

CS467/567 - Special/Advanced Topics in Algorithms

Assignment 3 Yi Ren (002269013)

Ghost busters and ghosts

1. Argue that there exists a line passing through one Ghostbuster and one ghost such that the number of Ghostbusters on one side of the line equals the number of ghosts on the same side. Describe how to find such a line in $O(n \log n)$ time.

- (1) Let the Ghostbusters to be the red points whereas the Ghosts to be the black points.
- (2) Find the lowest point P_0 as the origin, if there exists more than one lowest point, choose the left one.
- (3) Sort all the points(except for P_0) by their intersection angle with x coordinate axis.
- (4) Scan the points(which has different color with P_0) according to the sequence until it has the same number of the red points and black points under the scan line.

Complexity analysis:

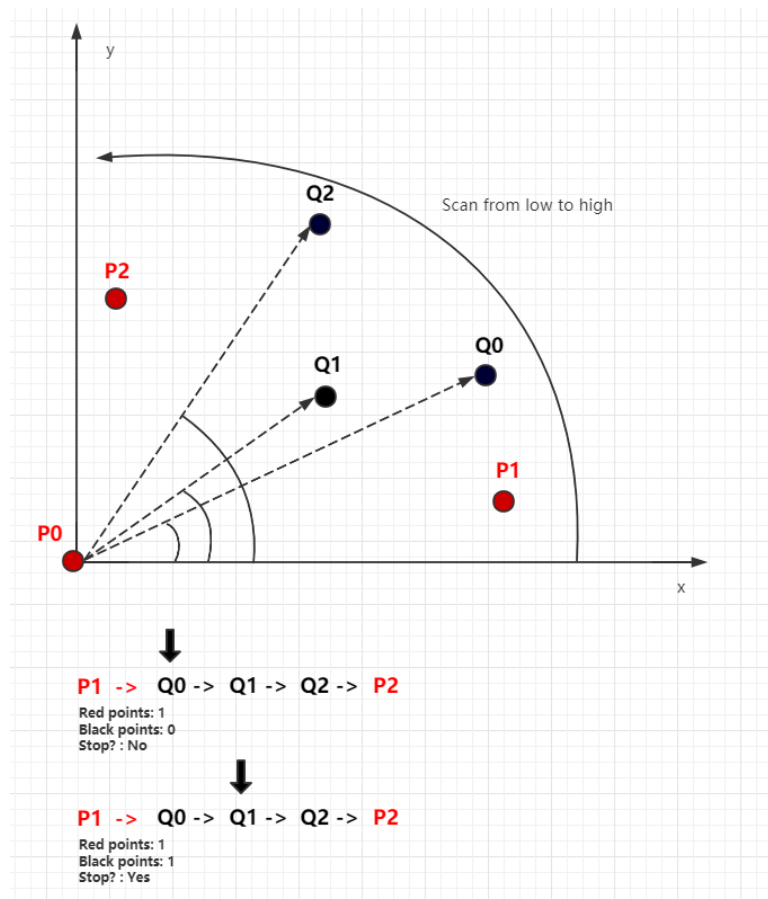
Pick up the lowest point: n

Sort all the points: $n \log n$

Scan the points: n

So the complexity is $O(n \log n)$

Example:



2. Give an $O(n^2 \log n)$ -time algorithm to pair Ghostbusters with ghosts in such a way that no streams cross.

- (1) Let the Ghostbusters to be the red points whereas the Ghosts to be the black points.
- (2) Suppose all the unpaired points are in the set A , for all the points in A , use Graham-Scan algorithm to figure out the convex hull.
- (3) For the adjacent points of the convex hull, if they have different color, pair these points and then remove them from set A .
- (4) If all the points of the convex hull have the same color, which means the Ghosts are surrounded by the Ghostbusters (or the Ghostbusters are surrounded), then every Ghostbuster will choose a Ghost nearby (minimum distance). Remove all the paired points from set A .
- (5) Repeat Step 2, 3 and 4 until set A is empty.

