Game Physics Notes 01

CSCI 321

WWU

November 21, 2017

References

http://chortle.ccsu.edu/vectorlessons/vectorindex.html

http://tutorial.math.lamar.edu/Classes/CalcII/VectorsIntro.aspx

https://www.mathsisfun.com/algebra/vectors.html

http://emweb.unl.edu/Math/mathweb/vectors/vectors.html

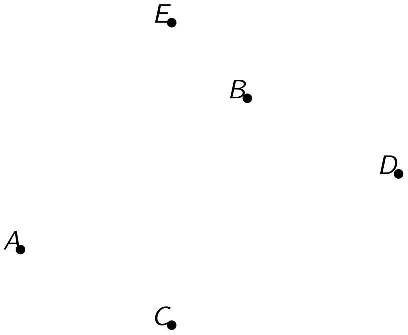
https://unity3d.com/learn/tutorials/topics/scripting/vector-maths

numpy

A nice python library for dealing with mathematical vectors and matrices.

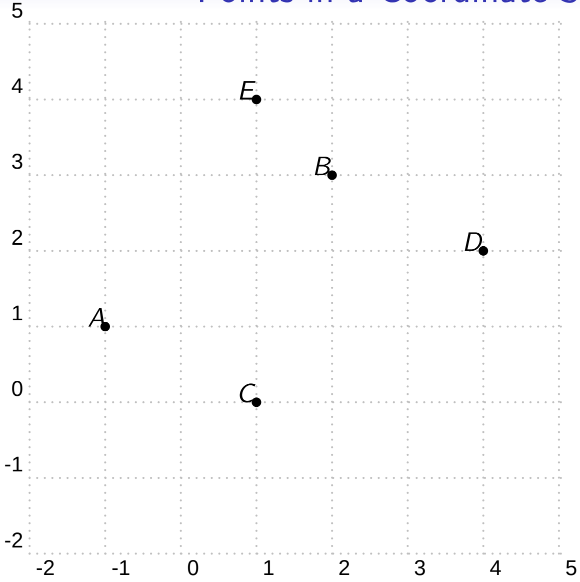
```
>>> import numpy
>>> x = (1,2,3)
>>> y = (3,2,1)
>>> 3*x
(1, 2, 3, 1, 2, 3, 1, 2, 3)
>>> x + y
(1, 2, 3, 3, 2, 1)
>>> xvec = numpy.array(x)
>>> yvec = numpy.array(y)
>>> 3*xvec
array([3, 6, 9])
>>> xvec + yvec
array([4, 4, 4])
>>> numpy.dot(xvec, yvec)
10
```

Points in Space



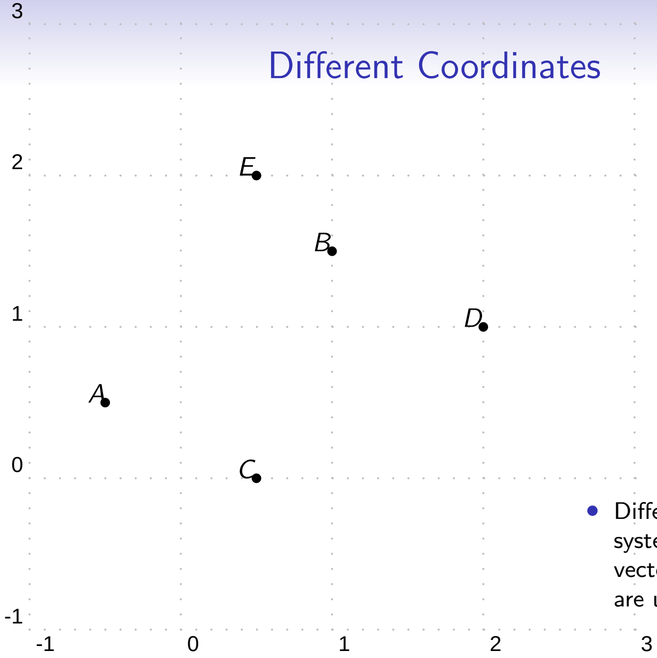
- Points exist in space without a coordinate system.
- But with only labels it's difficult to compute with them.

