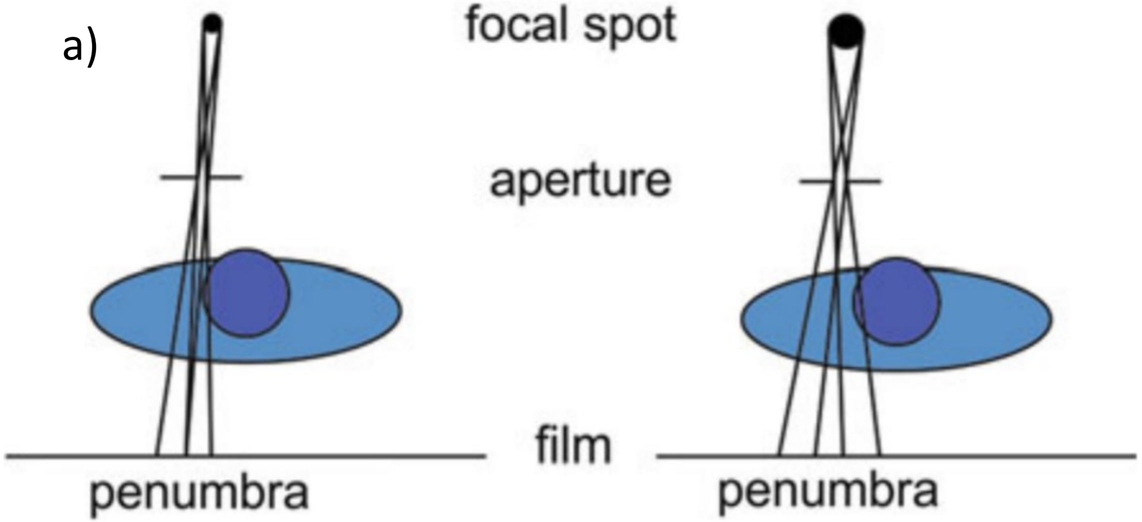
X-ray imaging

- X-ray imaging uses dependency of photoelectric absorption on atomic number to produce diagnostically meaningful image
- Equipment necessary to generate X-ray image:
  - 1) Cathode ray tube (CRT) for emitting X-rays
  - 2) Planar receptor (film, image intensifier, or detector)
  - 3) Person or object to place between CRT and receptor
- Image intensity at location (x, y) on receptor is proportional to attenuation along a ray from X-ray tube to the receptor:
- $I_{out} = I_{in} \cdot \exp(-\int_{s_0}^{s_1} \mu(s) ds)$
  - Where I_{in} is intensity of X-ray when entering body, s is a ray from X-ray source to (x, y) on imaging plane, s_1 is point where ray enters body, s_2 is point where ray exits body, $u(s)$ is the *attenuation* which is mainly caused by photoelectric absorption and Compton scatter

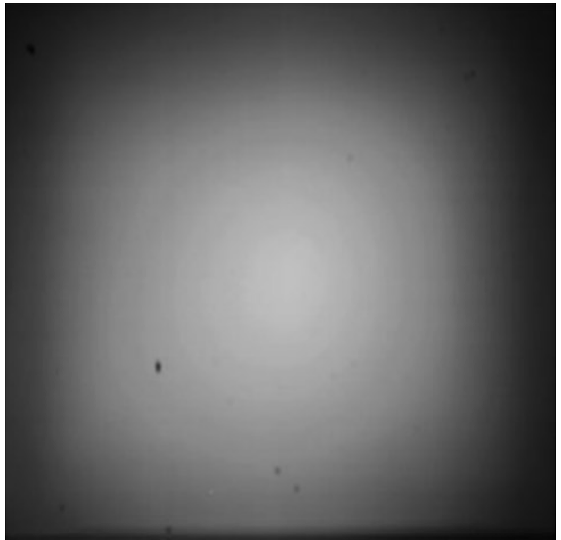
X-ray imaging

- We assumed that X-ray source is a *point source*
- In reality, *focal spot* of X-ray source covers a finite area, leading to loss of resolution due to *penumbrae* (a)
  - Regular X-ray CRTs have focal spot with 1mm diameter
- Vignetting caused by angle at which ray falls onto input screen (b)
  - Angle is perpendicular to screen in the center, energy distributed over smallest possible area
  - Angle decreases with distance to center, causing energy to distribute over wider area

a)



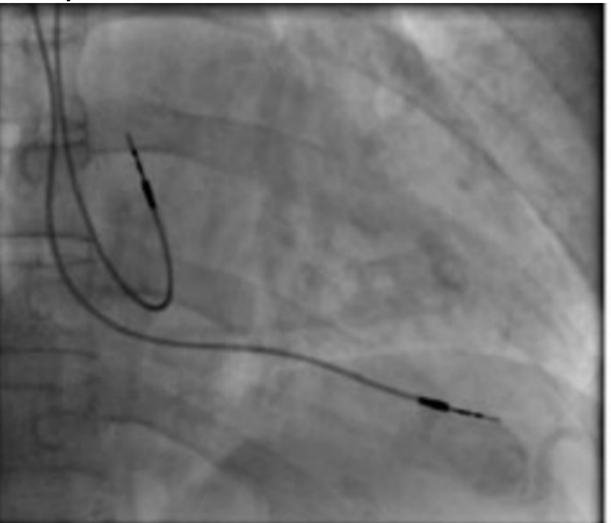
b)



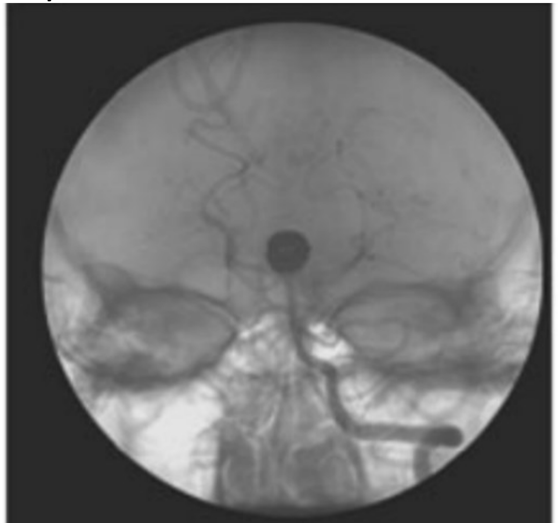
Fluoroscopy and Angiography

- *Fluoroscopy:* specific kind of X-ray imaging to visualize moving objects in body (a)
  - Follow heartbeat to detect abnormal function
  - Image cardiac or cerebral blood flow by using contrast agent
- In Fluoroscopy, X-rays turned into visible light using fluorescent screen
- Fluoroscopic imaging of vascular system using contrast agent is called *angiography*
- Angiographic images show anatomy with blood vessels enhanced, can be acquired in real-time and used to guide surgical intervention (b, d)
- Digital subtraction angiography – subtract image prior to injecting contrast agent from image after injecting contrast agent (c)

a)



b)

