Random Variables

1. (*) Suppose that, on average, Jonathan receives 10 emails each day, although the exact number varies from day to day. We'll treat the number of emails he recieves on a particular day as a random variable, X. Suppose the pmf p for X can be written as $p(k) = e^{-10}\frac{10^k}{k!}$ for $k = 0, 1, 2, \ldots$. (This is called a **Poisson distribution**, and is a surprisingly accurate model for this situation; we'll see why in Section 4.7). *Note: this problem is similar (but not identical) to HW problem 3.14.*

   (a) True or False? The probability that X takes the value $e^{-10}\frac{10^1}{1!}$ is 0.

   (b) What is a **mode** of X? (a mode is a value with the highest probability of occurring)

   (c) What is the probability that Jonathan recieves exactly 10 emails? Give both an exact value for the probability, as well as a decimal approximation.

   (d) Find $P(X \geq 1)$ and $P(X \geq 2)$ without summing infinite series. *Hint: What is the complement of the event $X \geq 1$?*

   (e) Define a function q on the positive integers by $q(k) = P(X = k \mid X \geq 1)$ (*This function is called the conditional pmf of X given $X \geq 1$.*) Give an explicit formula for q and discuss how the values of q relate to the values of p. Then create a story corresponding to a random variable whose pmf is q.

2. A national poll is conducted to estimate the U.S. president's approval rating. The polling firm randomly selects 100 U.S. voters and asks each the question "Do you approve or disapprove of the way Joe Biden is handling his job as president?". Suppose the polling firm samples *with replacement* from the population of U.S. voters, meaning that it is possible for a person to be selected for the sample more than once (in which case, their response will be recorded as many times as they appear in the sample). Let p denote the true proportion of U.S. voters who approve of the president.

   (a) What is the probability that the first person chosen for the sample approves of the president? Give your answer in terms of p.

   (b) What is the probability that at least 50 people in the sample approve of the president? Give your answer in terms of p.

   (c) Let X_1 be the random variable that takes the value 1 if the first person in the sample approves of the president, and that takes the value 0 otherwise. What is the name of the distribution of X_1? Be sure to specify the value of the parameter for this distribution as well.

   (d) Similarly, for each $k = 2, 3, \ldots, 100$, let X_k be the variable that takes the value 1 if the kth person in the sample approves of the president. What is the name of the distribution of X_k?

   (e) Let N be the random variable counting the number of people in the sample who approve of the president. Express N in terms of the random variables $X_1, X_2, \ldots, X_{100}$. What is the name of the distribution of N?