Points in a Coordinate System



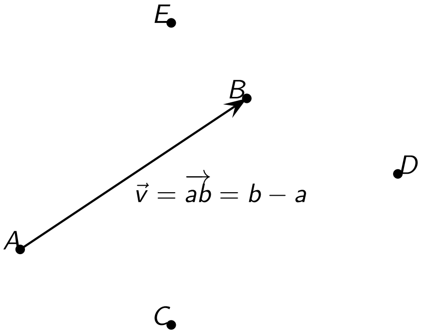
- A coordinate system gives positions to points.
- Relates *points* to *tuples of numbers*, or *mathematical vectors*.
- However, points are *not* vectors!

Different Coordinates



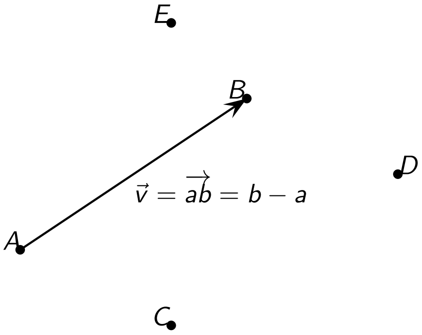
- Different coordinate systems give different vectors, but the *points* are unchanged.

Physical vectors are differences between points.



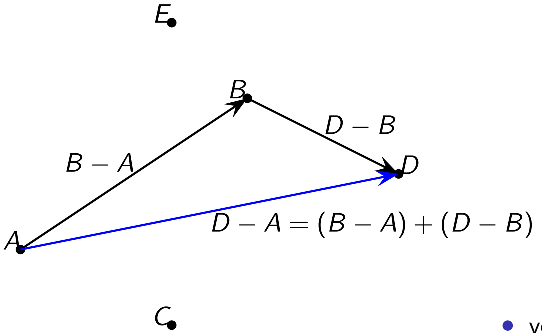
- Physical vectors are *not* mathematical vectors.
- But given a coordinate system, you can represent the points as mathematical vectors, and then subtract.
- But these mathematical vectors are not the same thing!
- Different coordinate systems will give you different mathematical vectors for the *same* physical vector.

Points and vectors are not the same thing!



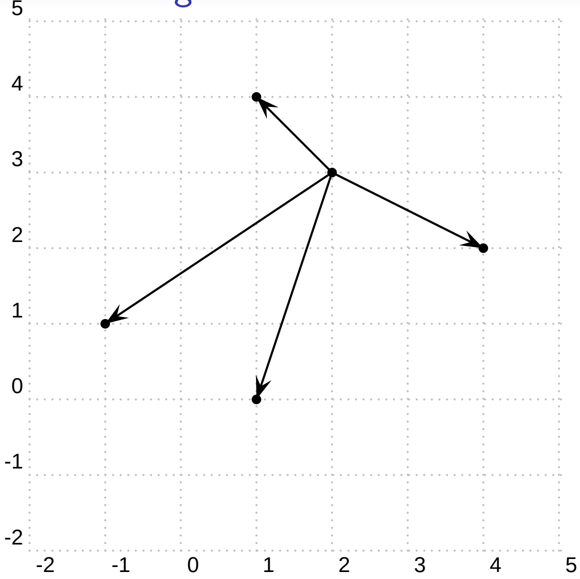
- A point is a position in space.
- A vector has a magnitude and a direction.
- The vector from a to b is the point difference: $\vec{v} = \overrightarrow{ab} = b - a$
- You can add two vectors, but you *cannot* add two points!
- You can add points and vectors: $b = a + \vec{v} = a + (b - a)$