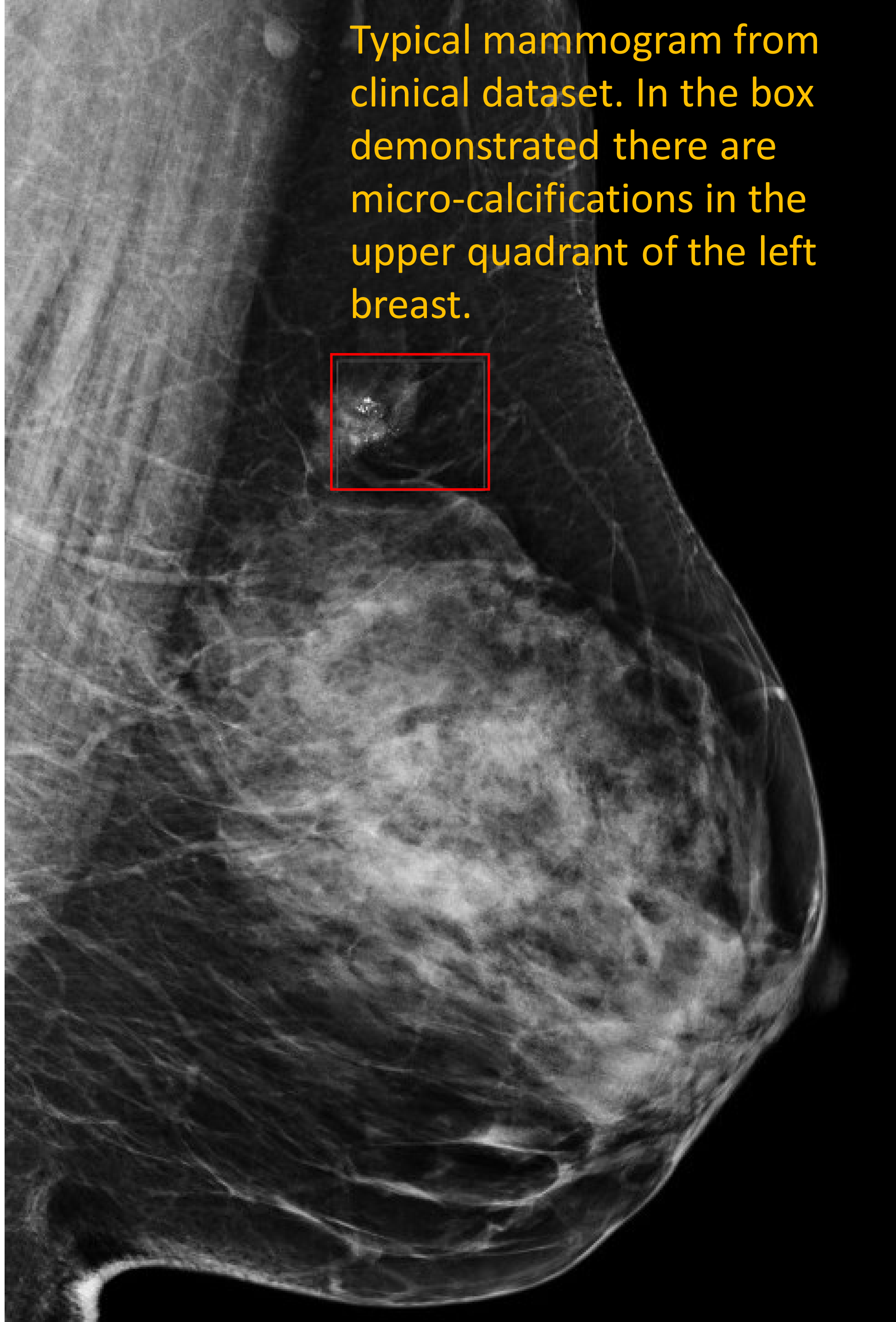


c)

d)

Mammography

- **Purpose of mammography**: detect small non-palpable lesions in female breast
- Requires much higher image quality than normal X-ray imaging (higher contrast and resolution)
- Contrast and resolution affected by scattering, so uses *mammography tubes* reduce scattering using filtering
- Mammography tubes also use material called molybdenum, which produces lower energy (17-19 keV) beams, increasing contrast between subtle differences in breast tissue
- Digital mammograms have potential for post-processing
    - Predict breast cancer risk and detect tumors
    - Estimate mammographic sensitivity
- "A Multi-million Mammography Image Dataset and Population-Based Screening Cohort for the Training and Evaluation of Deep Neural Networks—the Cohort of Screen-Aged Women (CSAW)":
    - https://link.springer.com/article/10.1007/s10278-019-00278-0

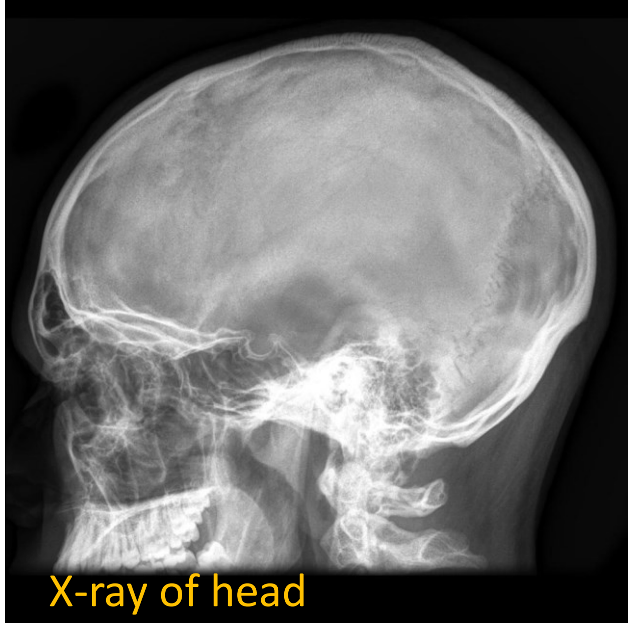


Typical mammogram from clinical dataset. In the box demonstrated there are micro-calcifications in the upper quadrant of the left breast.

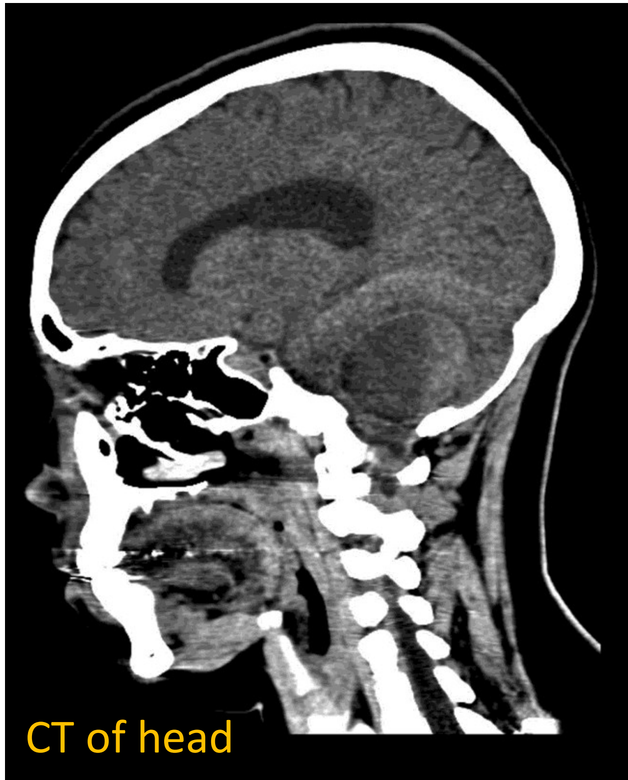
Computed Tomography (CT)

- Images from X-ray attenuation discussed so far are *projection images* – 2D images where structures are projected on top of each other.
  - High attenuating objects (bone) may hide other objects.
  - Example – skull hides most of details in a head X-ray (a)
- **Tomography** (Greek “tomos” = cut, slice) attempts to create an image of slices through the body
- Invented in 1970s, X-ray Computed Tomography (CT) provides a detailed 3d distribution of X-ray attenuation *per volume unit* (b)

a)



