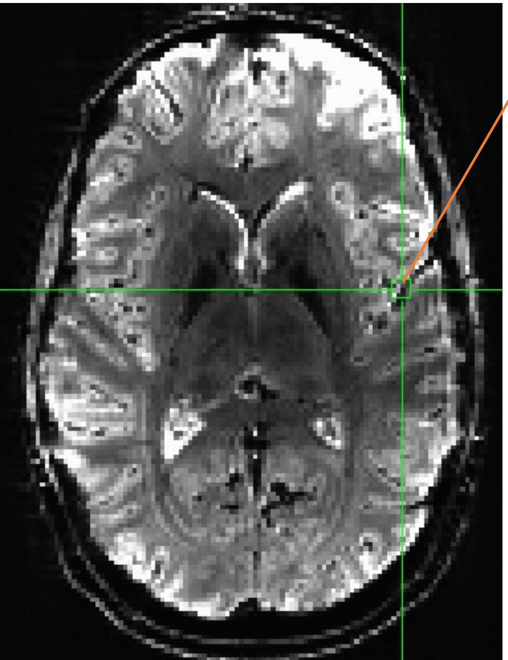
CS463/516

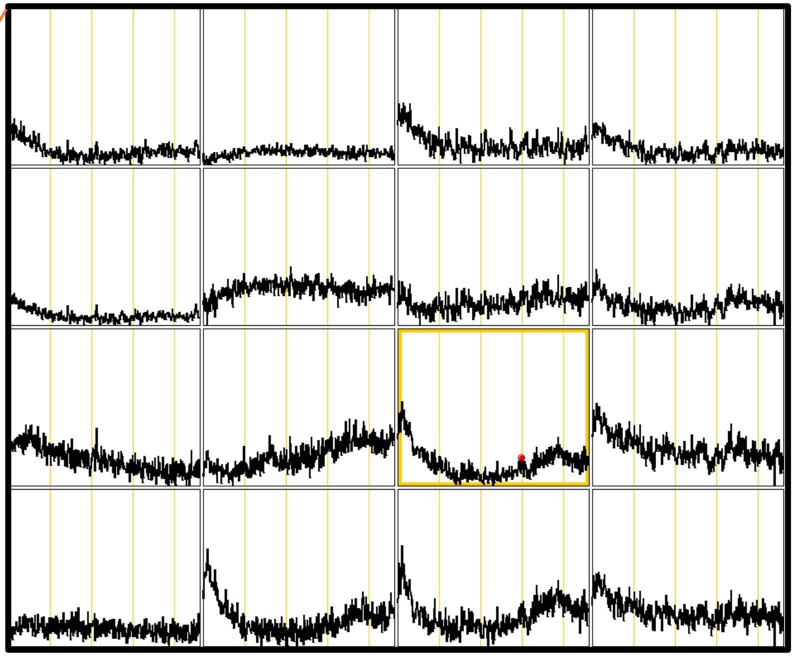
Lecture 10

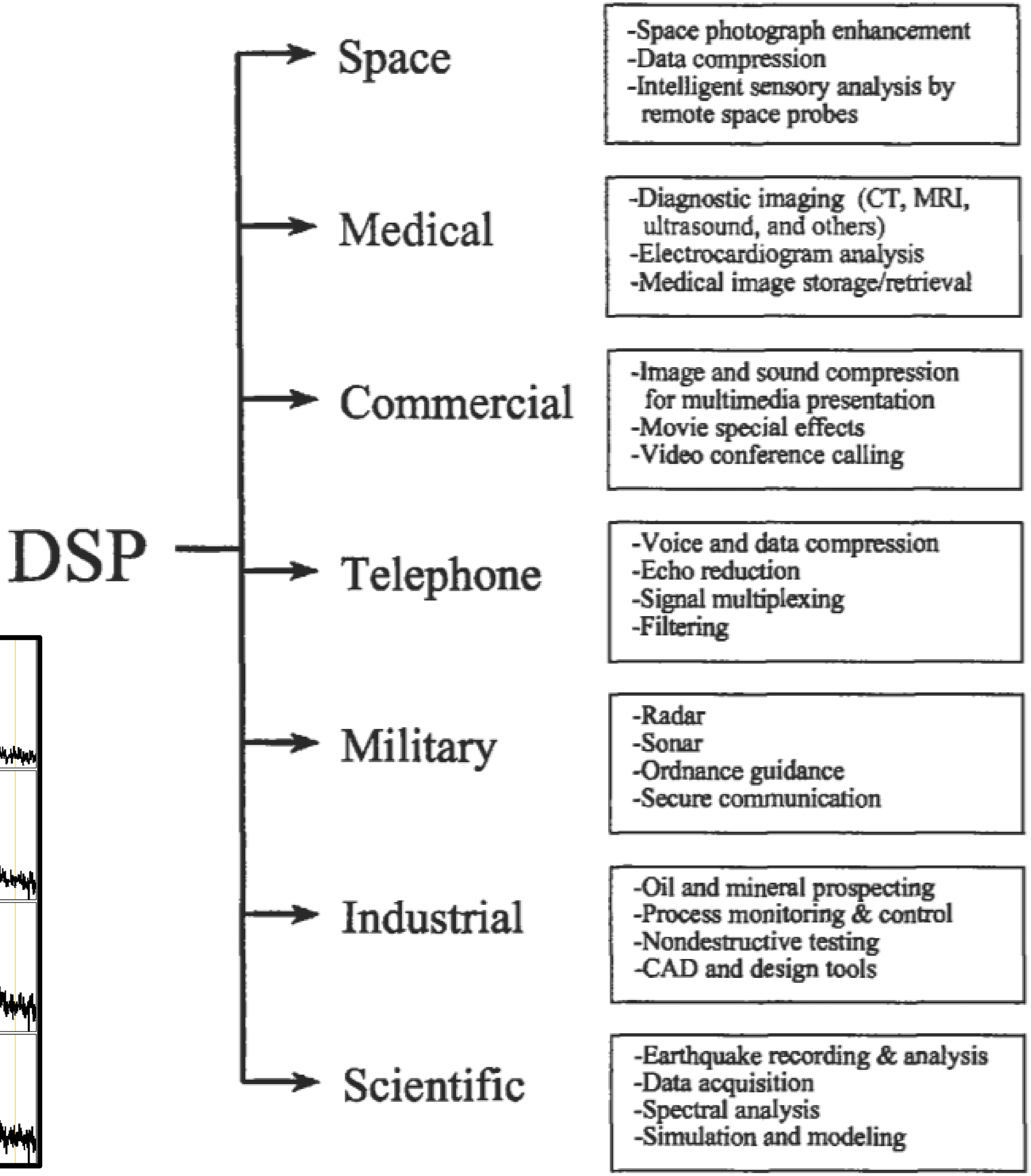
Basic digital signal processing and BOLD fMRI

Digital signal processing

- Digital signal processing is a basic skill needed by scientists/engineers in many fields
- Images are signals measured over space
- Medical *imaging* is an umbrella term for all methods of acquiring digital signals from human body
- Can have 1d, 2d, 3d, or even 4d signals. Example: BOLD fMRI: a 4d image, where each voxel has a time series (digital signal)



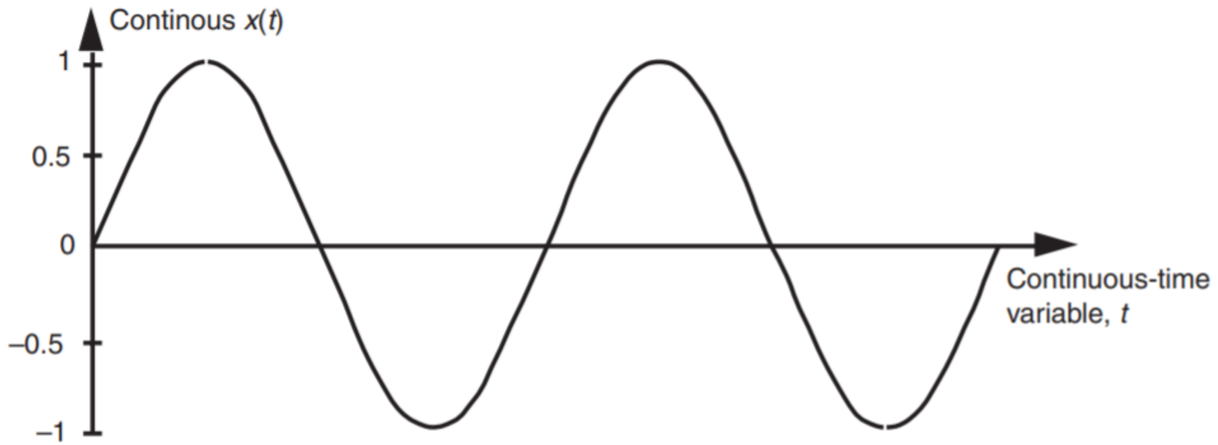




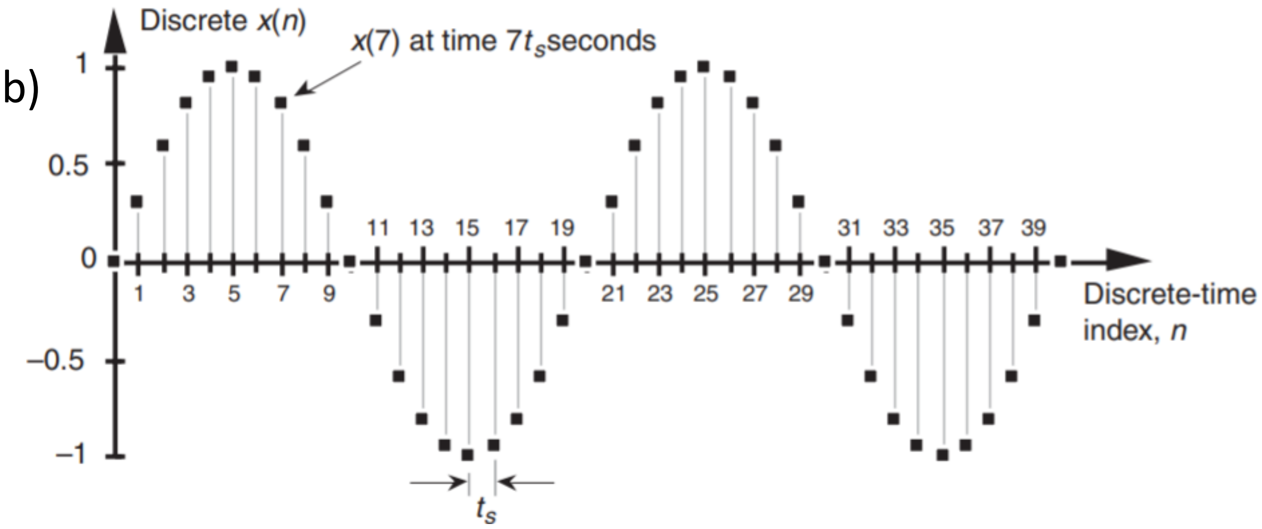
Continuous and discrete signals

- Continuous time signal: a function of an independent variable that is continuous
- 1d continuous time signal $x(t)$ expressed as function of time varying continuously from $-\infty$ to ∞
- a) example of continuous time function
- Discrete time signal: function defined only at discrete instants of time, undefined at all other times
  - Defined at equal intervals of time at $t = nT$ where T is a fixed interval in seconds known as sampling period and n an integer variable defined over $-\infty$ to ∞
- b) example of discrete-time function $x(n)$

a)



