

CS 467/567, Assignment 3

Due on 27 March at 11:59 pm

This assignment is individual. Submit your solutions by email as a document typeset to PDF. Include all your references (if any) and also cite them in the text as appropriate.

Late submissions will incur a penalty of 20% per day late.

Ghostbusters and ghosts

A group of n Ghostbusters is battling n ghosts. Each Ghostbuster carries a proton pack, which shoots a stream at a ghost, eradicating it. A stream goes in a straight line and terminates when it hits the ghost. The Ghostbusters decide upon the following strategy: They will pair off with the ghosts, forming n Ghostbuster-ghost pairs, and then simultaneously each Ghostbuster will shoot a stream at his chosen ghost. As we all know, it is *very* dangerous to let streams cross, and so the Ghostbusters must choose pairings for which no streams will cross.

Assume that the position of each Ghostbuster and each ghost is a fixed point in the plane and that no three positions are colinear.

1. Argue that there exists a line passing through one Ghostbuster and one ghost such that the number of Ghostbusters on one side of the line equals the number of ghosts on the same side. Describe how to find such a line in $O(n \log n)$ time.
2. Give an $O(n^2 \log n)$ -time algorithm to pair Ghostbusters with ghosts in such a way that no streams cross.