Complexity analysis:

Graham-Scan algorithm: $n \log n$

Select and remove the paired points: n

Loop the steps above for at most n times.

So the complexity is $O(n^2 \log n)$

Example:

Initialize with 10 points

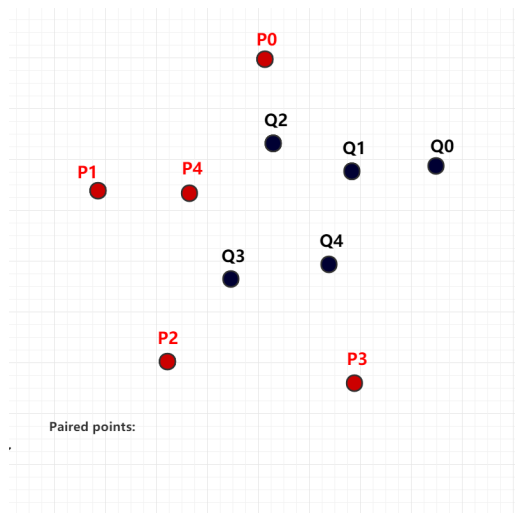
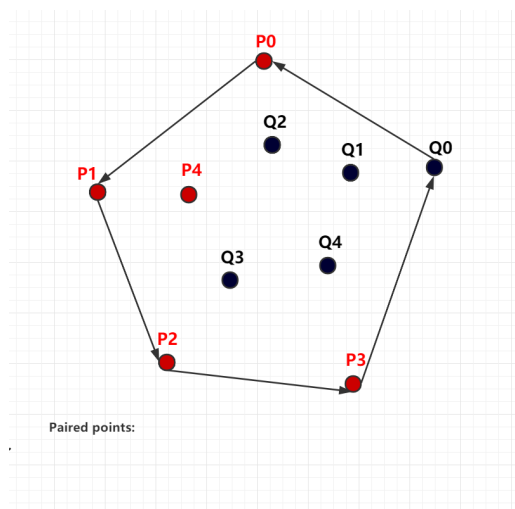


Figure out the convex hull



Remove P3 and Q0

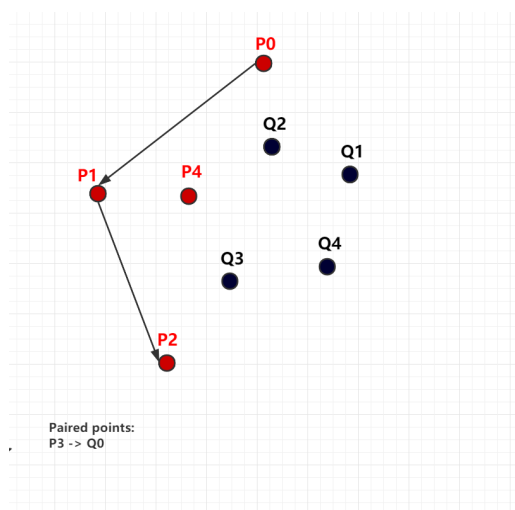
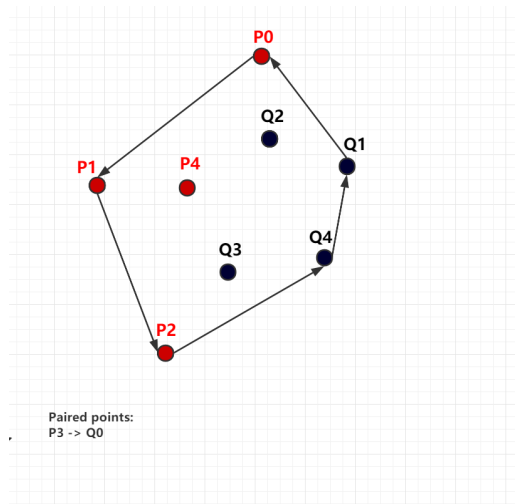