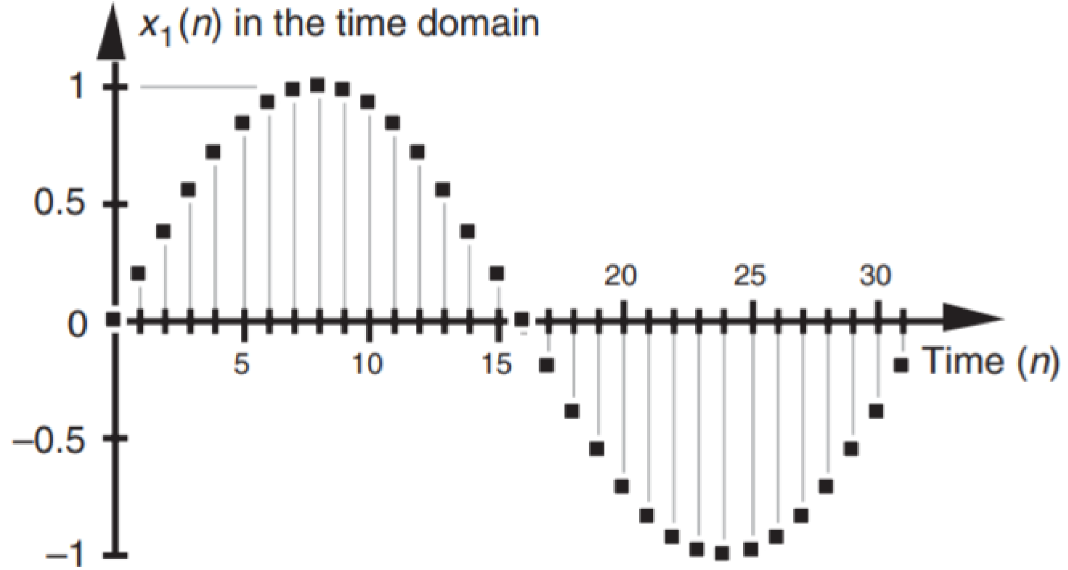
b)



Amplitude and magnitude

- Amplitude of a variable is a measure of how far, and in what direction, the variable differs from zero
- a) amplitude can be either positive or negative $x_1(n)$
- Magnitude of variable is measure of how far, regardless of direction, its quantity differs from zero
  - Always positive
- b) magnitude is equal to amplitude, but with the sign always positive, $|x_1(n)|$

a)



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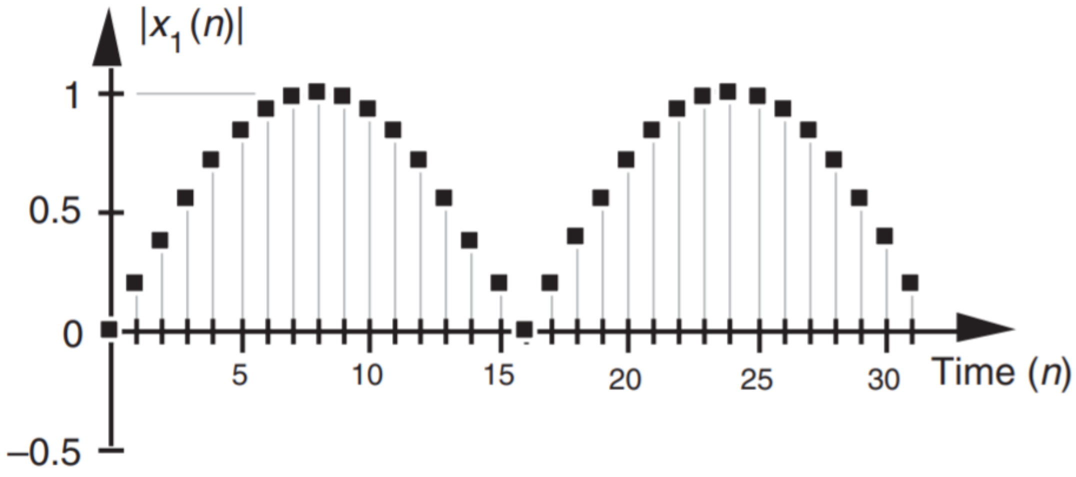
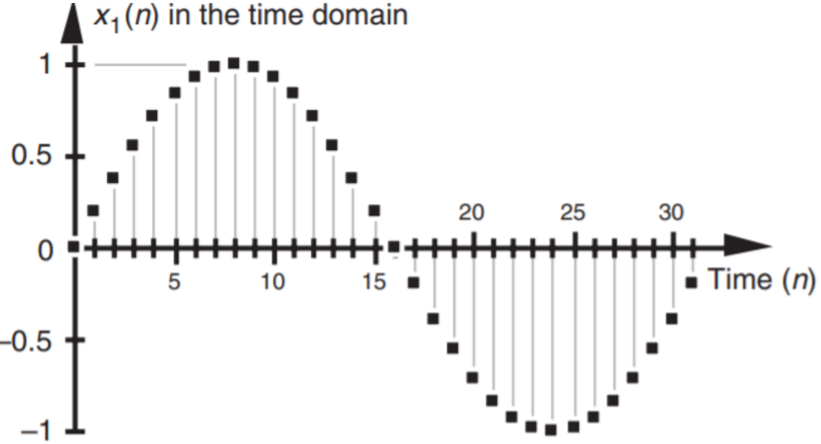
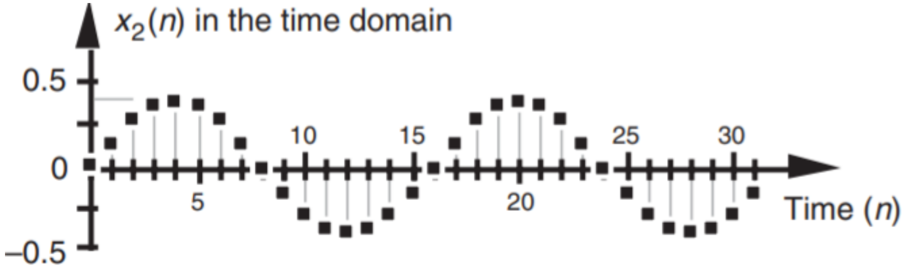
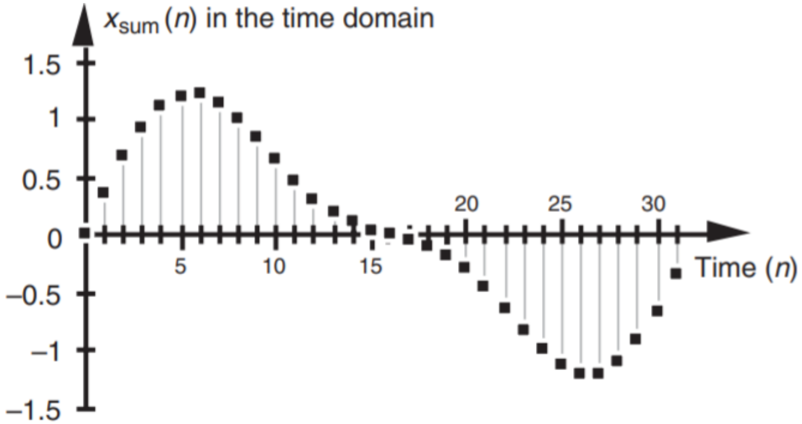
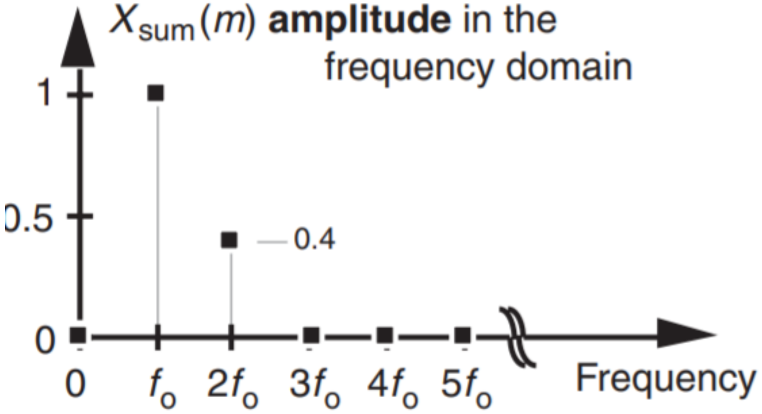