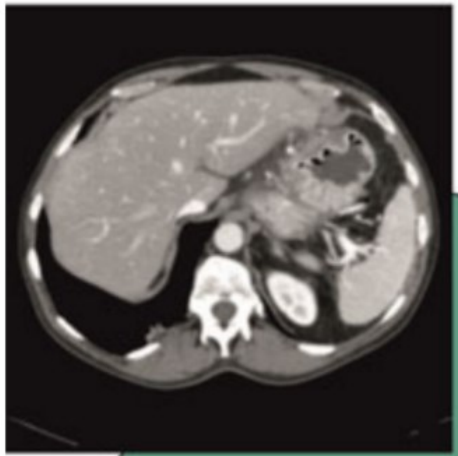
b)

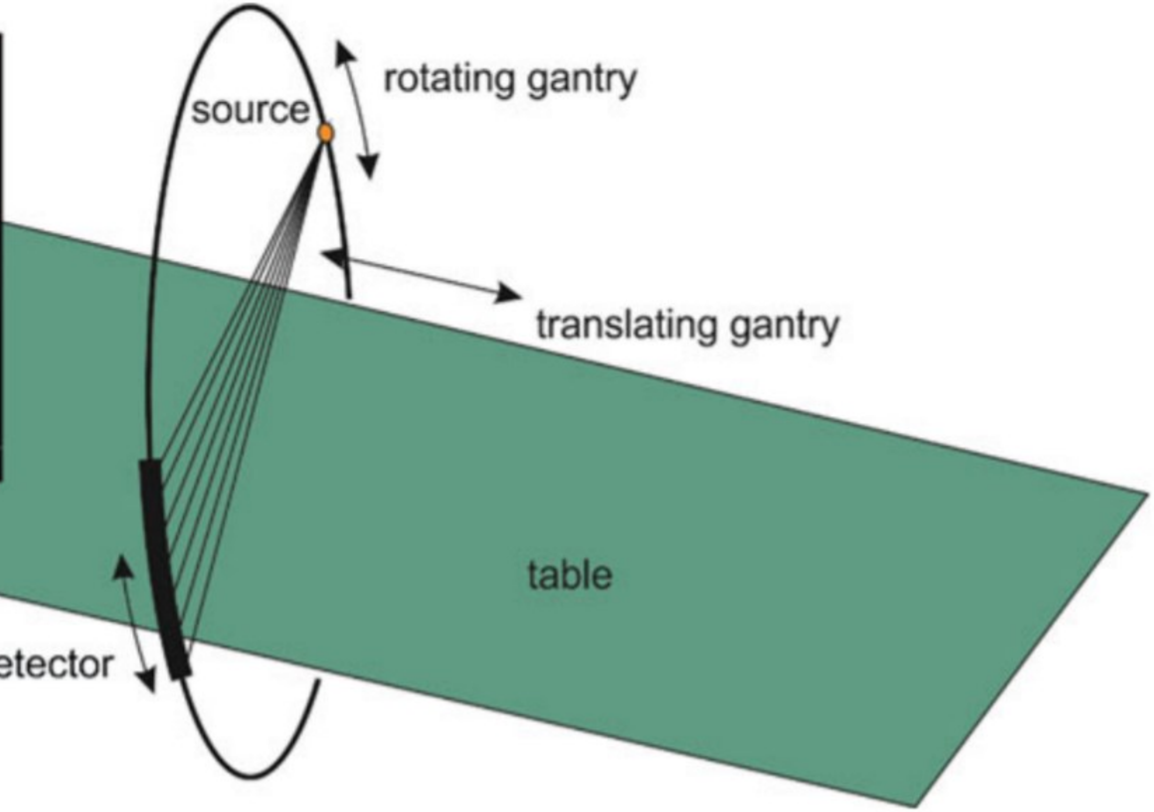


CT scanner

- Detector ring around which an X-ray source rotates
- Detections are made constantly during rotation
- Creates X-ray images from multiple angles

- Stack of slices produced by scanning a sequence of different slices
- Patient lies on table that moves into detector ring in pre-specified steps
- This movement is called translational *gantry*
- Step size of horizontal movement determines distance between slices
- Thickness of detector ring determines slice thickness
- **Early days**: slice thickness of 5mm, with 5mm gap between slices, typically <20 slices/scan
- **Today**: X-ray source rotates in continuous spiral while measuring absorption data, also, multi-slice detectors, can easily acquire 500-1000 slices in a single scan