Vector Addition



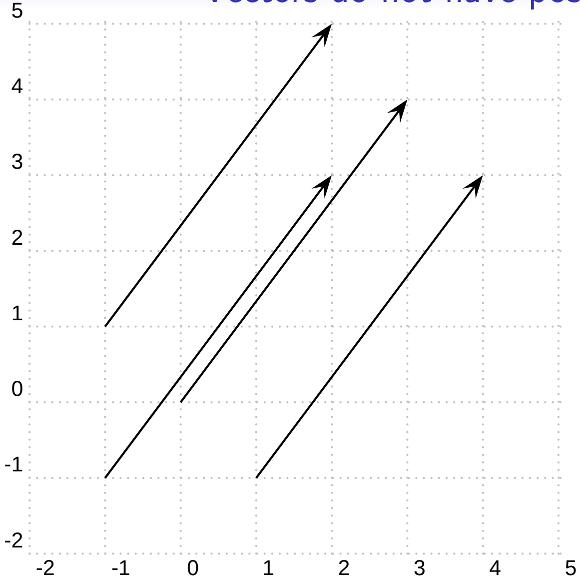
- vector + vector = vector
- point + vector = point
- point - point = vector
- point + point = *nonsense*

Coordinates give mathematical vectors to physical vectors.



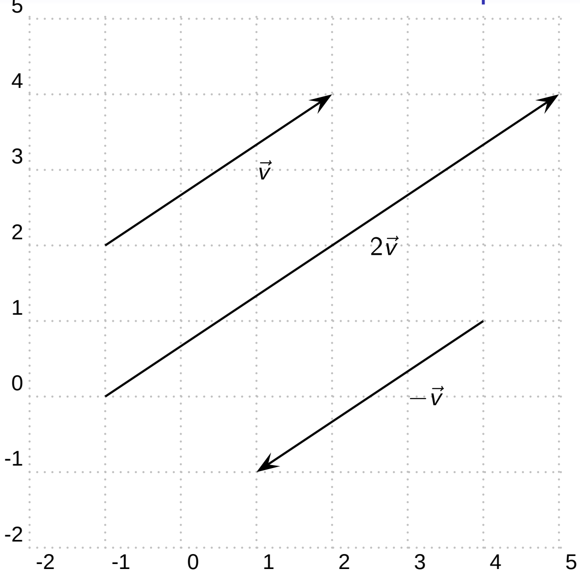
- Subtract the components.
- $(1,4) - (2,3) = (-1,1)$
- $(-1,1) - (2,3) = (-1,-2)$
- $(1,0) - (2,3) = (-1,-3)$
- $(4,2) - (2,3) = (2,-1)$
- Note: we subtract two *points* to get a *vector*.

Vectors do not have positions



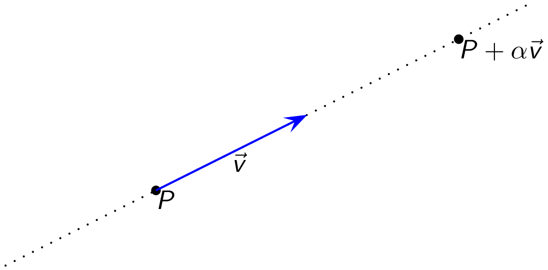
- Each of these vectors is the *same* vector.

Vectors can be multiplied by scalars



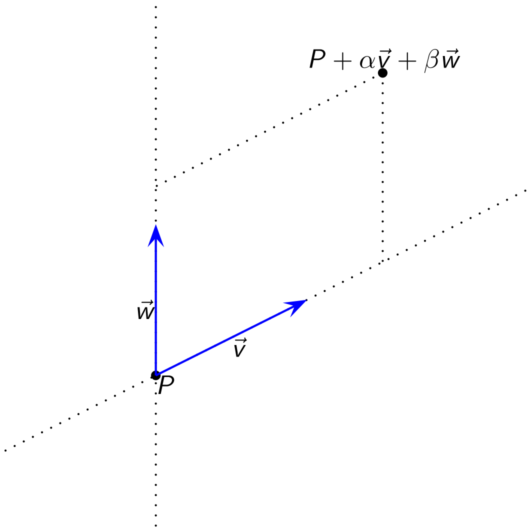
- Multiplication is repeated addition.