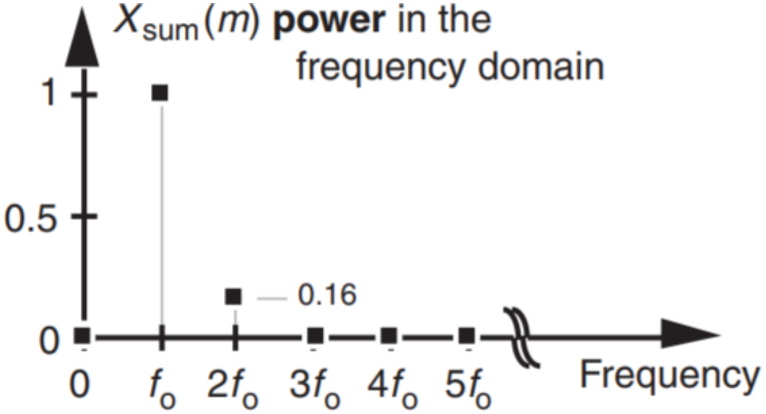
Power of a signal







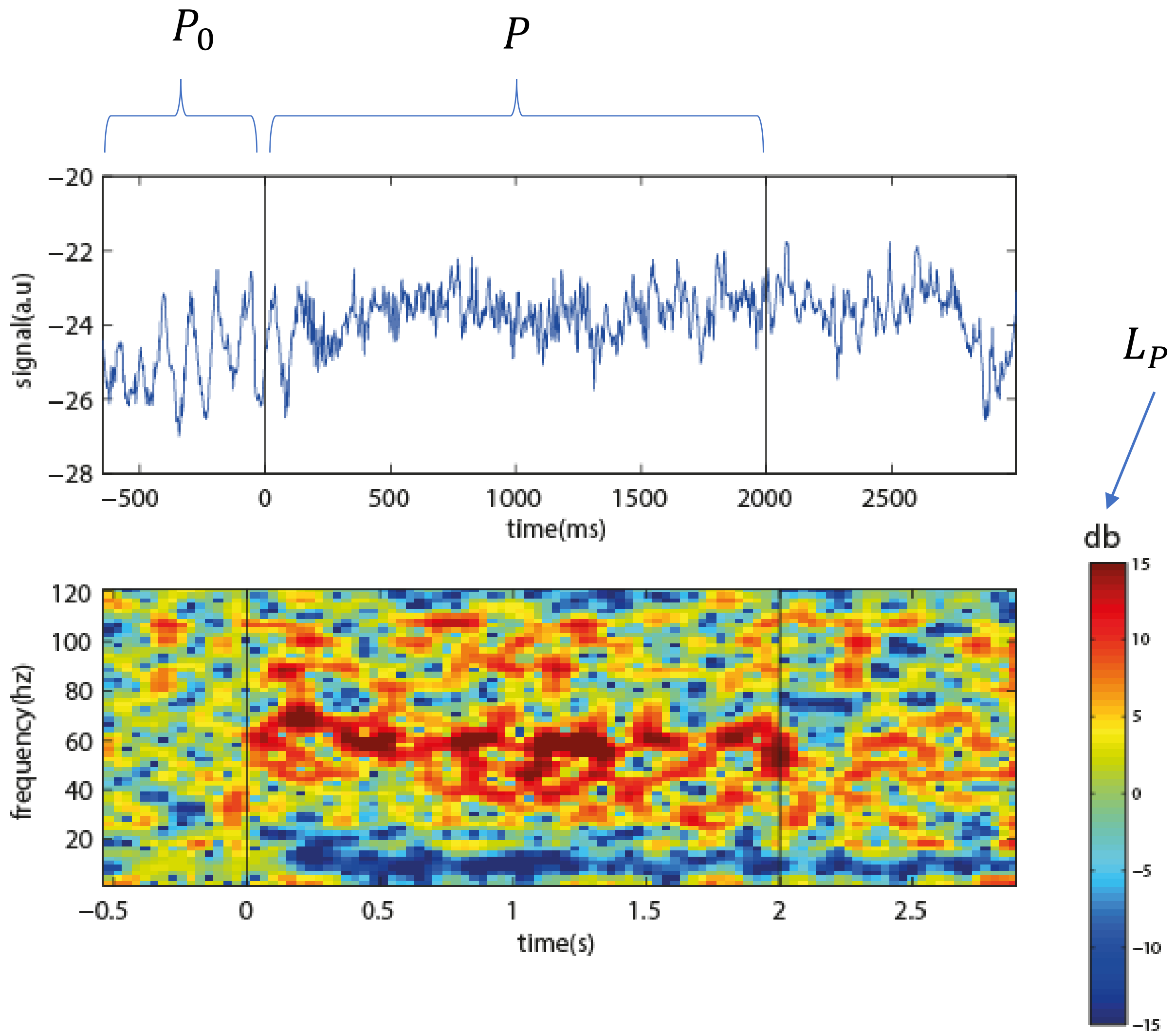




- When we examine signals in frequency domain, often interested in *power level* of a signal
- Power of signal is proportional to its amplitude (or magnitude) squared: $X_{pwr}(m) = |X(m)|^2$

Power quantities and decibels

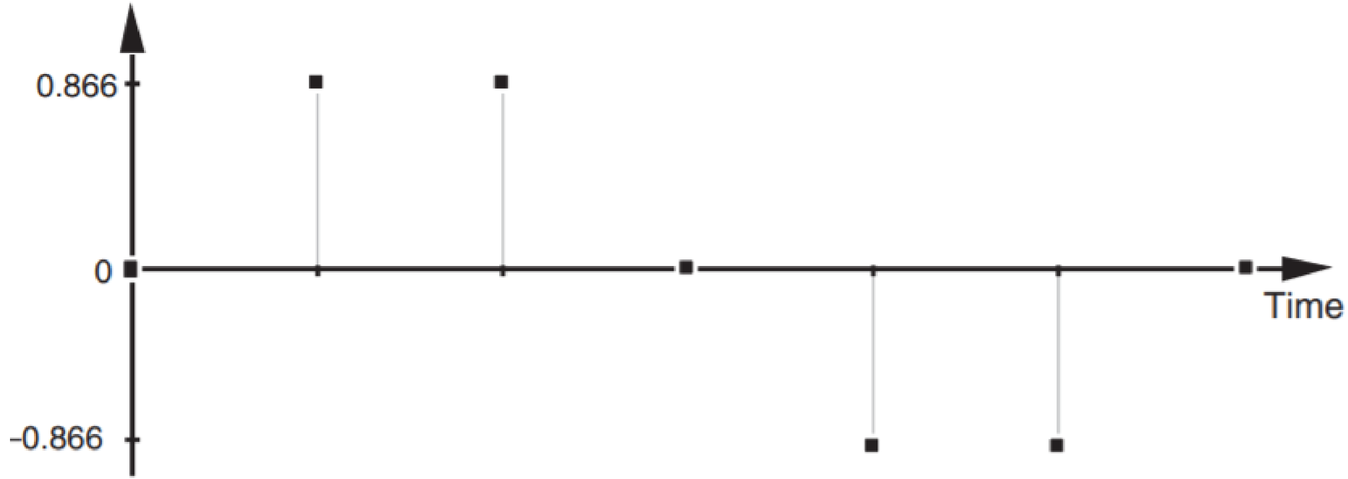
- When referring to measurements of power quantities, a ratio can be expressed as a level (logarithmic quantity) in decibels by evaluating 10 times the base-10 logarithm of the ratio of the measured quantity to a reference value
- Thus, ratio of P (measured power) to P_0 (reference power) is expressed in decibels by L_P :
- $L_P = 10\log_{10}\left(\frac{P}{P_0}\right) dB$
- If $P > \mathrm{P}_0$, L_P is positive
- If $P < \mathrm{P}_0$, L_P is negative

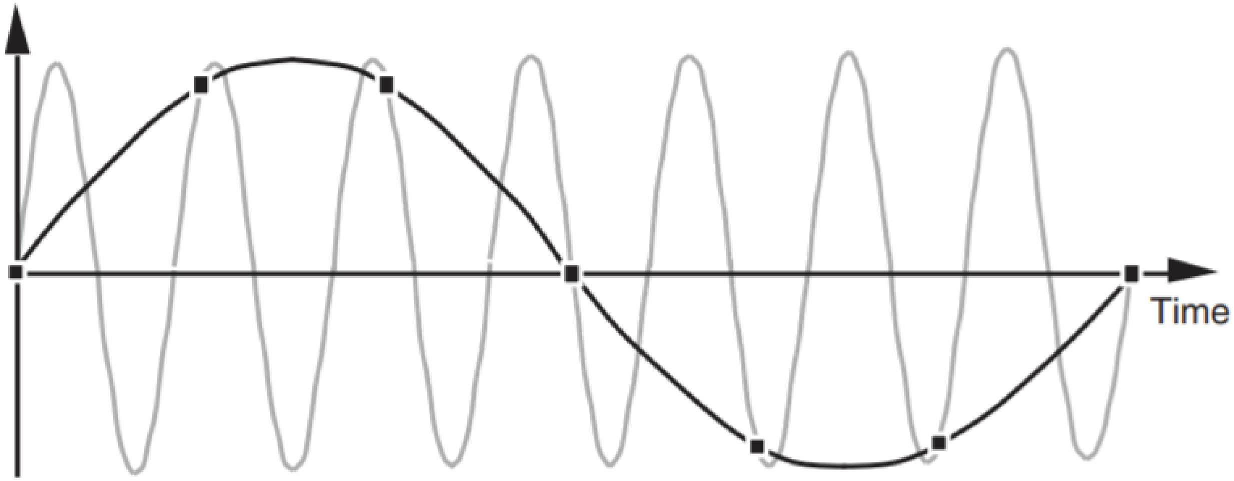


aliasing

aliasing: otherwise called **:** otherwise known as —used to indicate an additional name that a person (such as a criminal) sometimes uses
(from Merriam-Webster Dictionary)

