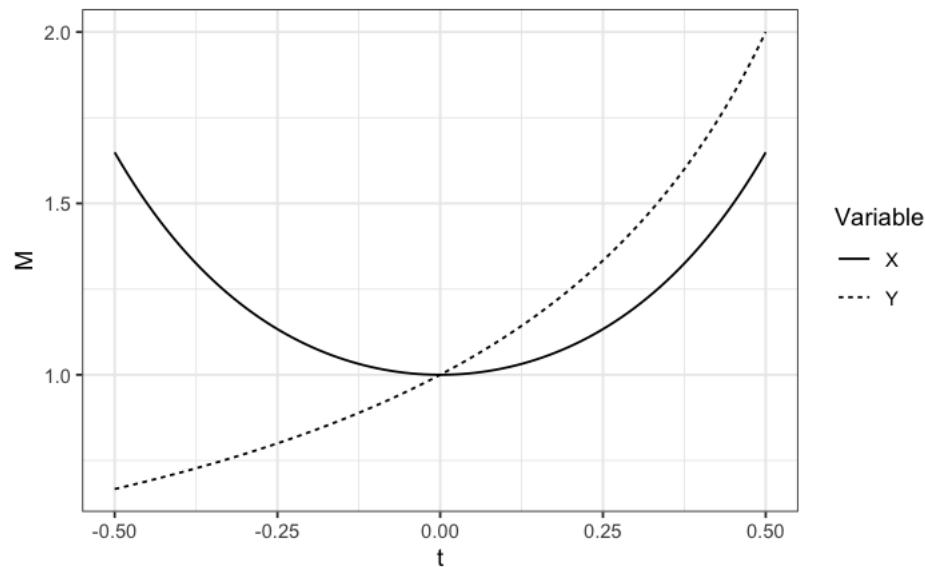


Moment Generating Functions

1. Let X_1, \dots, X_n be iid $\text{Expo}(\lambda)$. Show that $X_1 + \dots + X_n$ is not exponentially distributed.
2. The graphs of the MGFs M_X and M_Y for two random variable X and Y are shown below. Based on these graphs, which variable has the larger mean? Which has the larger variance? Is either variable necessarily asymmetric? Carefully explain how you know.

Note that the 2nd derivative of a function represents how quickly that function curves away from its tangent line. Although the second derivative of each function can be estimated from the graph, it would be somewhat tedious to do so. Instead, I've done this for you already and estimated $M_X''(0) = 4$ and $M_Y''(0) = 2$.



3. (*) Let $X \sim \text{DUnif}(\{-1, 1\})$ (i.e. $P(X = 1) = \frac{1}{2} = P(X = -1)$). Find the MGF of X , and then use the Taylor series expansion for the MGF to find **all** moments of X .
4. (Extra Practice:) Identify the variables (including parameters) which have the following moment generating functions:
 - (a) $M(t) = \frac{e^t - 1}{t}$ for $t \neq 0$, $M(0) = 1$.
 - (b) $M(t) = e^{t+t^2}$
 - (c) $M(t) = \frac{1}{1-\frac{t}{2}}$ for $t < 2$
 - (d) $M(t) = \frac{1}{8}(e^t + 1)^3$
 - (e) $M(t) = \frac{1}{2-e^t}$ for $t < \ln 2$.