

Change-of-Variables

1. (*) Suppose X and Y are iid $\text{Expo}(\lambda)$. In this problem, we will find the joint distribution of $X + Y$ and $\frac{X}{X+Y}$, as well as the marginal distribution of $\frac{X}{X+Y}$.
 - (a) Define a function $g : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ by $(u, v) = g(x, y) = \left(x + y, \frac{x}{x+y}\right)$. Find a formula for the inverse transformation $g^{-1}(u, v)$ and use it calculate the Jacobian of g^{-1} .
 - (b) Let $U = X + Y$ and $V = \frac{X}{X+Y}$. Use the change-of-variables formula to express the joint PDF $f_{U,V}$ of U, V in terms of the joint PDF $f_{X,Y}$ of X, Y .
 - (c) Based on your previous answer, are U and V independent?
 - (d) Find a formula for the marginal PDF of $U = \frac{X}{X+Y}$. What named distribution is this?