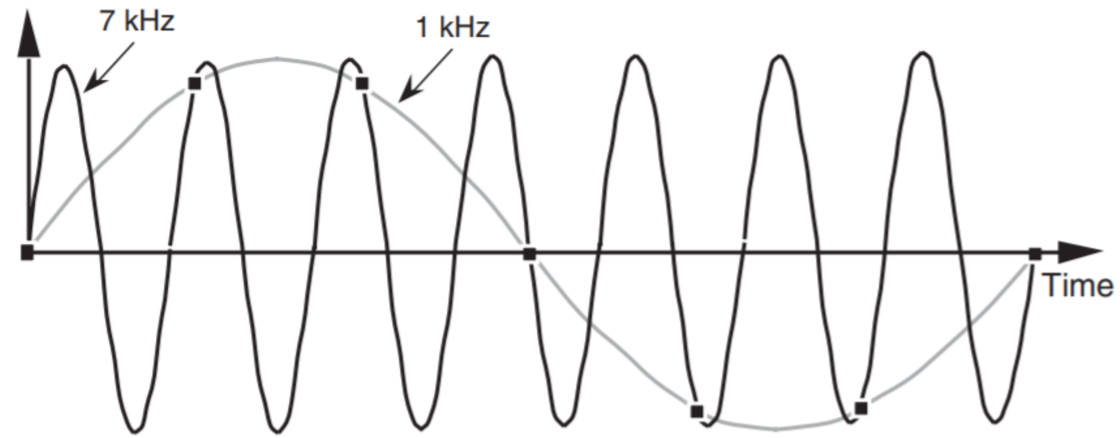
- Periodic sampling is the process of representing a continuous signal with a sequence of discrete data values
- Question: How fast do we need to sample (how many samples per second?)
- Answer: depends on the frequency of the signal we want to measure
- **Theorem:** when sampling at a rate of f_s samples/second, if k is any positive or negative integer, we cannot distinguish between the sampled values of a sinewave of f_0 Hz and a sinewave of $(f_0 + kf_s)$ Hz

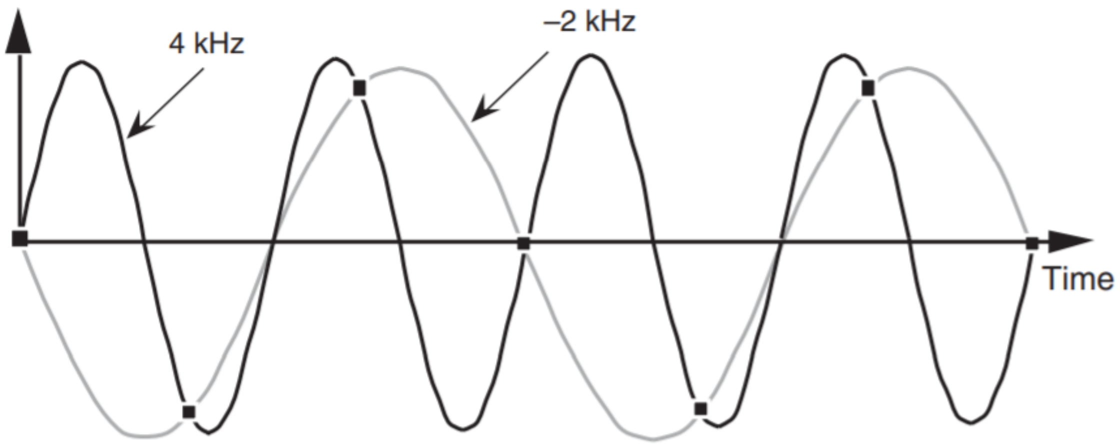




Frequency ambiguity effects

- a) sampling a 7 kHz sinewave at a sample rate of 6 kHz
  - Impossible to know if the sequence of sampled values (dots) came from a 7 kHz or 1 kHz sinewave
  - Trying to determine which frequency gave rise to the sampled points is like asking: “when I add two numbers, I get a sum of 4. what are the two numbers?”
- b) sampling a 4 kHz sinewave at a sampling rate of 6 kHz
  - The 4 kHz sinewave could be mistaken for a 2 kHz sinewave (impossible to know)





Prevent aliasing? Nyquist sampling theorem

- **Nyquist sampling theorem** is a theorem in the field of digital signal processing serving as a fundamental bridge between continuous-time signals and discrete-time signals
- Establishes sufficient condition for a *sampling rate* that permits a discrete sequence of samples to capture all information from a continuous time signal of finite bandwidth
- **Theorem**: if a function $x(t)$ contains no frequencies higher than B Hz, it is completely determined by giving its values at a series of points spaced $\frac{1}{2B}$ seconds apart.
- In other words, a sufficient sample-rate is anything larger than $2B$ samples per second. Equivalently, for a given sample rate f_s, perfect reconstruction is guaranteed for a bandwidth $B < \frac{f_s}{2}$
- Example: if signal contains a $1Hz$ wave, need to sample at rate $\geq 2\ Hz$

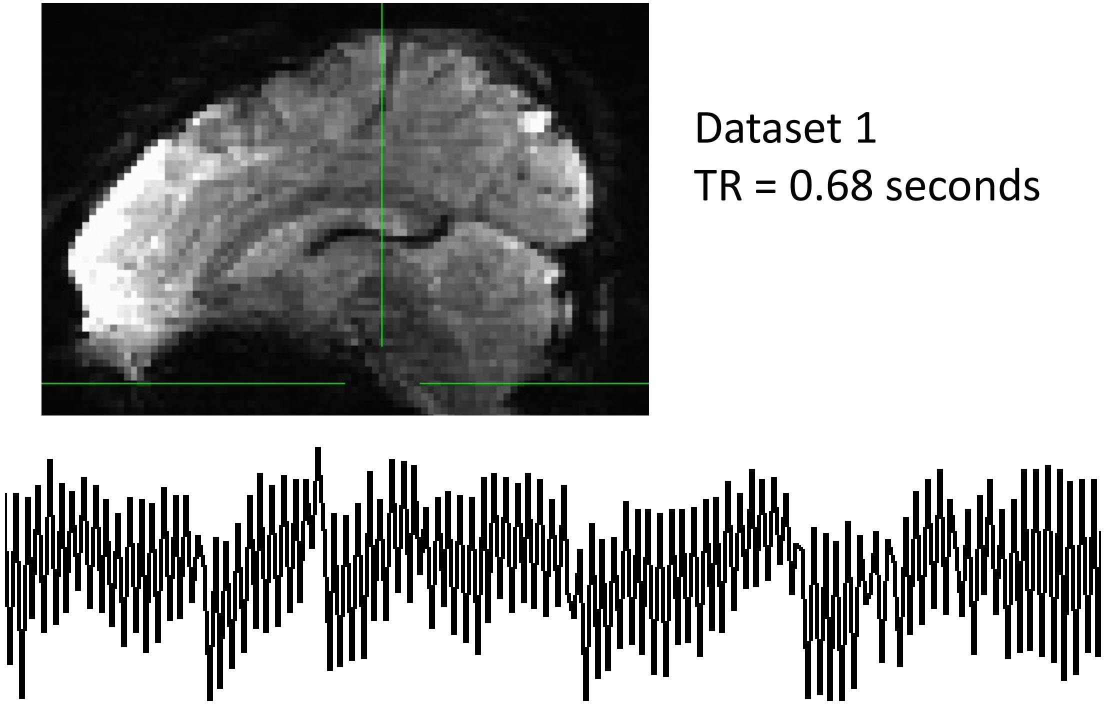
Aliasing in 2d

- In digital photography, if a repetitive pattern of high spatial frequency is sampled at low resolution, Moire patterns occur in the image
- a) picture of fur hat taken with anti-aliasing filter applied
- b) picture of fur hat taken with anti-aliasing filter removed
- This aliasing effect is also known as a Moire pattern
- Digital sound recording equipment contains low-pass filters that remove any signals above half the sampling frequency

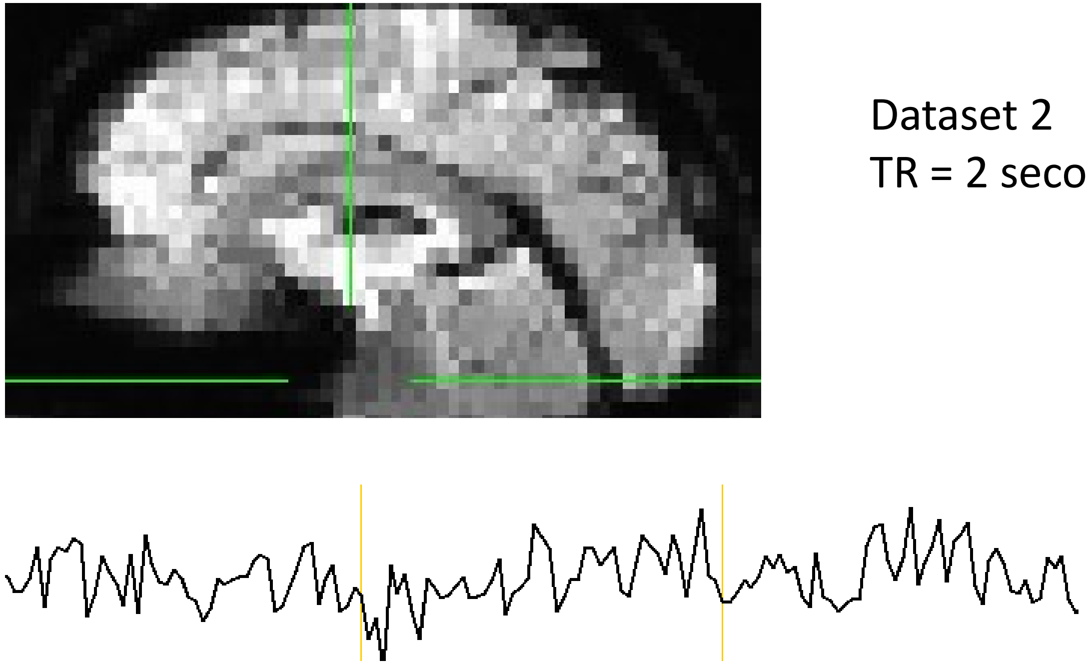
Example: BOLD fMRI

High temporal resolution fMRI by Laura Lewis:
https://www.youtube.com/watch?v=8BHufizQzC8

- BOLD images (3d volumes) are acquired every x milliseconds (typically $x \approx 1000$ ms)
- The time between each volume is called the *repetition time* or TR
- Problem: the heart beats at roughly 60 beats/minute (1 beat per second) which means we need a sampling rate of *at least* 2 Hz to prevent aliasing in BOLD signal
- Example from 2 datasets: signal in the carotid artery of neck (pulsates with heartbeat)



