Homework 6: 10/10 - 10/14

STA 335 Due 11:59pm Monday, October 24 Name:

Instructions: Write-up complete solutions to the following problems and submit answers on Gradescope. Your solutions should be neatly-written, show all work and computations, include figures or graphs where appropriate, and include some written explanation of your method or process (enough that I can understand your reasoning without having to guess or make assumptions). A rubric for homework problems appears on the final page of this assignment.

- Unless otherwise noted, problem numbers are taken from the 2nd edition of Blitzstein and Hwang's *Intro to Probability*.

Monday 10/10

Chapter 5

5, 13, 14

Additional Problems

AP1. The *Pareto distribution* with parameter $a > 0$ has PDF $f(x) = a/x^{a+1}$ for $x \geq 1$ (and 0 otherwise).

(a) Find the CDF of a Pareto r.v with parameter a.

(b) Pareto distributions are said to be *heavy-tailed*, which means they have relatively high probability of generating large values. For what values of a does a Pareto variable have a mean? A variance? Compute the mean and variance for those pareto variables where it makes sense to do so.

(c) R does not have a formula for generating Pareto random variables (unlike `rbinom` for the binomial distribution). But R does have a function to generating Uniform random variable (`runif`). Explain how to use `runif` to generate 100 samples of a variable with the Pareto$-a$ distribution.

Wednesday 10/12

Chapter 5

22, 28, 32

Additional Problems

AP2. We will prove later that if X, Y are independent with distribution $N(\mu, \sigma^2)$, then $W = X + Y$ is $N(2\mu, 2\sigma^2)$. For now, we'll verify this empirically.

(a) Suppose $\mu = 1, \sigma^2 = 4$. Sample from the distribution of W 1000 times by sampling from the distribution of X and from Y 1000 times each, and then adding the values of X and of Y together. Compute the mean and variance of the sample of W.

(b) Create of histogram of the sample of W, and plot the density function for $N(2, 8)$. How do the histogram and density function compare?

(c) Create a QQ plot for W versus the theoretical quantiles for $N(2, 8)$. To do so, let $a_1, a_2, \ldots, a_{1000}$ be values of the sample of W in increasing order, and let F^{-1} be the quantile function for $N(2, 8)$. Plot the points $(a_1, F^{-1}(1/1001)), (a_2, F^{-1}(2/1001)), \ldots, (a_{1000}, F^{-1}(1000/1001))$. Does the QQ-plot give evidence that $W \sim N(2, 8)$?

Friday 10/14

Chapter 5

36, 37, 46

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General Rubric

Points	Criteria
5	The solution is correct **and** well-written. The author leaves no doubt as to why the solution is valid.
4.5	The solution is well-written, and is correct except for some minor arithmetic or calculation mistake.
4	The solution is technically correct, but author has omitted some key justification for why the solution is valid. Alternatively, the solution is well-written, but is missing a small, but essential component.
3	The solution is well-written, but either overlooks a significant component of the problem or makes a significant mistake. Alternatively, in a multi-part problem, a majority of the solutions are correct and well-written, but one part is missing or is significantly incorrect
2	The solution is either correct but not adequately written, or it is adequately written but overlooks a significant component of the problem or makes a significant mistake.
1	The solution is rudimentary, but contains some relevant ideas. Alternatively, the solution briefly indicates the correct answer, but provides no further justification
0	Either the solution is missing entirely, or the author makes no non-trivial progress toward a solution (i.e. just writes the statement of the problem and/or restates given information)
Notes:	For problems with multiple parts, the score represents a holistic review of the entire problem. Additionally, half-points may be used if the solution falls between two point values above.