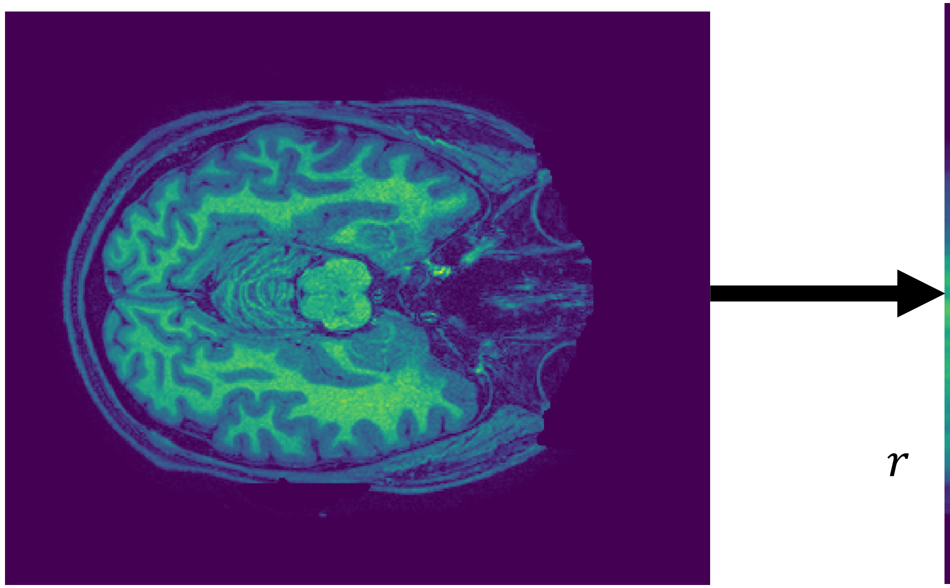




Radon transform

slice z

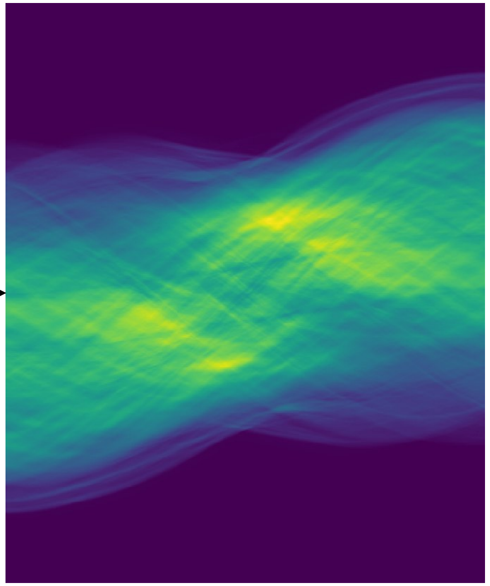
y



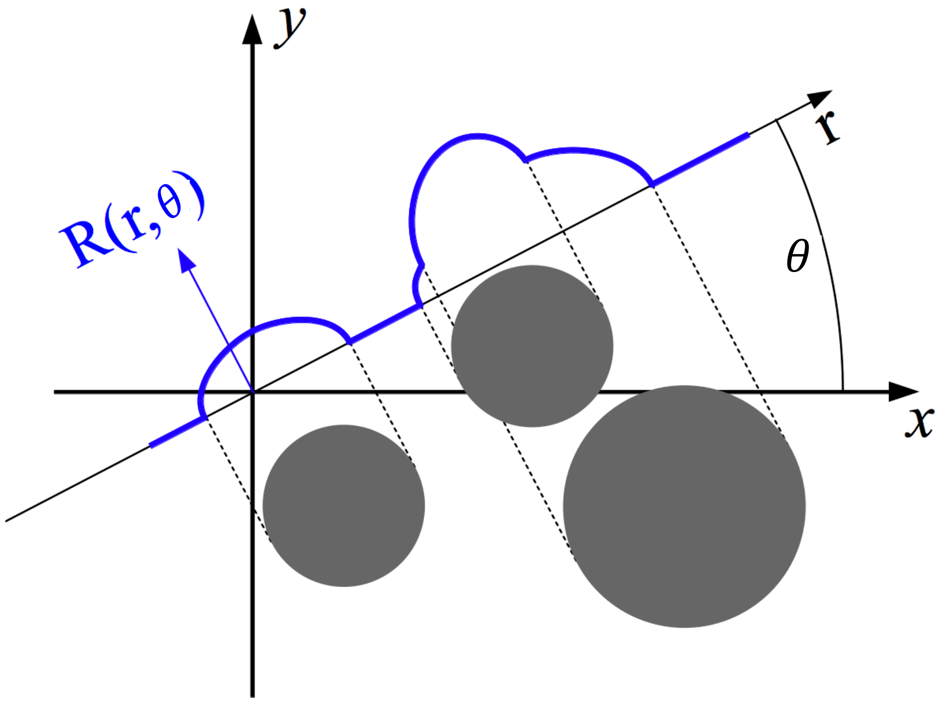
x

$R(z)$ (Radon transform of slice z)

r

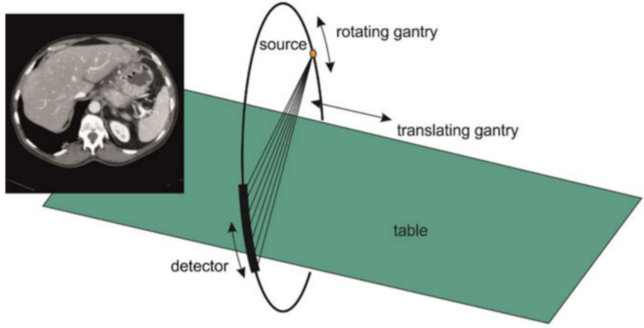


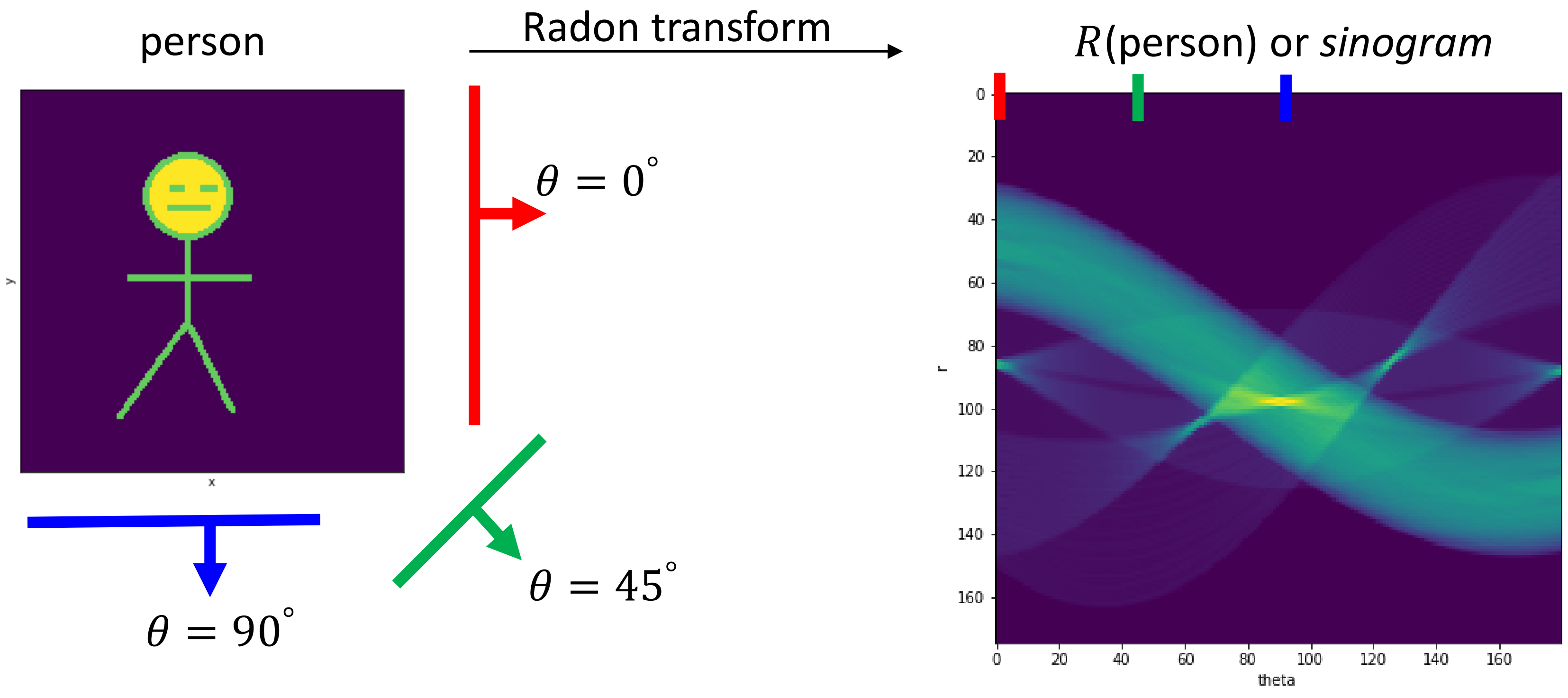
θ



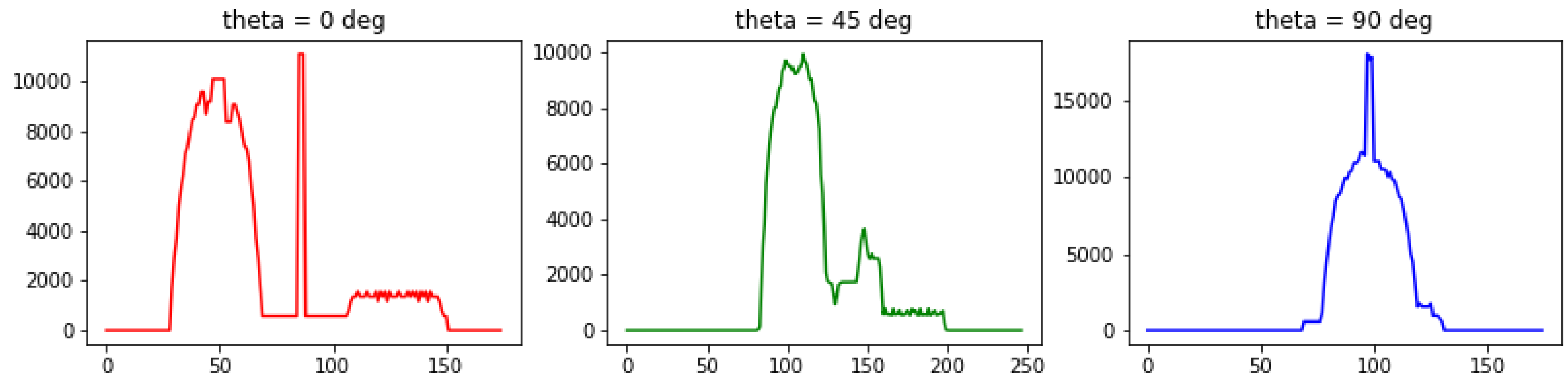
Each *column* in the Radon transform is a *projection* over the entire slice at a given angle θ.
r is the distance from the center of the projection