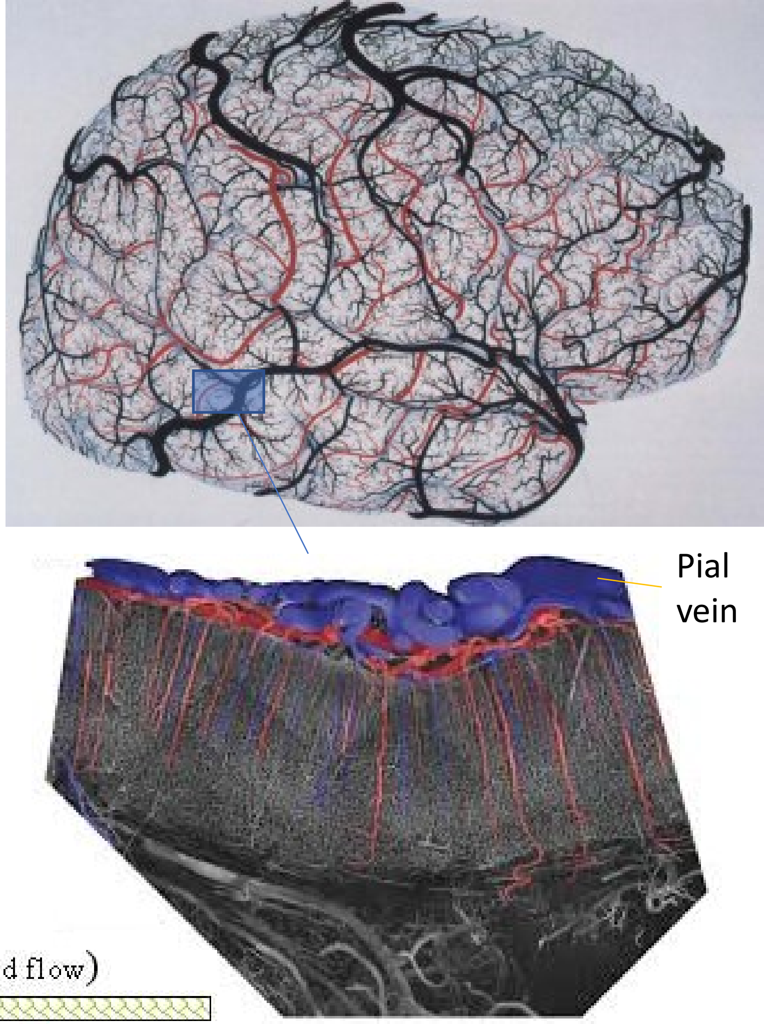
Dataset 1
TR = 0.68 seconds

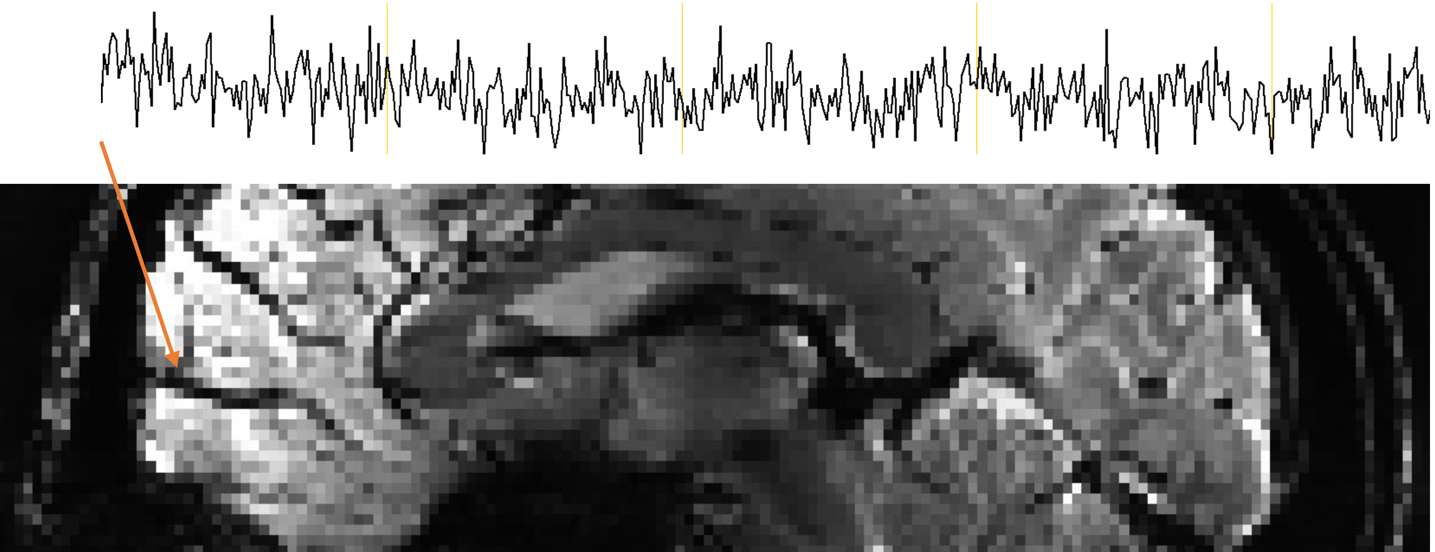


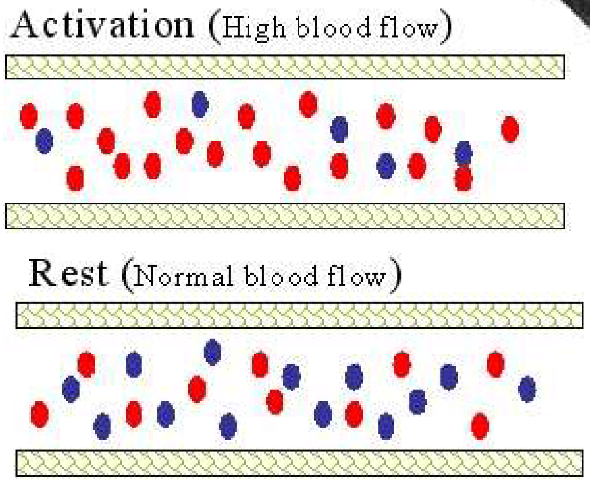
Dataset 2
TR = 2 seconds

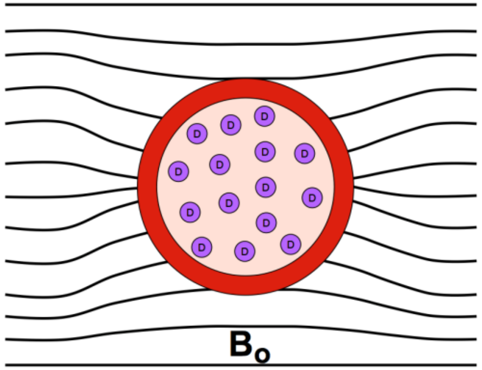
BOLD fMRI

- Since 1990, BOLD fMRI has evolved to become the dominant method for imaging human brain activity (thousands of articles published every year)
- a) The brain is heavily perfused (only 2.5% of total body weight but uses 20% of the body's energy!)
- b) arteries (red) bring fresh blood to brain carrying oxygen, oxygen is transferred to neurons in the brain tissue, veins (blue) take blood away from brain and back to heart
- c) when brain is activated by stimulus, blood flow increases relative to resting state, which results in decreased deoxyhemoglobin
- d) paramagnetic deoxyhemoglobin increases magnetic susceptibility around the vein, resulting in decreased BOLD signal (e) especially in large pial veins









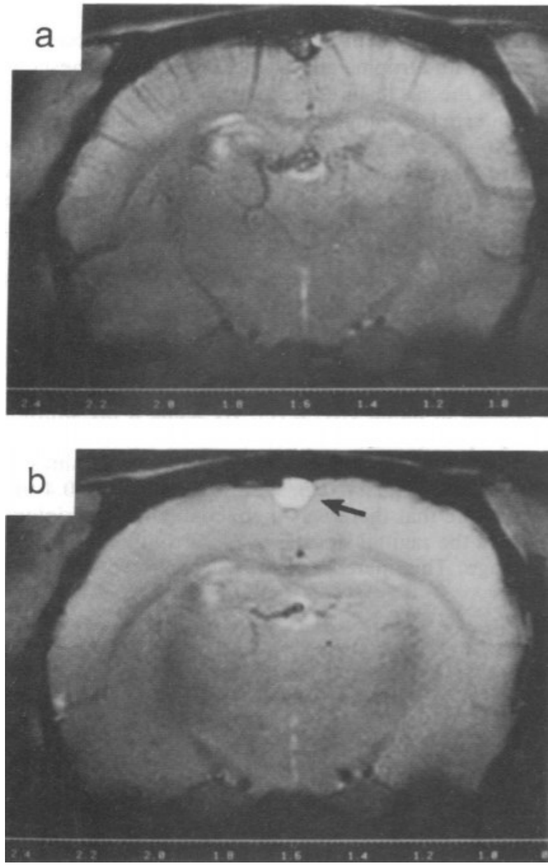
BOLD fMRI origins

Brain magnetic resonance imaging with contrast dependent on blood oxygenation

S Ogawa, T M Lee, A R Kay, and D W Tank

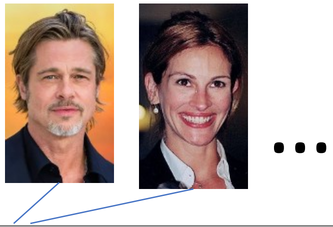
S. Ogawa

- **Blood oxygen level dependent** functional magnetic resonance imaging (BOLD fMRI)
- Discovered in 1990 by Seiji Ogawa who was working on rat brain
- "paramagnetic deoxyhemoglobin in venous blood is a naturally occurring contrast agent for MRI…BOLD contrast can be used to provide in-vivo real time maps of blood oxygenation in the brain under normal physiological conditions"
- a) coronal slice of rat brain, rat was inhaling 100% O_2 gas
- b) coronal slice of same rat, except rat was inhaling gas mixture of 90% O_2 and 10% CO_2
- Arrow points to sagittal sinus, showing increased signal intensity
  - Basically, in this experiment CO_2 inhaled by the rat caused *hypercapnia* (increased blood flow) which washes away the deoxyhemoglobin more quickly, increasing the BOLD signal in that region where arrow points (b)