Lines



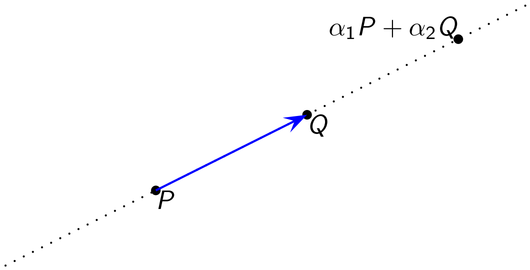
- The line through P in the direction v is the set of all points $P + \alpha v$ for some $\alpha \in \mathbb{R}$

Planes (in 3 dimensions)



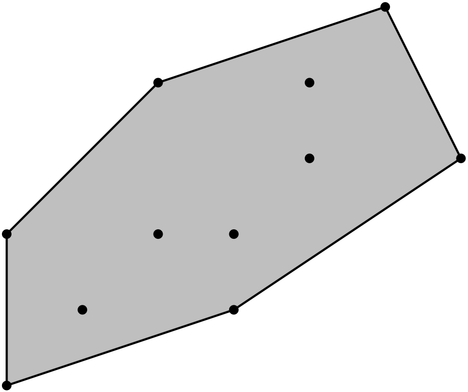
- The plane through P spanned by v and w is the set of all points $P + \alpha v + \beta w$ for some $\alpha, \beta \in \mathbb{R}$

Affine sums



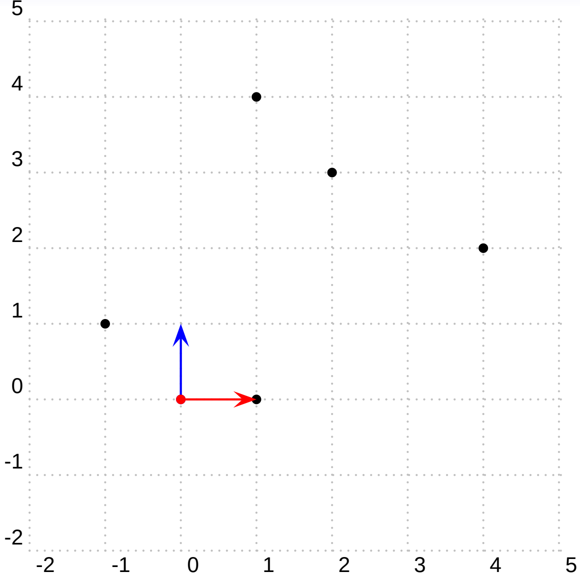
- $P + \alpha(Q - P)$
  $= (1 - \alpha)P + \alpha Q$
  $= \alpha_1 P + \alpha_2 Q$
- $\alpha_1 + \alpha_2 = 1$
- Think of each point as the vector from some arbitrary point:
  $P \equiv P - O$
  $Q \equiv Q - O$
- If $0 \leq \alpha_i$ then the point is between P and Q.

Convex hull



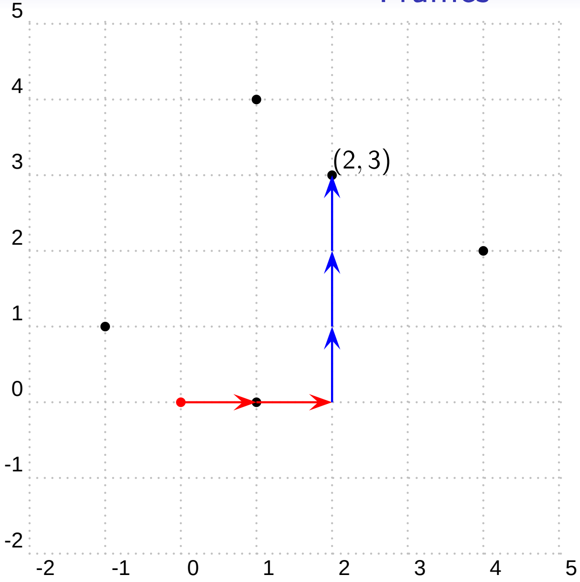
- $P = \alpha_1 P_1 + \alpha_2 P_2 + \ldots + \alpha_n P_n$
- $\alpha_1 + \alpha_2 + \ldots + \alpha_n = 1$
- $0 \leq \alpha_i$