Radon transform: concept





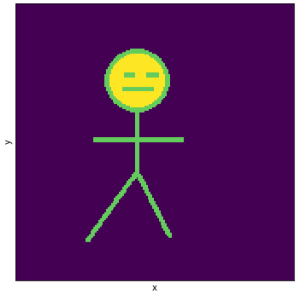
- The CT scanner rotates around the slice, taking projections at various angles
- Each column in the R(person) is a projection or X-ray image through the person at a certain angle.
- R(person) is what the CT scanner outputs (typically, many slices are scanned so there are hundreds of images like this)
- The problem is then: **how to reconstruct the person given all the projections?**
- We need to *invert* the Radon transform, or find $R(person)^{-1}$

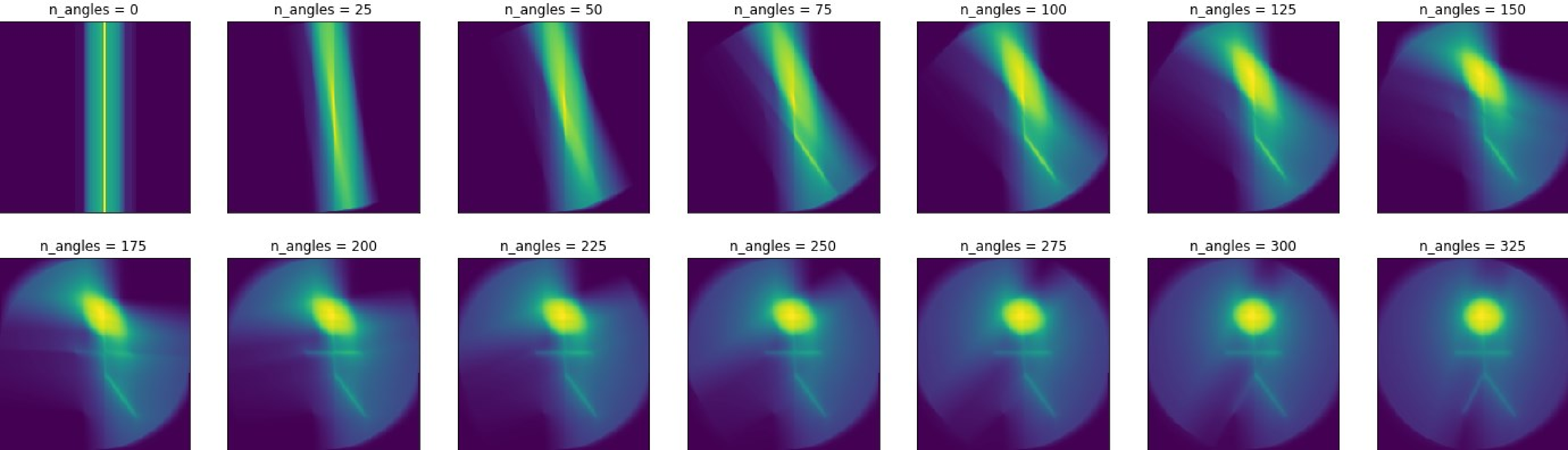


←projections at 3 different angles

Inverting the Radon transform: **backprojection**

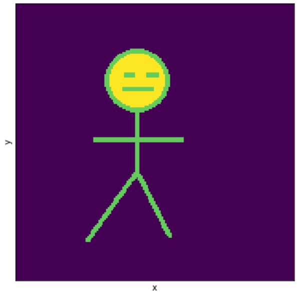
- CT scanner gives projections at multiple angles, still need to reconstruct image
- Problem: low frequencies over-represented
  - Cannot see details (higher frequencies)

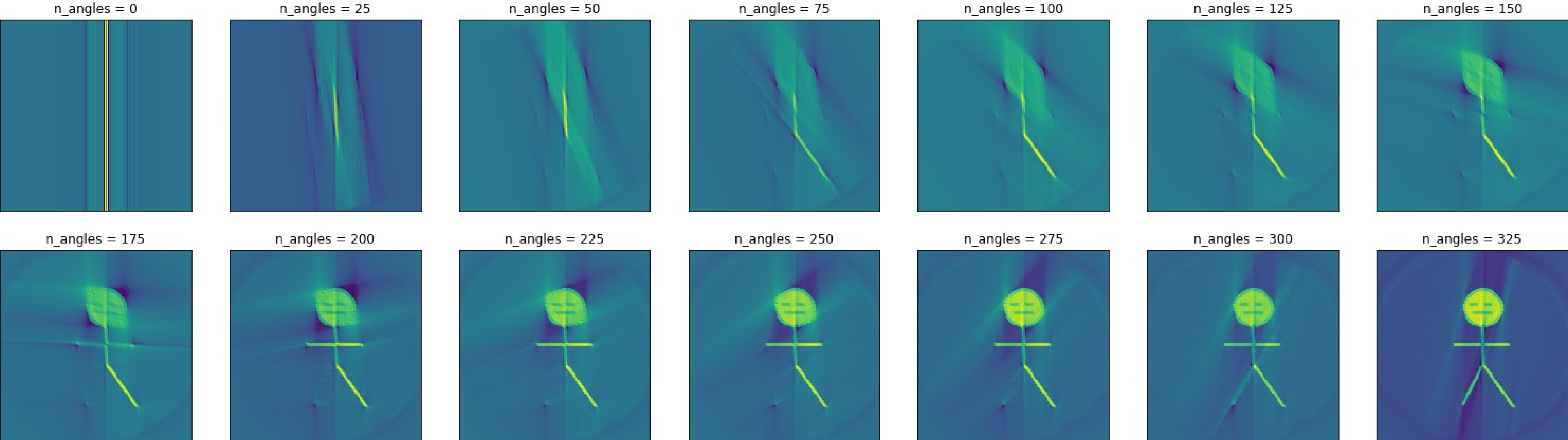




filtered backprojection

- Can reduce low frequency power by multiplying in frequency domain
- Result more closely matches ground truth after filtered backprojection
- More angles = better reconstruction (but also more radiation!)