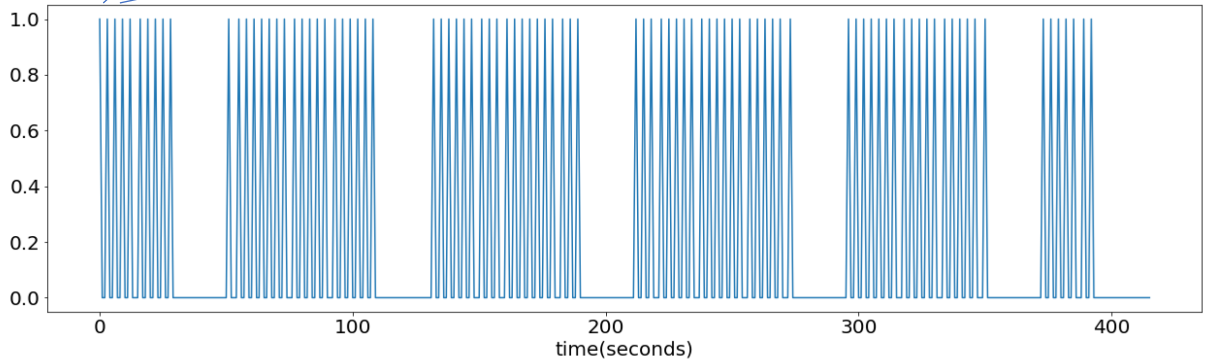


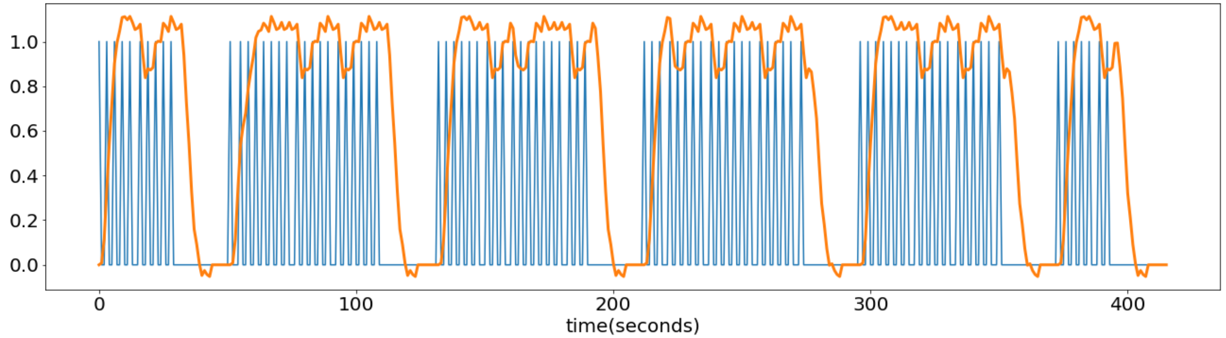
Task-induced BOLD fMRI

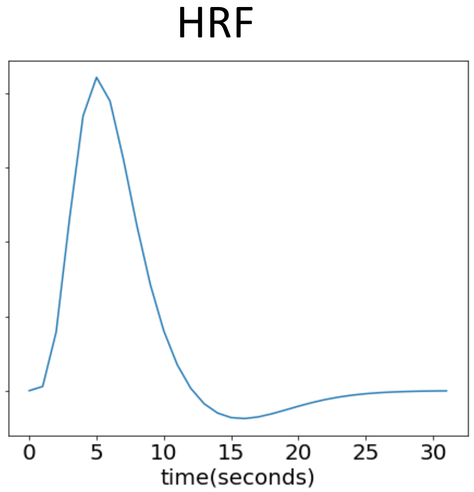
- BOLD fMRI most commonly used to localize which brain area responds to a task
- Example: show subject image of face while in scanner, what region responds?

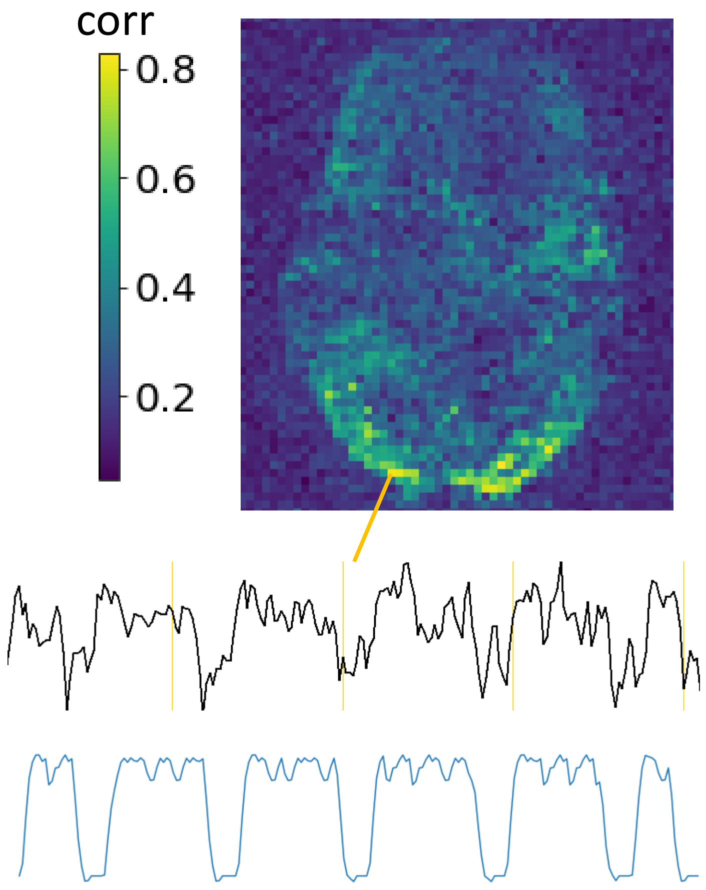


Basic processing pipeline:
a) Get stimulus triggers
b) Convolve with hemodynamic response function (hrf)
c) Correlate convolved triggers with signal in each voxel





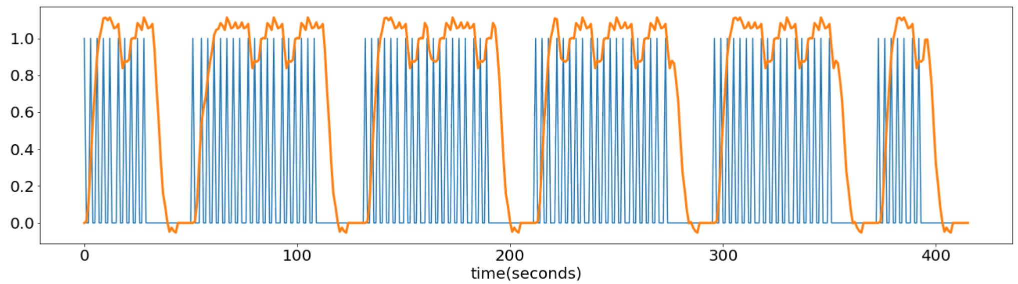




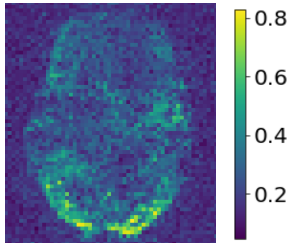
Task-induced BOLD fMRI (simple python example)

```python
import numpy as np
import nibabel as nib
import matplotlib.pyplot as plt
import pandas as pd
import scipy.stats as stats
import scipy.signal as signal
from scipy.ndimage import rotate

plt.rcParams.update({'font.size': 20})


fmri = nib.load('C:/shared/faces/fmri.nii.gz');
events = pd.read_csv('C:/shared/faces/events.tsv',delimiter='\t');
events = events.to_numpy()
tr = fmri.header.get_zooms()[3]

ts = np.zeros(int(tr*fmri.shape[3]))

for i in np.arange(0,events.shape[0]):
    if events[i,3]=='FAMOUS' or events[i,3]=='UNFAMILIAR' or
events[i,3]=='SCRAMBLED':
        ts[int(events[i,0])] = 1

plt.plot(ts); plt.xlabel('time(seconds)')
```

```
In [69]: events
Out[69]:
     onset  duration  ...  response_time       stim_file
0    0.000     0.908  ...          2.158    func/f013.bmp
1    3.273     0.962  ...          1.233    func/f013.bmp
2    6.647     0.825  ...          1.183    func/u014.bmp
3    9.838     0.968  ...          0.930    func/u014.bmp
4   12.978     0.904  ...          1.068    func/u016.bmp
..     ...       ...  ...            ...              ...
94 382.786     0.995  ...          1.003    func/u015.bmp
95 385.993     0.976  ...          1.536    func/u005.bmp
96 389.300     0.906  ...          1.078    func/u009.bmp
97 392.508     0.957  ...          1.223    func/f016.bmp
98 395.264     0.012  ...          0.000  func/Circle.bmp

[99 rows x 8 columns]
```