Frames



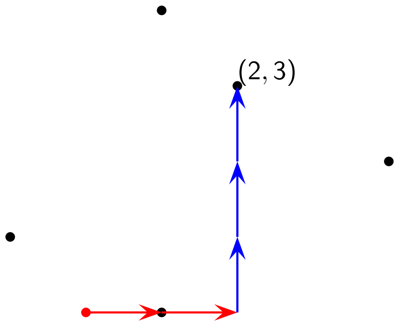
- A coordinate system can be thought of as a single point, the *origin*, and a set of *basis vectors*.
- Such a set is called a *frame*.

Frames



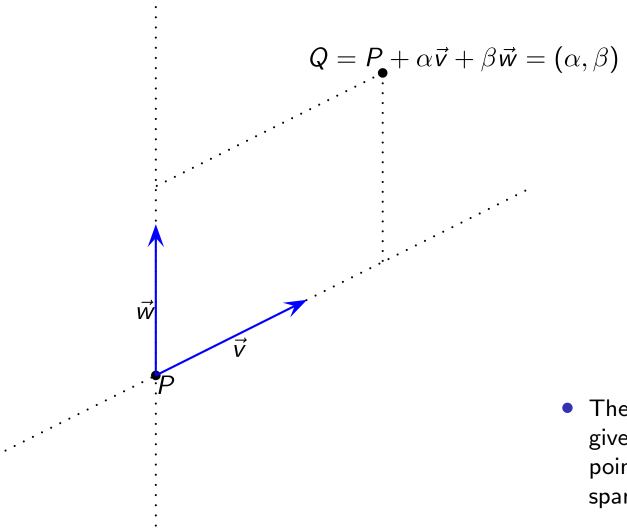
- The coordinates of a point are how many copies of the basis vectors you have to add to the origin.

Frames



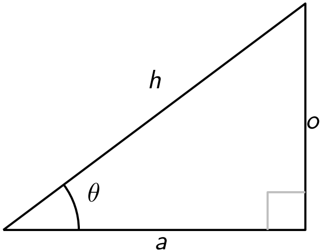
- Note that a frame gives sense to coordinates without anything other than points and vectors.
- A coordinate system is nothing more than an origin and a set of basis vectors, a *frame*.
- An *orthonormal* frame is one in which all the vectors are of unit length and perpendicular to each other.

Frames do not have to be orthonormal



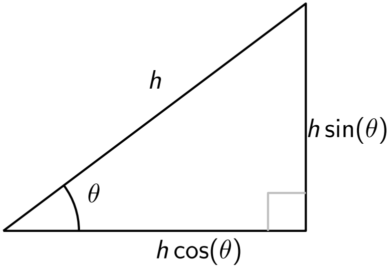
- The frame $F = (P, \vec{v}, \vec{w})$ gives coordinates to any point in the plane it spans.

