CS 467/567, Assignment 1

Report due on 6 February

1 Your task

Choose one of the problems below and provide a complete proof of NP-completeness. Also state (without proof or justification) what is the optimization problem from which your decision problem is derived.

Trivial variants (such as the size of the solution being equal rather than smaller or larger than the given constant or the other way around) are acceptable. The suggested reductions are just that: suggestions.

1. SET COVER: A set covering instance is (X, F) for a finite set X and $F \subseteq 2^X$ such that $X = \bigcup_{S \in F} S$. We say that S covers its elements. Given a set covering instance (X, F) and an integer k, is there a subset $C \subseteq F$, $|C| = k$, whose members cover X.

   Suggested reduction: from SAT or 3-SAT

2. KNAPSACK: Given n objects with volumes v_i and prices s_i, $1 \leq i \leq n$, a knapsack of volume V, and a constant $S > 0$, is there a subset $N' \subseteq \{1, \ldots, n\}$ such that $\sum_{i \in N'} v_i < V$ (the objects in N' can be placed in the knapsack) and $\sum_{i \in N'} s_i \geq S$ (their price exceeds the given limit S)?

   Suggested reduction: from SET COVER

3. PARTITION: Given a set of nonnegative integers $\{a_1, \ldots, a_n\}$, is there a subset $P \subseteq \{1, \ldots, n\}$ such that $\sum_{i \in P} a_i = \sum_{i \notin P} a_i$?

   Suggested reduction: from KNAPSACK

4. BIN PACKING: Having an infinite number of bins each of volume V, can we put a set of n objects of volumes v_i, $1 \leq i \leq n$ into less than k bins for a given constant k?

   Suggested reduction: from PARTITION

5. INDEPENDENT SET: Given a graph $G = (V, E)$ and a constant k, is there a subset $V' \subseteq V$ such that $|V'| \geq k$ and $\forall u, v \in V' : (u, v) \notin E$.

   Suggested reduction: from CLIQUE or 3-SAT

6. DOMINATING SET: Given a graph $G = (V, E)$ and a constant k, is there a subset $V' \subseteq V$, $|V'| \leq k$ such that $\forall v \in V \setminus V' : \exists u \in V' : (u, v) \in E$.

   Suggested reduction: from VERTEX COVER

7. SUBSET SUM: Given a set S of integers, is there a subset $S' \subseteq S$ such that $\sum_{i \in S'} i = 0$.

   Suggested reduction: From KNAPSACK

8. GRAPH 3-COLORING: Given a graph $G = (V, E)$, is there a function $f : V \to \{\text{red}, \text{green}, \text{blue}\}$ such that $f(x) \neq f(y)$ whenever $(x, y) \in E$.

   Suggested reduction From 3-SAT

9. TWO-MACHINE SCHEDULING: Given the execution time $a_1, a_2, \ldots, a_n$ of n independent tasks and a deadline D, is it possible to schedule the n tasks on two identical machines in a non-preemptive fashion (meaning that once a task starts executing it must run to completion) such that all the task complete by the deadline D.

   Suggested reduction: from KNAPSACK

2 Deliverables

Your solution for this assignment consists of three steps:

1. Form a team of exactly three students (both enrolled in the course) and send an email to your instructor naming the members of the team and ranking the problems above in the order of your preference. It is OK not to have any preference but please say so if this is the case.

2. Write a report that contains your proof of NP-completeness. Cite all your sources. In particular the report should cite at least one technical reference (journal paper, conference paper, technical report, or textbook). Enciclopaedic reference (such as to Wikipedia articles) and references to course lecture notes are acceptable as secondary references but are not considered technical references. All the references *must* be cited in the report.

3. Present you proof in class to your colleagues in a 7-minute talk.

3 Grading and deadlines

If you send your preferences by Tuesday, 28 January then I will do my best to match them. No matching can be guaranteed afterward.

Your report is due on 6 February. It will be marked for completeness (no pun intended), clarity, and references (and their citation in the text). The report must be typeset to PDF and must be released under a Creative Commons Attribution-Non Commercial-Share Alike license (so that is can be included on the course's Web site).

The presentation will be scheduled the week after the due date of the report. One team member must present the proof of membership in NP and another the proof of NP-hardness. The third team member must say a few words about the equivalent optimization problem (if any). The presentation will be marked for completeness and clarity.

The report marks will contribute 80% toward your grade for this assignment while the presentation will contribute the remaining 20%.

Late submissions will incur a penalty of 20% per day late up to the day of the presentation and a penalty of 100% afterward.