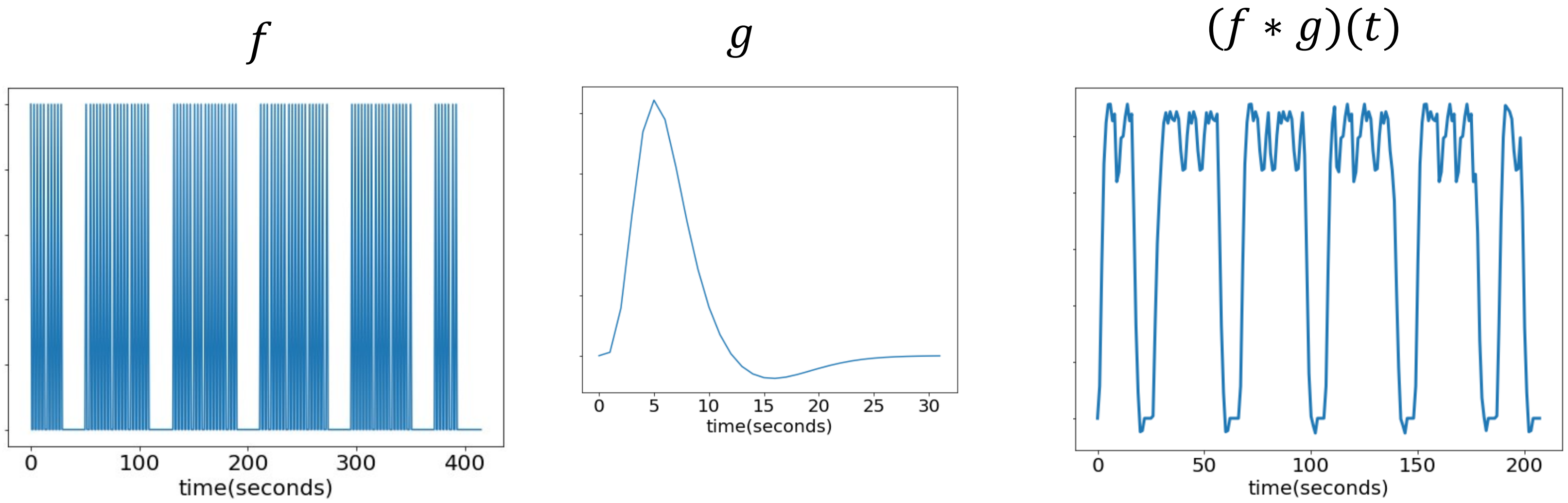


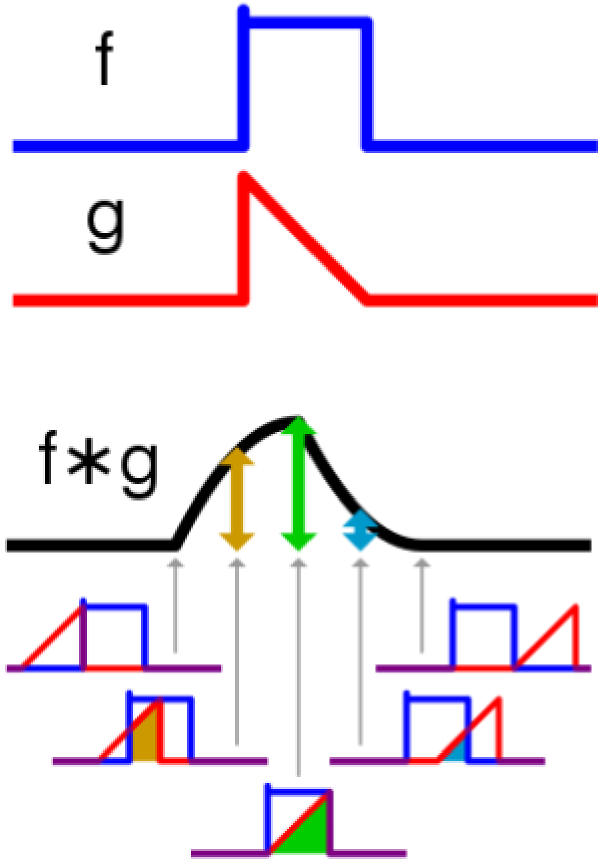

```python
conved = signal.convolve(ts,hrf,mode='full')
conved = conved[0:ts.shape[0]]
plt.plot(ts)
plt.plot(conved*3.2,lineWidth=3); plt.xlabel('time(seconds)')

conved = conved[0::2]

img = fmri.get_data()

meansub_img = img - np.expand_dims(np.mean(img,3),3)
meansub_conved = conved - np.mean(conved)

corrs = np.sum(meansub_img*meansub_conved,3)/   \

(np.sqrt(np.sum(meansub_img*meansub_img,3))*np.sqrt(np.sum(meansub_conved*meansub_conved)))

plt.imshow(np.rot90(np.max(corrs,axis=2))) ;plt.colorbar()
```



convolution

- Convolution is a mathematical operation on two functions f and g that produces a third function expressing how the shape of one is modified by the other
- $(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau$



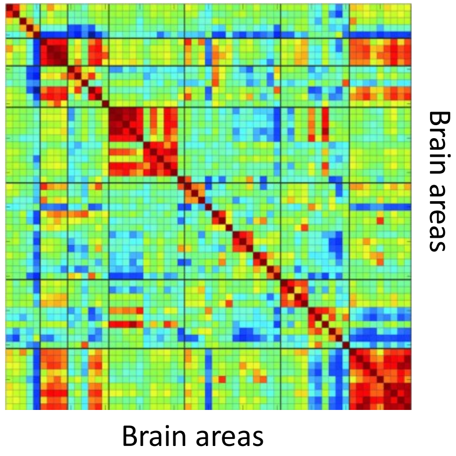


- In assignment 1, we actually performed a convolution by *point-wise multiplication in the frequency domain*

Resting state BOLD fMRI

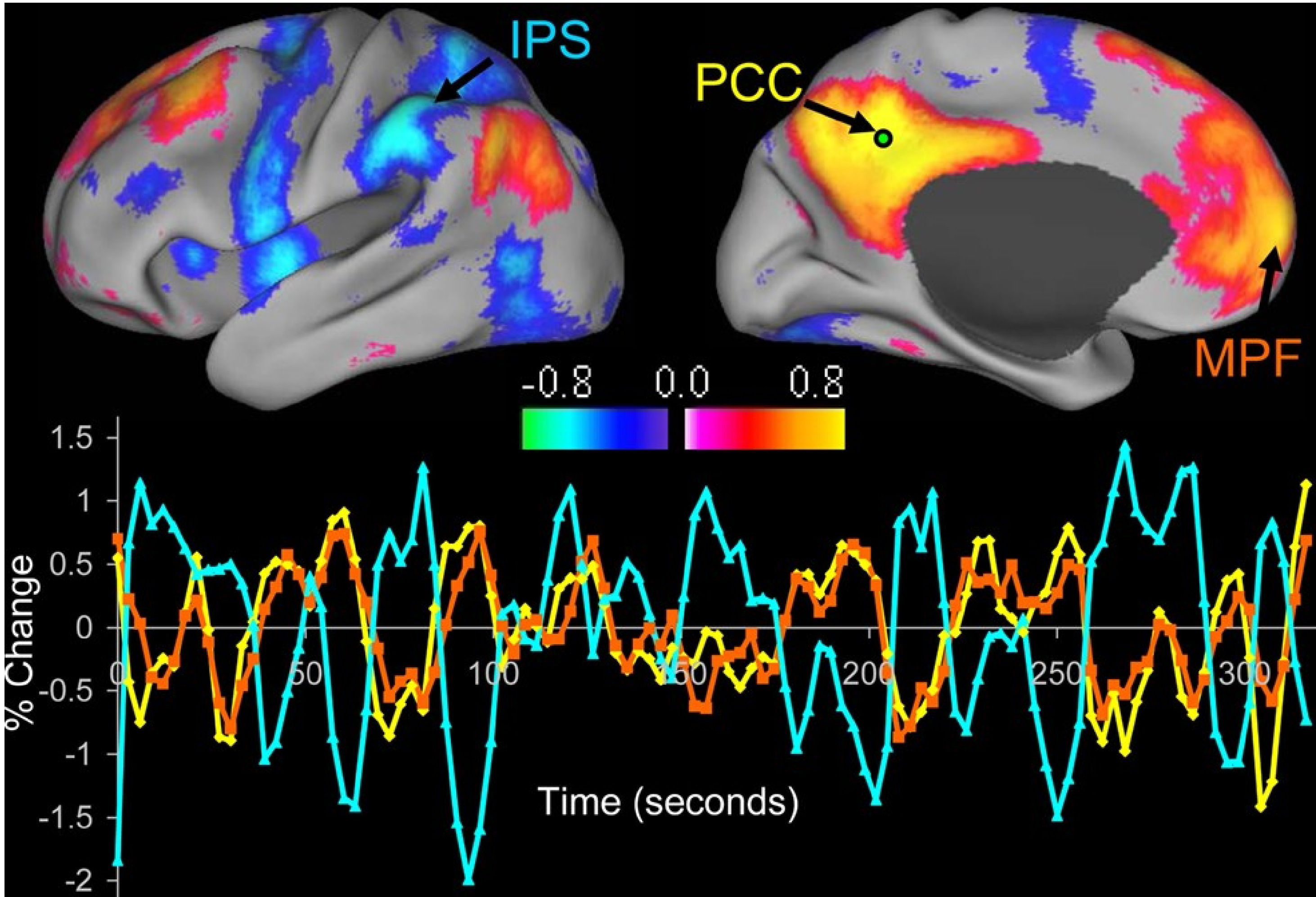
- More recently, *resting-state* BOLD fMRI has become very popular
- No task, just scan the subject while they lay in scanner awake (or watch a movie)
- Also called 'resting state *functional connectivity'* because it involves computing correlation of BOLD fMRI signals across different brain areas
- Has become very popular for analyzing differences across individuals, and examining disease states (although still no clinical applications)

Functional connectivity matrix



Introduction to functional connectivity:
https://www.youtube.com/watch?v=Ij0rcPtOUOc&feature=youtu.be

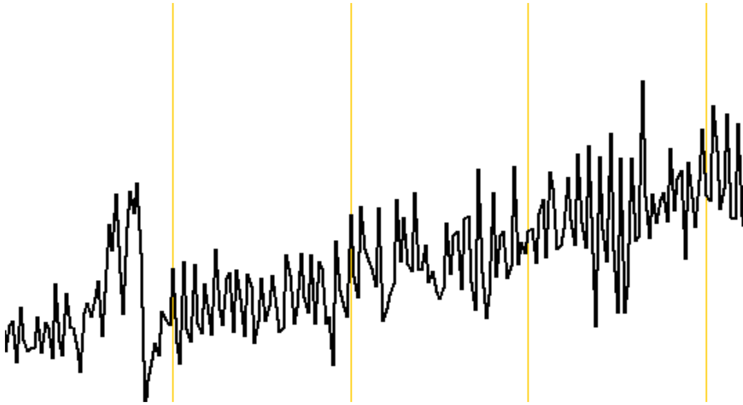
https://www.annualreviews.org/doi/abs/10.1146/annurev-neuro-071013-014030