Trigonometry



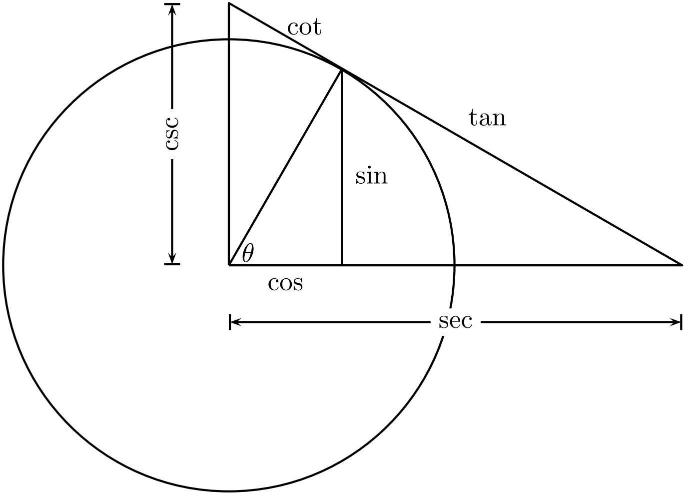
- $\sin(\theta) = o/h$
- $\cos(\theta) = a/h$
- $\tan(\theta) = o/a$

Trigonometry

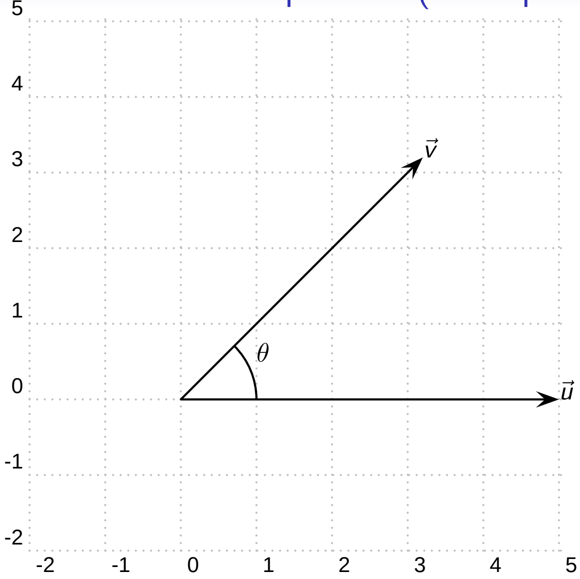


- $\tan(\theta) = \sin(\theta)/\cos(\theta)$

Trigonometry

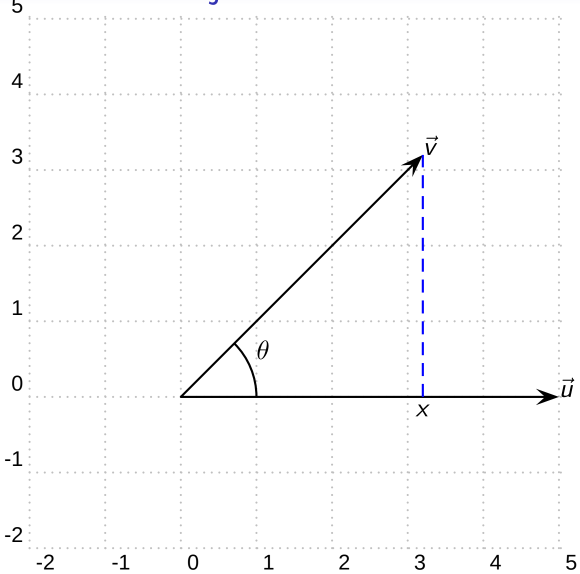


Dot product (Inner product)



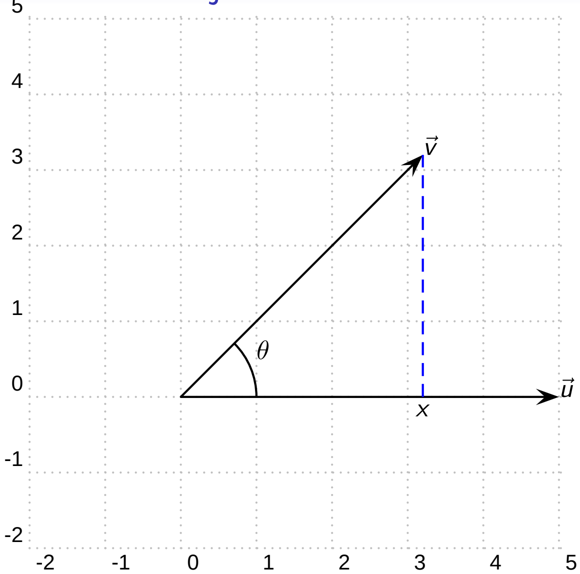
- $u \cdot v = \cos(\theta)|u||v|$
- $|u| = \sqrt{u \cdot u}$