Figure out the convex hull



Remove P2, Q4, P0 and Q1

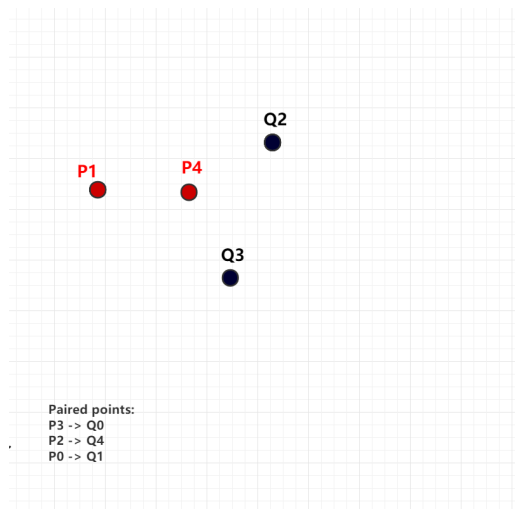
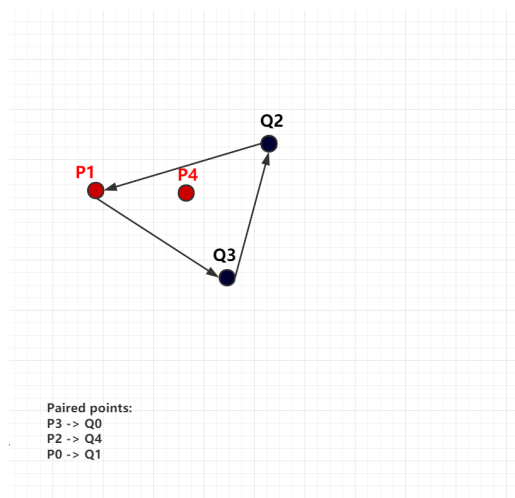


Figure out the convex hull



Remove P1 and Q3

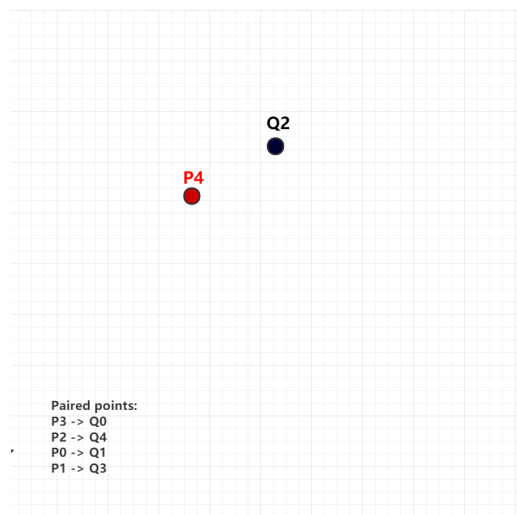
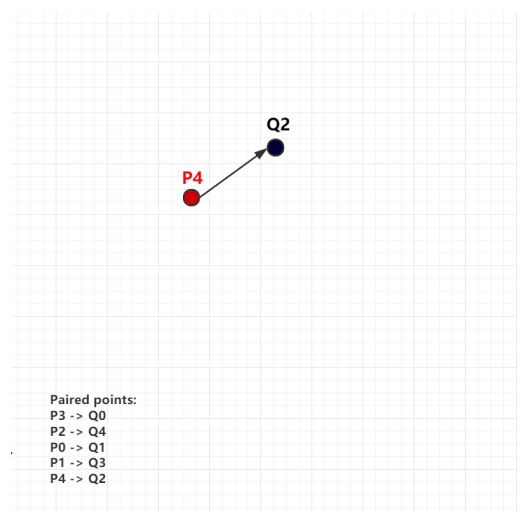


Figure out the convex hull



With all the points paired, here's the result