Artifact and structured noise in BOLD fMRI

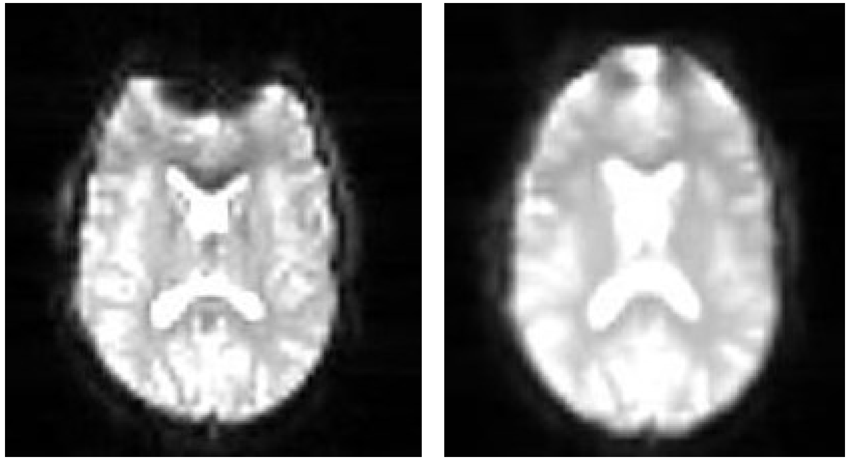
- BOLD is susceptible (pun intended) to a wide variety of artifacts and noise
- Some are caused by acquisition hardware (scanner)
    - a) Echo-planar imaging (EPI) distortion artifact
    - b) Magnetic susceptibility artifact (signal dropout)
    - c) Gradient coil heating artifact (low frequency drift)

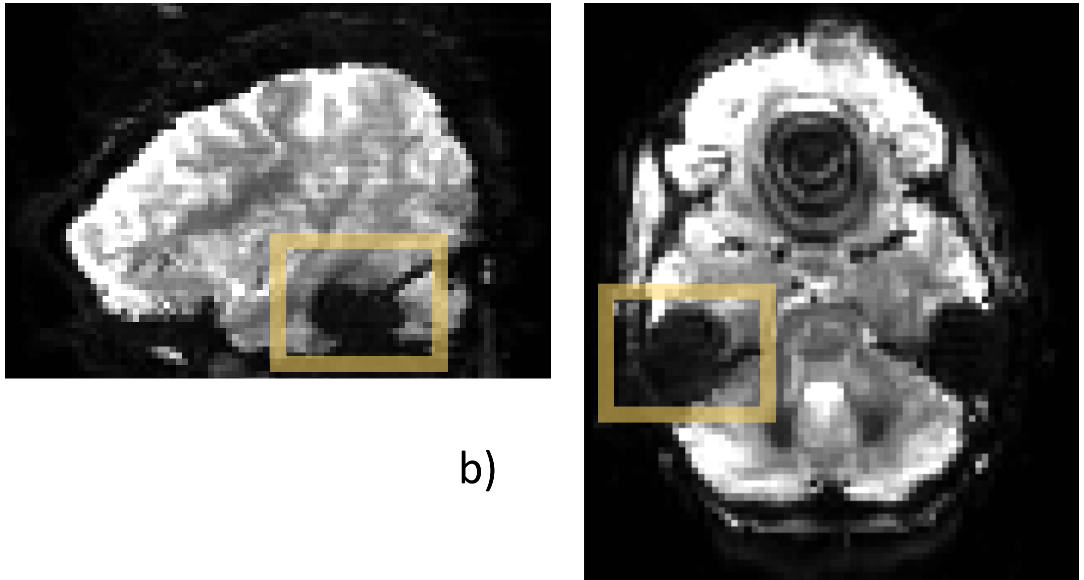
c)



uncorrected corrected

a)

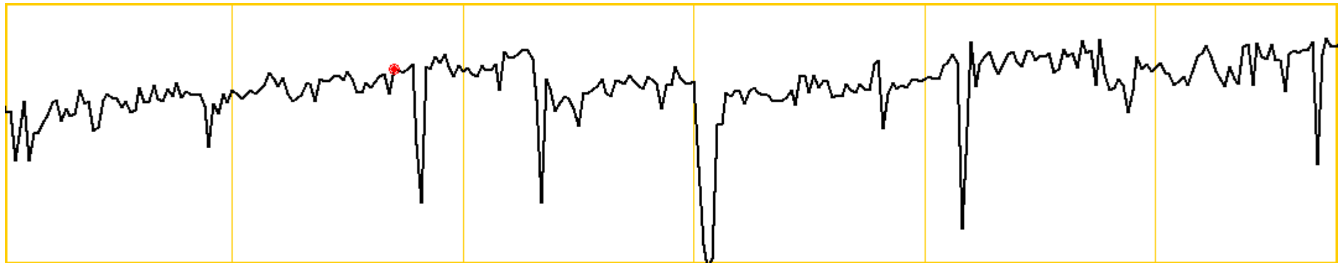




b)

Artifact and structured noise in BOLD fMRI

- Other types of artifact are caused by the subject themselves
  - Heartrate changes, pulsation of blood
  - Respiratory changes
  - Rigid body head motion



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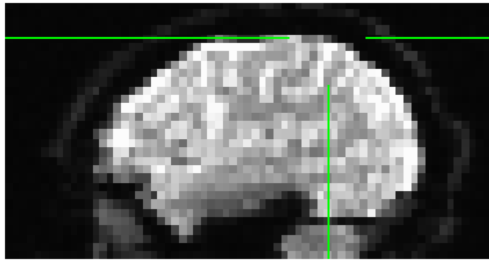

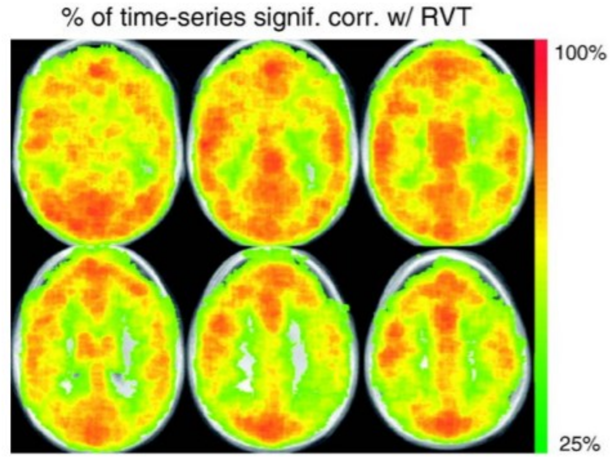

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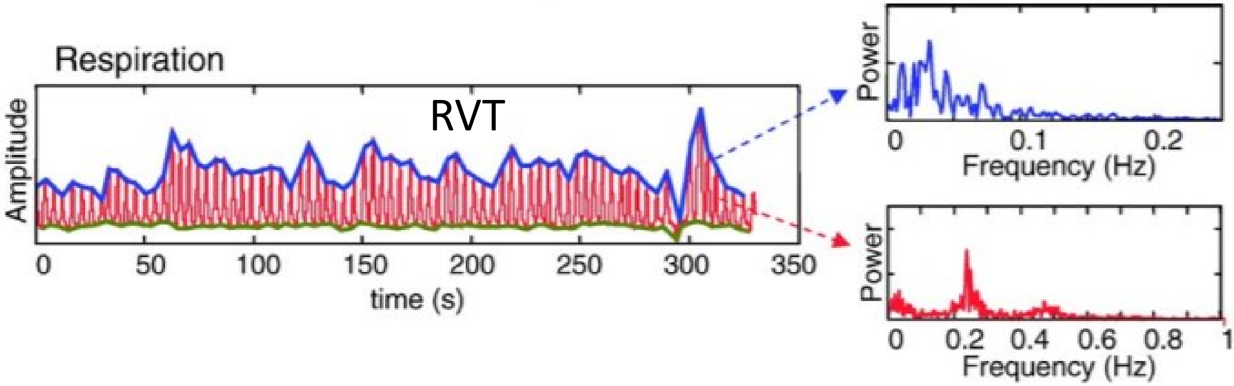

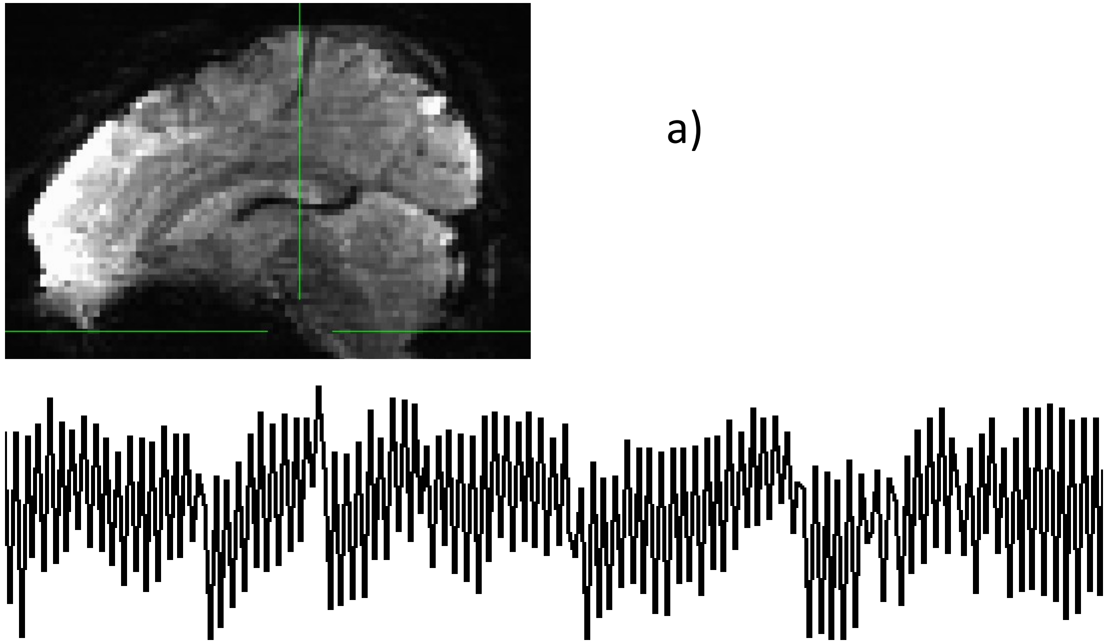

a)