Projection of one vector on another



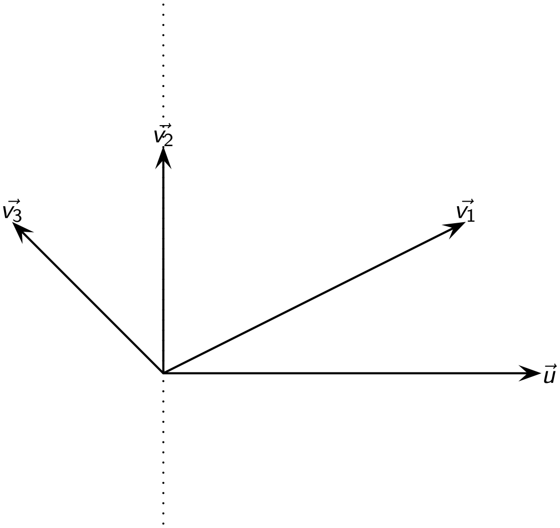
- What is x?

Projection of one vector on another



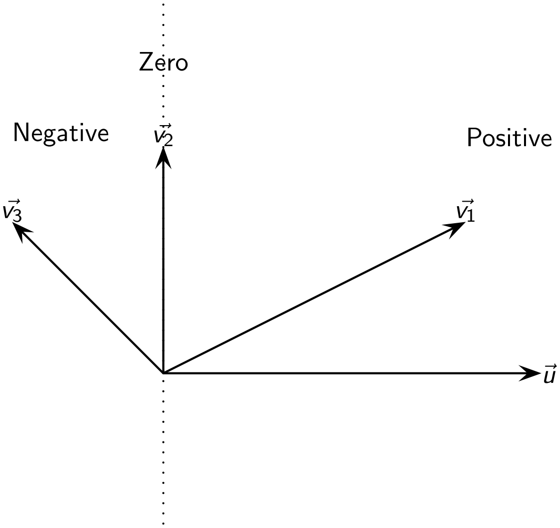
- $x = \cos(\theta)|v|$
- $x = u \cdot v / |u|$

Same direction, opposite direction



- What is the sign of $u \cdot v_i$?

Same direction, opposite direction



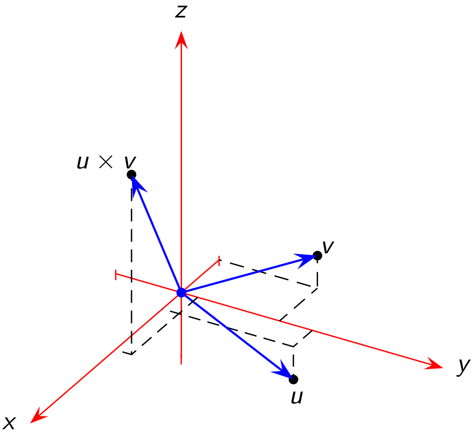
- Sign of $u \cdot v_i$

AMAZING theorem about the dot product.

- In any coordinate system whatsoever:

$$\begin{aligned} u \cdot v &= (u_x, u_y, u_z) \cdot (v_x, v_y, v_z) \\ &= u_x v_x + u_y v_y + u_z v_z \\ &= [u_x \;\; u_y \;\; u_z] \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \\ &= u^T v \end{aligned}$$

Cross product (vector product)



- A vector at right angles to u and v.
- $u \times v = (u_2 v_3 - u_3 v_2, u_3 v_1 - u_1 v_3, u_1 v_2 - u_2 v_1)$
- Mnemonic:

$$u \times v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

- $|u \times v| = |u||v| \sin(\theta)$