CS 467/567, Assignment 2

Due on 9 March at 11:59 pm

This assignment can be solved in teams of no more than two students. Submit your solutions by email as a document typeset to PDF. I recommend that you solve the problems by yourself with no external references, but if references are used then they must be provided and also cited in the text.

Late submissions will incur a penalty of 20% per day late.

Problem 1: Approximate Maximum Clique

Let $G = (V, E)$ be an undirected graph. For $k \geq 1$ define $G^{(k)} = (V^{(k)}, E^{(k)})$ such that $V^{(K)} = \{(v_1, v_2, \ldots, v_k) : v_i \in V, 1 \leq i \leq k\}$ and $((v_1, v_2, \ldots, v_k), (w_1, w_2, \ldots, w_k)) \in E^{(k)}$ iff either $(v_i, w_i) \in E$ or $v_i = w_i$ for all $1 \leq i \leq k$.

1. Prove $|C^{(k)}| = |C|^k$, where $C^{(k)}$ and C are the maximum cliques of $G^{(k)}$ and G, respectively.
2. Argue that the existence of an approximation algorithm for finding the maximum clique with a constant approximation ratio implies the existence of a polynomial-time approximation scheme for finding the maximum clique.

Problem 2: Linear Inequality Feasibility

Given a set of m linear inequalities over n variables, the linear inequality feasibility problem asks whether there exists a setting of the variables that satisfies all the linear inequalities simultaneously.

1. Prove that we can use an algorithm for linear programming to solve linear inequality feasibility problems. The number of variables and constraints used in the linear programming problem must be polynomial in n and m.
2. Prove that we can use an algorithm for the linear inequality feasibility problem to solve linear programming problems. The number of variables and linear inequalities must be polynomial in the number of variables and constraints of the linear program.