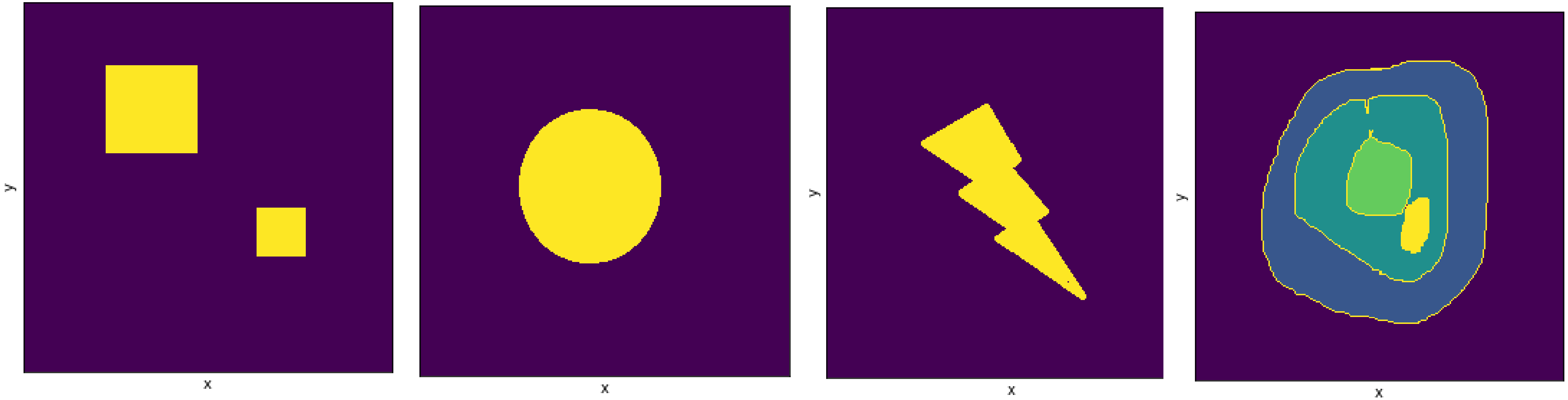




Quiz: match the $R(z)$ to z

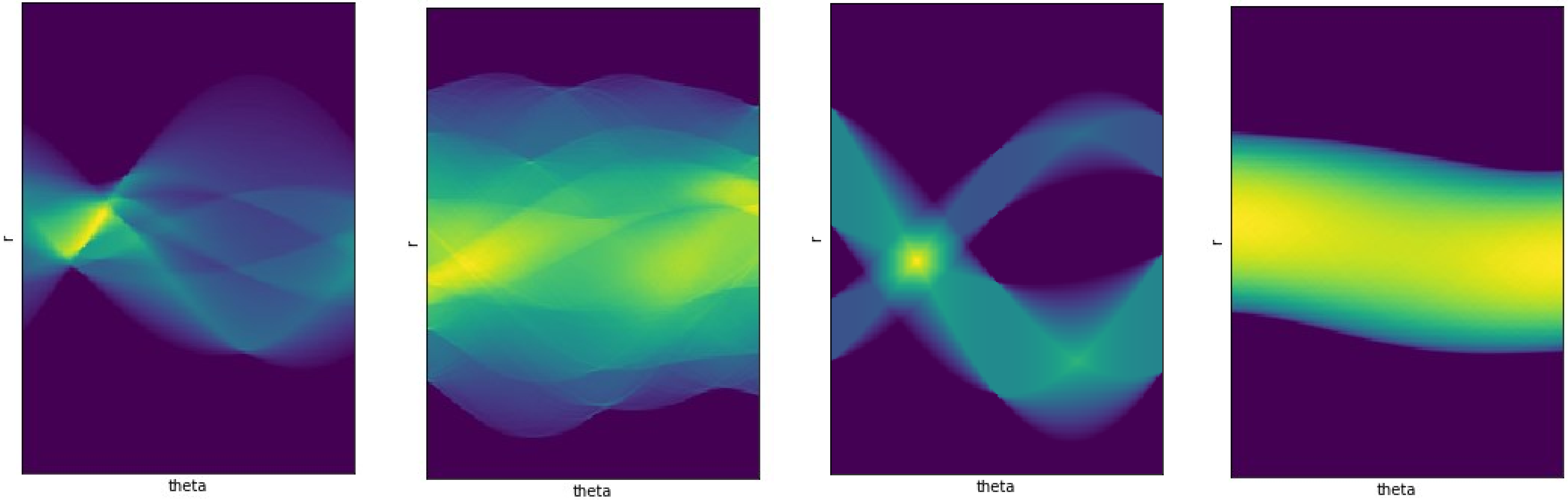
(solution on next slide)
Will have similar question on final ☺

a b c d



z

1 2 3 4



$R(z)$

Computed Tomography (CT) and Radon Transform

- CT produces solution of inverse Radon Transform from projections in a slice
- For 2D attenuation function $\mu(x, y)$ describing X-ray attenuation per unit volume in some slice z in human body, Radon transform is given by all line integrals through this function:
- $R(s, \theta)[\mu(x, y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu(x, y) \delta(s - x cos\theta - y sin\theta) dx dy$
  - Where δ is the Dirac-Delta function
  - for given angle θ, Radon transform produces projections onto line s along rays perpendicular to s with angle θ to the x-axis
- Since Radon transform is invertible, $\mu(x, y)$ may be reconstructed from all projections onto lines s for all angles $0^{\circ} < \theta < 180^{\circ}$

Previous slide quiz solution: $a \rightarrow 3, \mathrm{b} \rightarrow 4, \mathrm{c} \rightarrow 1, \mathrm{d} \rightarrow 2$

Computed Tomography (CT) and Radon Transform

- From X-ray imaging: $I_{out} = I_{in} \cdot \exp(-\int_{t_0}^{t_1} \mu(t)dt)$, therefore we have:
- $\int_{t_0}^{t_1} \mu(t)dt = -\ln(\frac{I_{out}}{I_{in}})$, with $t = s - xcos\theta - ysin\theta$
- Assuming no attenuation outside $[t_1, t_2]$, can extend integral bounds to infinity:
- $$R(s,\theta)[\mu(x,y)] = \int_{-\infty}^{\infty} \mu(t)dt$$
  $$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu(x,y)\delta(s - xcos\theta - ysin\theta)$$
  - Can be computed from radiographic projections at angles $0^{\circ} < \theta < 180^{\circ}$
  - With enough of these projections, Radon transform can be inverted, reconstruction 3d distribution of *attenuation coefficients*

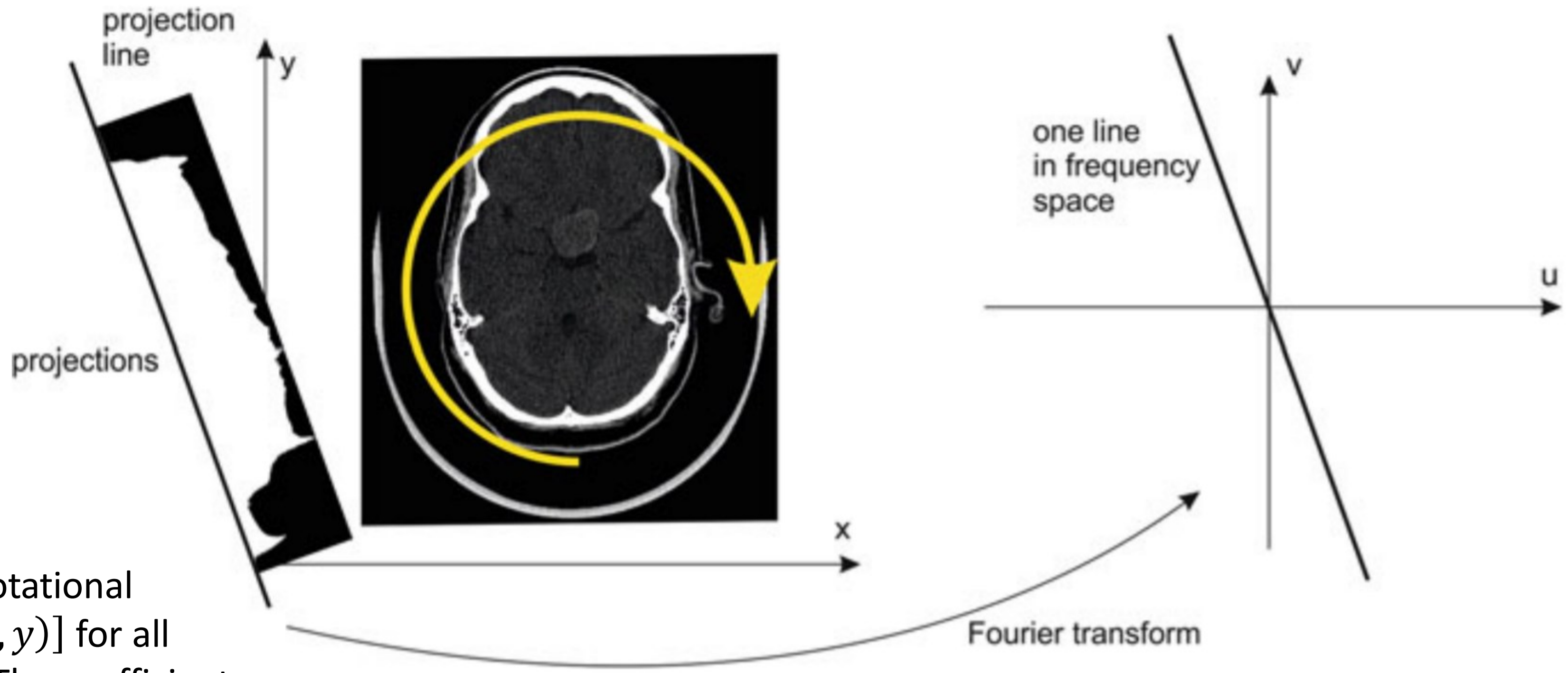
CT image reconstruction

(We will cover the Fourier transform in detail later, for now just accept it)

- **Central slice theorem:** the Fourier transform $M(u, v)$ of a projection of a function $\mu(x, y)$ in a direction with angle θ to the x-axis equals the coefficients on a line with this angle θ in frequency space of the Fourier transform of μ

Example with $\theta = 0$:

$$FFT(R(s,0)(\mu(x,y))$$
$$= FFT[\int_{-\infty}^{\infty} \mu(x,0)dx]$$
$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \mu(x,0)\, dx\right] \exp(-ivy)\, dy$$
$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu(x,0) x exp(-i(0u + vy)dxdy$$
$$= M(x,0)$$

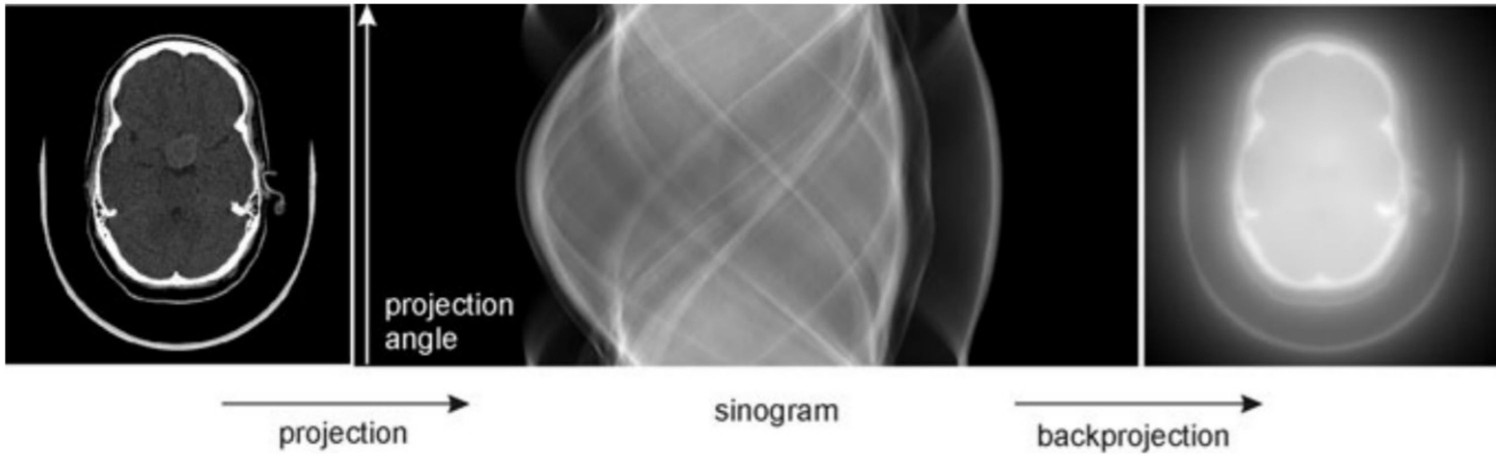


The theorem is true for all other angles θ as well (rotational property of FFT). Hence, projections of $R(s,\theta)[\mu(x,y)]$ for all angles θ are transformed into 1d frequency space. The coefficients for a projection $R(s,\theta)$ are mapped on the corresponding line with angle θ in 2d frequency space. The inverse Fourier transform is then applied for computing the reconstructed image.

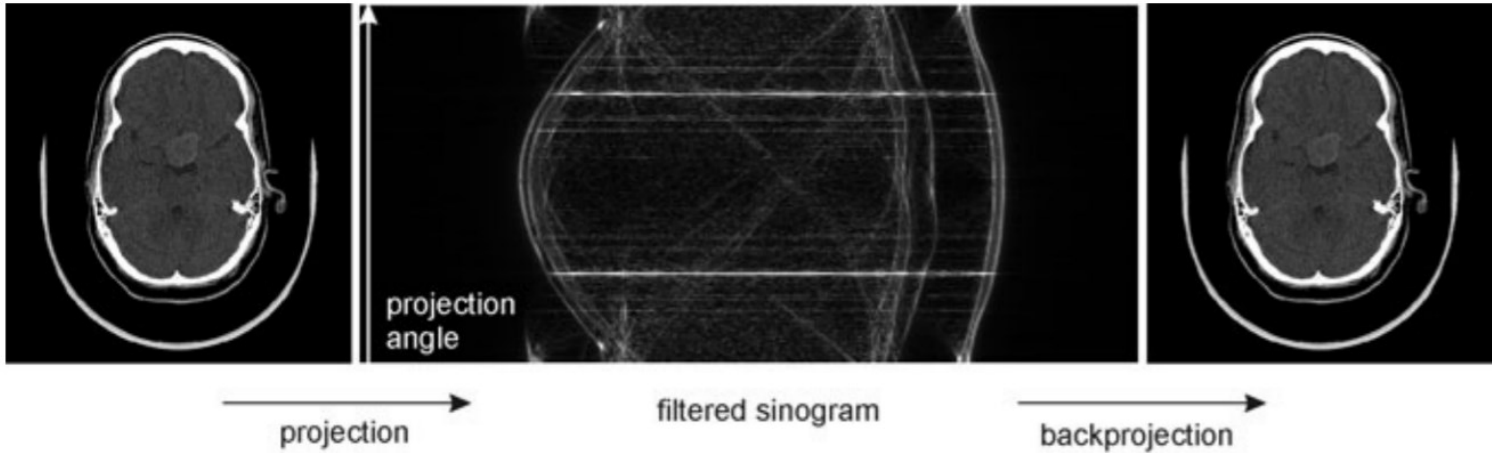
CT image reconstruction

- **Problem with approach on previous slide**:
- Fourier transform from projections is defined in a polar coordinate system, but we require coefficients for carrying out inverse transform to be in cartesian coordinates.
- Need to account for difference in size of a unit area in polar and cartesian coordinates, otherwise low-frequency components in image would be over-emphasized (a).
- Correct for this by multiplying Fourier coefficients with $1/r$ where r is radial distance of given location (u, v) to origin, this is also known as a *ramp filter.* (b)

a)



b)



CT image reconstruction

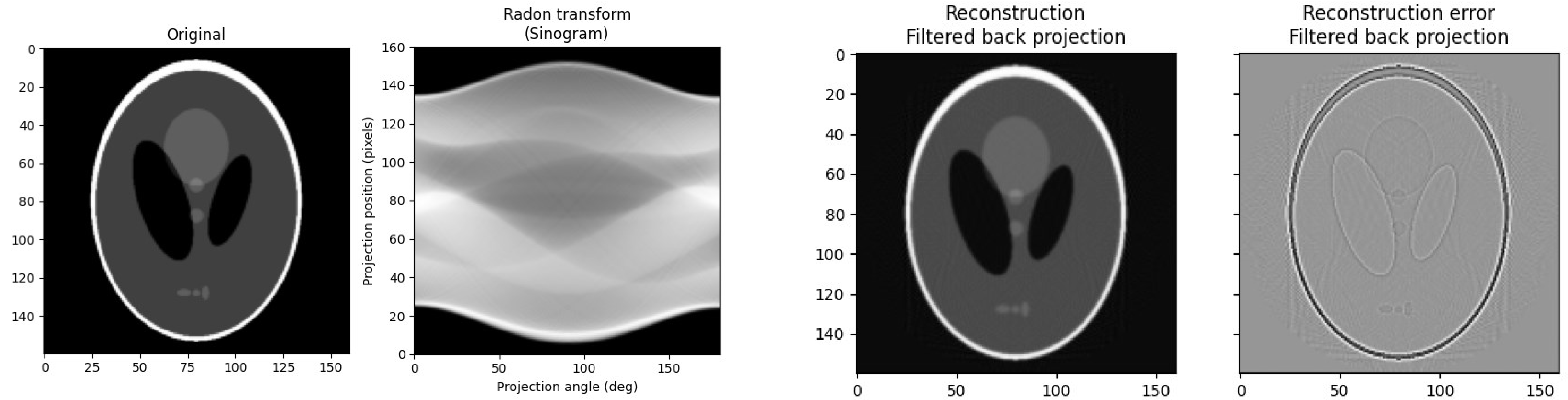
- Reconstruction results in digital image of attenuation coefficients for each voxel
- Common CT image sizes are 512x512 or 256x256
- Size of voxel in x,y axes depends on field-of-view (FOV) which depends on opening angle of cone beam scanning the patient
- For brain images, with FOV=30x30 cm^2, in-plane voxel sizes for a 512x512 image are 0.5x0.5 mm^2
- Attenuation coefficients normalized based on attenuation μ_{water} of water and μ_{air} of air: $HU(\mu) = 1000 \cdot \frac{\mu - \mu_{water}}{\mu_{water} - \mu_{air}}$
  - Air has -1000 HU, and water has 0 HU.
  - Hounsfield units mapped to integers, usually represented in range -1000 to 3000

Hounsfield units for different tissue types:

Air	Fat	Water	Blood	Muscle	White matter	Gray matter	CSF	Bone
−1000	−100	0	30–45	40	20–30	37–45	15	>150

Radon transform in scikit-image

- https://scikit-image.org/docs/dev/auto_examples/transform/plot_radon_transform.html



"A projection is formed by drawing a set of parallel rays through the 2D object of interest, assigning the integral of the object's contrast along each ray to a single pixel in the projection. A single projection of a 2D object is one dimensional. To enable computed tomography reconstruction of the object, several projections must be acquired, each of them corresponding to a different angle between the rays with respect to the object. A collection of projections at several angles is called a sinogram, which is a linear transform of the original image."

(for inverting Radon transform)

iradon function

```
"""
Inverse radon transform.

Reconstruct an image from the radon transform, using the filtered
back projection algorithm.

Parameters
----------
radon_image : array_like, dtype=float
    Image containing radon transform (sinogram). Each column of
    the image corresponds to a projection along a different angle. The
    tomography rotation axis should lie at the pixel index
    ``radon_image.shape[0] // 2`` along the 0th dimension of
    ``radon_image``.
theta : array_like, dtype=float, optional
    Reconstruction angles (in degrees). Default: m angles evenly spaced
    between 0 and 180 (if the shape of `radon_image` is (N, M)).
output_size : int, optional
    Number of rows and columns in the reconstruction.
filter : str, optional
    Filter used in frequency domain filtering. Ramp filter used by default.
    Filters available: ramp, shepp-logan, cosine, hamming, hann.
    Assign None to use no filter.
interpolation : str, optional
    Interpolation method used in reconstruction. Methods available:
    'linear', 'nearest', and 'cubic' ('cubic' is slow).
circle : boolean, optional
    Assume the reconstructed image is zero outside the inscribed circle.
    Also changes the default output_size to match the behaviour of
    ``radon`` called with ``circle=True``.

Returns
-------
reconstructed : ndarray
    Reconstructed image. The rotation axis will be located in the pixel
    with indices
    ``(reconstructed.shape[0] // 2, reconstructed.shape[1] // 2)``.

References
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.. [1] AC Kak, M Slaney, "Principles of Computerized Tomographic
       Imaging", IEEE Press 1988.
.. [2] B.R. Ramesh, N. Srinivasa, K. Rajgopal, "An Algorithm for Computing
       the Discrete Radon Transform With Some Applications", Proceedings of
       the Fourth IEEE Region 10 International Conference, TENCON '89, 1989

Notes
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It applies the Fourier slice theorem to reconstruct an image by
multiplying the frequency domain of the filter with the FFT of the
projection data. This algorithm is called filtered back projection.

"""
```

https://github.com/scikit-image/scikit-image/blob/master/skimage/transform/radon_transform.py

iradon code implementing filtered back-projection

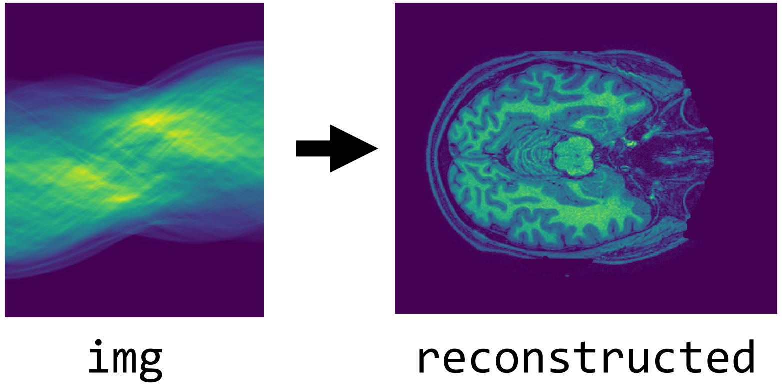
```
fourier_filter = _get_fourier_filter(projection_size_padded, filter_name)
projection = fft(img, axis=0) * fourier_filter
radon_filtered = np.real(ifft(projection, axis=0)[:img_shape, :])

reconstructed = np.zeros((output_size, output_size))
radius = output_size // 2
xpr, ypr = np.mgrid[:output_size, :output_size] - radius
x = np.arange(img_shape) - img_shape // 2

for col, angle in zip(radon_filtered.T, np.deg2rad(theta)):
    t = ypr * np.cos(angle) - xpr * np.sin(angle)
    if interpolation == 'linear':
        interpolant = partial(np.interp, xp=x, fp=col, left=0, right=0)
    else:
        interpolant = interp1d(x, col, kind=interpolation,
                               bounds_error=False, fill_value=0)
    reconstructed += interpolant(t)

if circle:
    out_reconstruction_circle = (xpr ** 2 + ypr ** 2) > radius ** 2
    reconstructed[out_reconstruction_circle] = 0.

return reconstructed * np.pi / (2 * angles_count)
```



https://github.com/scikit-image/scikit-image/blob/master/skimage/transform/radon_transform.py

Summary slide: