Dynamics (continued)

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During the lecture
I will mute the audience
Ask questions through ’Chat’

Last Time

Kinematics
Redundancy Resolution

Dynamics
Rigid Body Dynamics: equation of motion
Inverse Dynamics Control: given $\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}^*$ find τ

Error Dynamics

Task-space Dynamics

Redundancy Resolution in Dynamics

Force Control

Outline

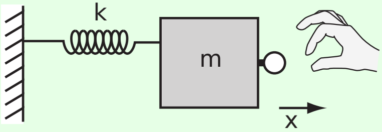
What are stiffness and damping

Question: What are stiffness and damping and how to tune it?

Let’s start with our high school/UG physics!

A mass-spring system

An object attached to a spring, on a frictionless table



m: mass

k: spring constant

$\tilde{x}$: displacement from an equilibuirum

A mass-spring system
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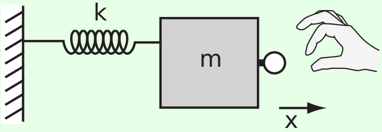

m: mass
k: spring constant
$\tilde{x}$: displacement from an equilibuirum

Pull and release the object! Without friction and gravity, the system will oscillate!

Conservative System

Question: What happened?

$$\text{Kinetic Energy}\ \ \mathbf{K} = \frac{1}{2} m\dot{\tilde{x}}^2,$$
$$\text{Potential Energy}\ \ \mathbf{P} = \frac{1}{2} m\tilde{x}^2$$
$$\text{Euler Lagurange equation}\ \ \frac{d}{dt}\frac{\partial L}{\partial \dot{\mathbf{q}}} - \frac{\partial L}{\partial \mathbf{q}} = m\ddot{\tilde{x}} + k\tilde{x} = 0$$

Conservative System

Question: What happened?

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At Rest

$\mathbf{K} = 0, \mathbf{P} = 0$

Question: What happened?

$$\text{Kinetic Energy}\ \ \mathsf{K} = \frac{1}{2} m\dot{\tilde{x}}^2,$$

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At Rest
$\mathsf{K} = 0, \mathsf{P} = 0$

Hold it
$\mathsf{K} = 0, \mathsf{P} \Uparrow$

Conservative System

Question: What happened?

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At Rest
$\mathsf{K} = 0, \mathsf{P} = 0$

Hold it
$\mathsf{K} = 0, \mathsf{P} \Uparrow$

Release it
$\mathsf{K} \Uparrow, \mathsf{P} \Downarrow$

Conservative System

Question: What is the relationship between the oscillation and m, k

From $m\ddot{\tilde{x}} + k\tilde{x} = 0$

Bigger $m \rightarrow$ smaller $\ddot{\tilde{x}} \rightarrow$ smaller frequency

smaller $k \rightarrow$ smaller $\tilde{x} \rightarrow$ smaller frequency

Question: What is the relationship between the oscillation and m, k
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smaller $k \rightarrow$ smaller $\tilde{x} \rightarrow$ smaller frequency

Natural Frequency
The frequency at which a system oscillates in the absence of any force.

$$\omega_N = \sqrt{\frac{k}{m}}$$

Oscillation A transfer between potential and kinetic energy
Without friction, the object will oscillate with a frequency of ω_N

Conservative System

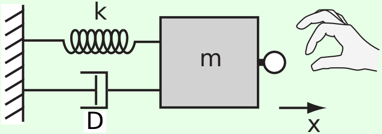
Oscillation A transfer between potential and kinetic energy
Without friction, the object will oscillate with a frequency of ω_N

Dynamics in terms of natural frequency

$$\ddot{\tilde{x}} + \omega_N^2 \tilde{x} = 0$$

A mass-spring-damper System

An object attached to a spring, on a table with friction



m: mass
k: spring constant
d: viscous damping coefficient
$\tilde{x}$: displacement from an equilibuirum

A mass-spring-damper System

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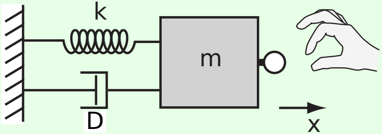

m: mass
k: spring constant
d: viscous damping coefficient
$\tilde{x}$: displacement from an equilibuirum

Still oscillates, but friction slows down the system

$$f_{friction} = -d\dot{\tilde{x}}$$

Dissipative System

Question: What happened?

$$\text{Kinetic Energy} \quad \mathbf{K} = \frac{1}{2} m \dot{\tilde{x}}^2,$$
$$\text{Potential Energy} \quad \mathbf{P} = \frac{1}{2} m \tilde{x}^2$$
$$\text{Euler Lagurange equation} \quad m\ddot{\tilde{x}} + k\tilde{x} = f_{friction}$$

Dissipative System

Question: What happened?

$$\text{Kinetic Energy} \quad \mathbf{K} = \frac{1}{2} m \dot{\tilde{x}}^2,$$
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$$\text{Euler Lagurange equation} \quad m\ddot{\tilde{x}} + k\tilde{x} = f_{friction}$$

Second-Order Error Dynamics

$$m\ddot{\tilde{x}} + k\tilde{x} = -d\dot{\tilde{x}}$$
$$m\ddot{\tilde{x}} + d\dot{\tilde{x}} + k\tilde{x} = 0$$

Question: What is the relationship between the oscillation and d
From $m\ddot{\tilde{x}} + d\dot{\tilde{x}} + k\tilde{x} = 0$
bigger $d \rightarrow$ system may not return to the equilibuirum
smaller $d \rightarrow$ system still oscillates but eventually stops

Question: What is the relationship between the oscillation and d
From $m\ddot{\tilde{x}} + d\dot{\tilde{x}} + k\tilde{x} = 0$
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Natural Damping Ratio ζ
A parameter that characterizes the frequency response of a second-order error dynamics

$$\zeta_N = \frac{d}{2\omega_N m}, \quad \begin{cases} \zeta > 1, \text{over-damped} \\ \zeta = 1, \text{critically-damped} \\ \zeta < 1, \text{under-damped} \end{cases}$$

Second-Order Error Dynamics

Damping coeffient with a Critically-damped Ratio

$$\zeta_N = \frac{d}{2\omega_N m} = 1, \;\; d = 2m\omega_N = 2\sqrt{km}$$

Why?

$$\ddot{\tilde{x}} + 2\zeta_N \omega_N \dot{\tilde{x}} + \omega_N^2 \tilde{x} = 0$$

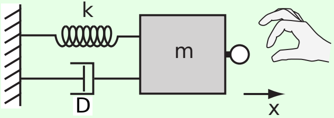
$$x(t) = ce^{-\zeta_N \omega_N t} \cos(\omega_N t\sqrt{1-\zeta_N^2} + \phi)$$

$$if \, \zeta_N = 1, cos(0) = 0, \text{ only the exponentail function left}$$

$$\text{the system oscillates with a frequency } \omega_N t\sqrt{1-\zeta_N^2}$$

Proportional-Derivative Control (PD)

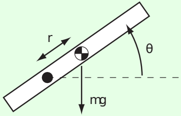
mass-spring-damper



$$m\ddot{\tilde{x}} + d\dot{\tilde{x}} + k\tilde{x} = 0$$

$$\ddot{\tilde{x}} + \frac{d}{m}\dot{\tilde{x}} + \frac{k}{m}\tilde{x} = 0$$

PD controller



$$\tilde{\theta} = \theta - \theta^{ref}$$

$$\ddot{\tilde{\theta}} + \frac{\mathsf{D}}{m}\dot{\tilde{\theta}} + \frac{\mathsf{K}}{m}\tilde{\theta} = 0$$

PD controller imitatess the mass-spring-damper system We have an imaginary spring at the joint, that brings the joint to the desired position. we need to select k and d

Question: How to choose the parameters?

- Choose an equilibuirum point
- Choose your k, depending how fast the system should react to the system
- Calculate the natural frequency $\omega_N = \sqrt{\frac{k}{m}}$
- We want a critically damped system: $\zeta_N = 1$

$$\zeta_N = \frac{d}{2\omega_N m} = 1, \quad \rightarrow d = \sqrt{2km}$$

How to choose the parameters?

Single joint: find k and d as 2 constant
Multiple joints: find $\mathbf{K}$ and $\mathbf{D}$ as 2 diagonal matrices

Proportional-Derivative Control (PD) for Multiple Joints

How to choose the parameters?

Single joint: find k and d as 2 constant
Multiple joints: find $\mathbf{K}$ and $\mathbf{D}$ as 2 diagonal matrices

In reality?

- Start with a small k, $\mathbf{K} = k\mathbf{I}$
- Slowly increase k
- The joint friction is relatively higher, normally $d < \sqrt{2km}$

Let's try

Question: Given Task-space trajectory, how to find the torques?
Can we describe the equation of motion in terms of the task-space positions, velocities, and accelerations

Task space

$$\mathbf{x} \in \mathbb{R}^6 = \begin{bmatrix} \theta \\ \mathbf{p} \end{bmatrix}, \quad \dot{\mathbf{x}} \in \mathbb{R}^6 = \begin{bmatrix} \omega \\ \dot{\mathbf{p}} \end{bmatrix}, \quad \ddot{\mathbf{x}} \in \mathbb{R}^6 = \begin{bmatrix} \dot{\omega} \\ \ddot{\mathbf{p}} \end{bmatrix}$$

Task-space Dynamics

Task space

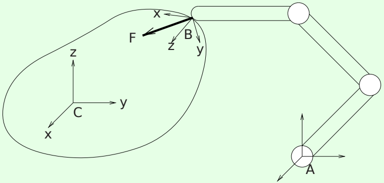
$$\mathbf{x} \in \mathbb{R}^6 = \begin{bmatrix} \theta \\ \mathbf{p} \end{bmatrix}, \quad \dot{\mathbf{x}} \in \mathbb{R}^6 = \begin{bmatrix} \omega \\ \dot{\mathbf{p}} \end{bmatrix}, \quad \ddot{\mathbf{x}} \in \mathbb{R}^6 = \begin{bmatrix} \dot{\omega} \\ \ddot{\mathbf{p}} \end{bmatrix}$$

Spatial Force/ Wrench

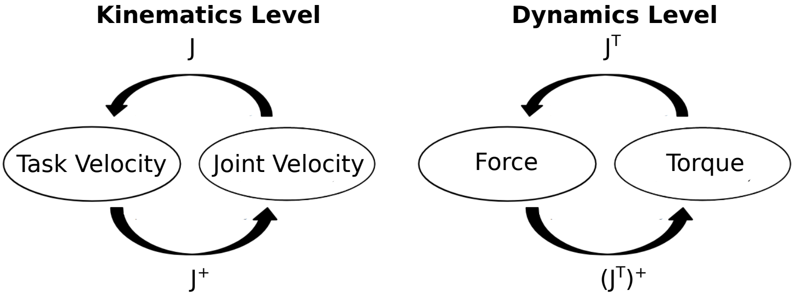
$$\mathbf{F} \in \mathbb{R}^6 = \begin{bmatrix} \mathbf{m} \\ \mathbf{f} \end{bmatrix}$$

$\mathbf{m} \in \mathbb{R}^3$ angular force / moment

$\mathbf{f} \in \mathbb{R}^3$ linear force



Question: What is the relationship between f and τ?



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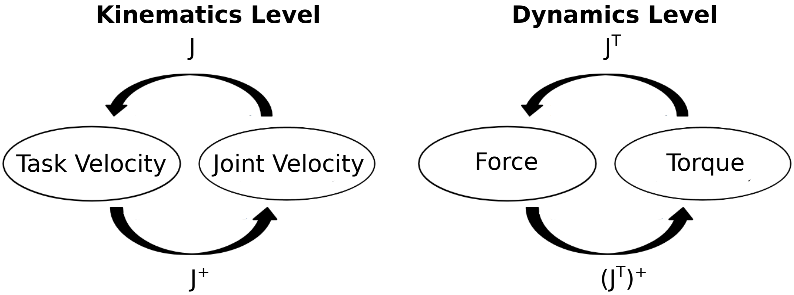

Question: Can we describe the rigid-body dynamics in terms of f and $\ddot{\mathrm{x}}$?

Configuration $\rightarrow$ Task-space

$$\dot{\mathbf{x}} = \mathbf{J}\dot{\mathbf{q}}$$

$$\ddot{\mathbf{x}} = \mathbf{J}\ddot{\mathbf{q}} + \dot{\mathbf{J}}\dot{\mathbf{q}}$$

Task-space Dynamics

Configuration $\rightarrow$ Task-space

$$\dot{x} = J\dot{q}$$

$$\ddot{x} = J\ddot{q} + \dot{J}\dot{q}$$

Task-space $\rightarrow$ Configuration

$$\dot{q} = J^{\dagger}\dot{x}$$

$$\ddot{q} = J^{\dagger}\ddot{x} - J^{\dagger}\dot{J}\dot{q}$$

Task-space Dynamics

Configuration $\rightarrow$ Task-space

$$\dot{\mathbf{x}} = \mathbf{J}\dot{\mathbf{q}}$$
$$\ddot{\mathbf{x}} = \mathbf{J}\ddot{\mathbf{q}} + \dot{\mathbf{J}}\dot{\mathbf{q}}$$

Task-space $\rightarrow$ Configuration

$$\dot{\mathbf{q}} = \mathbf{J}^{\dagger}\dot{\mathbf{x}}$$
$$\ddot{\mathbf{q}} = \mathbf{J}^{\dagger}\ddot{\mathbf{x}} - \mathbf{J}^{\dagger}\dot{\mathbf{J}}\dot{\mathbf{q}}$$

Substitute $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ into the equation of motion $\tau = \mathbf{M}\ddot{\mathbf{q}} + \mathbf{h}$

$$\tau = \mathbf{M}(\mathbf{J}^{\dagger}\ddot{\mathbf{x}} - \mathbf{J}^{\dagger}\dot{\mathbf{J}}\dot{\mathbf{q}}) + \mathbf{h}$$

Configuration $\rightarrow$ Task-space

$$\dot{\mathbf{x}} = \mathbf{J}\dot{\mathbf{q}}$$
$$\ddot{\mathbf{x}} = \mathbf{J}\ddot{\mathbf{q}} + \dot{\mathbf{J}}\dot{\mathbf{q}}$$

Task-space $\rightarrow$ Configuration

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Substitute $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ into the equation of motion $\tau = \mathbf{M}\ddot{\mathbf{q}} + \mathbf{h}$

$$\tau = \mathbf{M}(\mathbf{J}^{\dagger}\ddot{\mathbf{x}} - \mathbf{J}^{\dagger}\dot{\mathbf{J}}\dot{\mathbf{q}}) + \mathbf{h}$$

Premultiply both sides by $(\mathbf{J}^{\top})^{\dagger}$, we get

$$(\mathbf{J}^{\top})^{\dagger}\tau = (\mathbf{J}^{\top})^{\dagger}\mathbf{M}\mathbf{J}^{\dagger}\ddot{\mathbf{x}} - (\mathbf{J}^{\top})^{\dagger}\mathbf{M}\mathbf{J}^{\dagger}\dot{\mathbf{J}}\dot{\mathbf{q}} + (\mathbf{J}^{\top})^{\dagger}\mathbf{h}$$

Task-space Dynamics

$$(\mathbf{J}^\top)^\dagger \tau = (\mathbf{J}^\top)^\dagger \mathbf{M}\mathbf{J}^\dagger \ddot{\mathbf{x}} - (\mathbf{J}^\top)^\dagger \mathbf{M}\mathbf{J}^\dagger \dot{\mathbf{J}}\dot{\mathbf{q}} + (\mathbf{J}^\top)^\dagger \mathbf{h}$$

Task-space Dynamics

$$(\mathbf{J}^\top)^\dagger \tau = (\mathbf{J}^\top)^\dagger \mathbf{M}\mathbf{J}^\dagger \ddot{\mathbf{x}} - (\mathbf{J}^\top)^\dagger \mathbf{M}\mathbf{J}^\dagger \dot{\mathbf{J}}\dot{\mathbf{q}} + (\mathbf{J}^\top)^\dagger \mathbf{h}$$

Expressing $\mathbf{F} = (\mathbf{J}^\top)^\dagger \tau$

Task-space Dynamics

$$(\mathbf{J}^\top)^\dagger \tau = (\mathbf{J}^\top)^\dagger \mathbf{M}\mathbf{J}^\dagger \ddot{\mathbf{x}} - (\mathbf{J}^\top)^\dagger \mathbf{M}\mathbf{J}^\dagger \dot{\mathbf{J}}\dot{\mathbf{q}} + (\mathbf{J}^\top)^\dagger \mathbf{h}$$

Expressing $\mathbf{F} = (\mathbf{J}^\top)^\dagger \tau$

Dynamics equation expressed in taskspace

$$\mathbf{F} = \mathbf{\Lambda}\ddot{\mathbf{x}} + \eta$$
$$\text{where } \mathbf{\Lambda} = (\mathbf{J}^\top)^\dagger \mathbf{M}\mathbf{J}^\dagger$$
$$\eta = (\mathbf{J}^\top)^\dagger \mathbf{h} - \mathbf{\Lambda}\dot{\mathbf{J}}\dot{\mathbf{q}}$$

Task-space Inverse Dynamics Controller

Inputs:

- from motion plannar: $\mathbf{x}^{ref}, \dot{\mathbf{x}}^{ref}, \ddot{\mathbf{x}}^{ref}$
- from forward kineamtics: $\mathbf{x}, \dot{\mathbf{x}}$

Task-space Inverse Dynamics Controller

Inputs:

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Task-space Inverse Dynamics Controller

$$\ddot{\mathbf{x}}^{*} = \ddot{\mathbf{x}}^{ref} + \mathbf{D}(\dot{\mathbf{x}}^{ref} - \dot{\mathbf{x}}) + \mathbf{K}(\mathbf{x}^{ref} - \mathbf{x})$$

with feedback

Task-space Inverse Dynamics Controller

Inputs:

- from motion plannar: $\mathbf{x}^{ref}, \dot{\mathbf{x}}^{ref}, \ddot{\mathbf{x}}^{ref}$
- from forward kineamtics: $\mathbf{x}, \dot{\mathbf{x}}$

Task-space Inverse Dynamics Controller

$$\ddot{\mathbf{x}}^{*} = \ddot{\mathbf{x}}^{ref} + \mathbf{D}(\dot{\mathbf{x}}^{ref} - \dot{\mathbf{x}}) + \mathbf{K}(\mathbf{x}^{ref} - \mathbf{x}) \qquad \text{with feedback}$$

$$\mathbf{\Lambda} = (\mathbf{J}^{\top})^{\dagger}\mathbf{M}\mathbf{J}^{\dagger} \qquad \text{task-space mass matrix}$$

Task-space Inverse Dynamics Controller

Inputs:

- from motion plannar: $\mathbf{x}^{ref}, \dot{\mathbf{x}}^{ref}, \ddot{\mathbf{x}}^{ref}$
- from forward kineamtics: $\mathbf{x}, \dot{\mathbf{x}}$

Task-space Inverse Dynamics Controller

$\ddot{\mathbf{x}}^{*} = \ddot{\mathbf{x}}^{ref} + \mathbf{D}(\dot{\mathbf{x}}^{ref} - \dot{\mathbf{x}}) + \mathbf{K}(\mathbf{x}^{ref} - \mathbf{x})$	with feedback
$\boldsymbol{\Lambda} = (\mathbf{J}^{\top})^{\dagger}\mathbf{M}\mathbf{J}^{\dagger}$	task-space mass matrix
$\eta = (\mathbf{J}^{\top})^{\dagger}\mathbf{h} - \boldsymbol{\Lambda}\dot{\mathbf{J}}\dot{\mathbf{q}}$	task-space **h**

Task-space Inverse Dynamics Controller

Inputs:

- from motion plannar: $\mathbf{x}^{ref}, \dot{\mathbf{x}}^{ref}, \ddot{\mathbf{x}}^{ref}$
- from forward kineamtics: $\mathbf{x}, \dot{\mathbf{x}}$

Task-space Inverse Dynamics Controller

$\ddot{\mathbf{x}}^* = \ddot{\mathbf{x}}^{ref} + \mathbf{D}(\dot{\mathbf{x}}^{ref} - \dot{\mathbf{x}}) + \mathbf{K}(\mathbf{x}^{ref} - \mathbf{x})$	with feedback
$\mathbf{\Lambda} = (\mathbf{J}^\top)^\dagger \mathbf{M} \mathbf{J}^\dagger$	task-space mass matrix
$\eta = (\mathbf{J}^\top)^\dagger \mathbf{h} - \mathbf{\Lambda} \dot{\mathbf{J}} \dot{\mathbf{q}}$	task-space $\mathbf{h}$
$\mathbf{F} = \mathbf{\Lambda} \ddot{\mathbf{x}}^* + \eta$	task-space force to achieve $\ddot{\mathbf{x}}^*$

Task-space Inverse Dynamics Controller

Inputs:

- from motion plannar: $\mathbf{x}^{ref}, \dot{\mathbf{x}}^{ref}, \ddot{\mathbf{x}}^{ref}$
- from forward kineamtics: $\mathbf{x}, \dot{\mathbf{x}}$

Task-space Inverse Dynamics Controller

$\ddot{\mathbf{x}}^* = \ddot{\mathbf{x}}^{ref} + \mathbf{D}(\dot{\mathbf{x}}^{ref} - \dot{\mathbf{x}}) + \mathbf{K}(\mathbf{x}^{ref} - \mathbf{x})$	with feedback
$\mathbf{\Lambda} = (\mathbf{J}^\top)^\dagger \mathbf{M}\mathbf{J}^\dagger$	task-space mass matrix
$\eta = (\mathbf{J}^\top)^\dagger \mathbf{h} - \mathbf{\Lambda}\dot{\mathbf{J}}\dot{\mathbf{q}}$	task-space **h**
$\mathbf{F} = \mathbf{\Lambda}\ddot{\mathbf{x}}^* + \eta$	task-space force to achieve $\ddot{\mathbf{x}}^*$
$\tau = \mathbf{J}^\top \mathbf{F}$	torque to achieve $\ddot{\mathbf{x}}^*$

Example (Kinova Jaco arm)

Move hand from position A to position B
$\mathbf{q} \in \mathbb{R}^7$: joints
$\mathbf{x} \in \mathbb{R}^6$: hand position

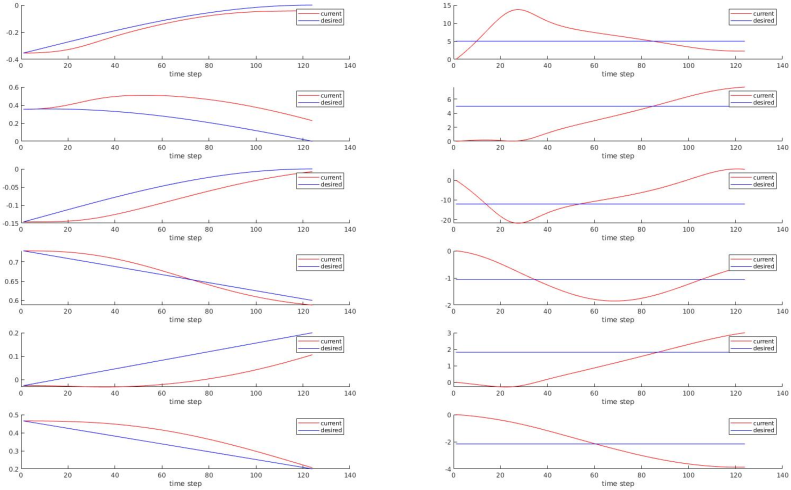
Example

Example (Kinova Jaco arm)

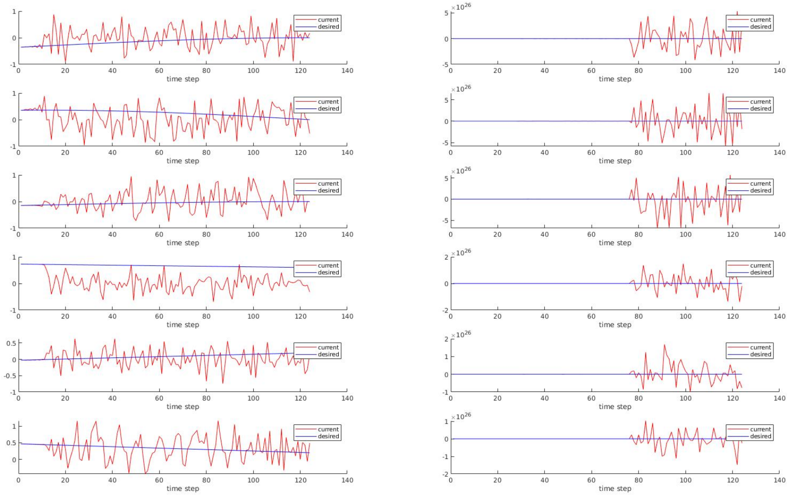
Move hand from position A to position B
$\mathbf{q} \in \mathbb{R}^7$: joints
$\mathbf{x} \in \mathbb{R}^6$: hand position

We need to choose K and D again
K and D in task-space is not the same as K and D in joint-space
Trial-and-error to get the appropriate PD gains

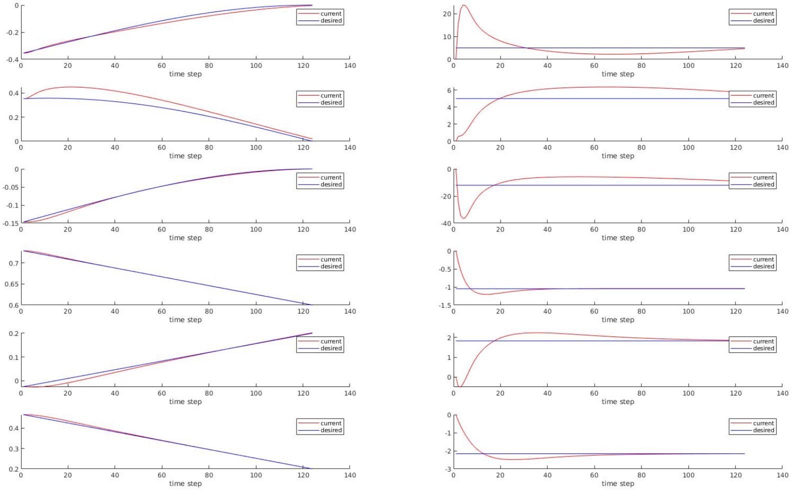
Example: Low PD gains



Example: High PD gains



Example: Good PD gains



Example

Low Gains

Good Gains

High Gains

Outline

Error Dynamics

Task-space Dynamics

Redundancy Resolution in Dynamics

Force Control

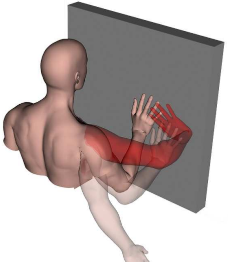
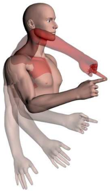
What is Redundancy?

Redundancy
More degree-of-freedom than what is required for your task
There are more than one solutions to achieve the desired task

Redundancy
More degree-of-freedom than what is required for your task
There are more than one solutions to achieve the desired task

Example

- keep your finger tip and the same position and move your elbows
- joint-space velocity but no task-space velocity

Question: What do we want?

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Kinematics

$$\dot{\mathbf{q}}_{total} = \dot{\mathbf{q}}_{task} + \dot{\mathbf{q}}_{null}$$

$$\dot{\mathbf{q}}_{task} = \mathbf{J}^{\dagger}\mathbf{x}$$

$$\dot{\mathbf{q}}_{null} = \mathbf{N}\dot{\mathbf{q}}^{0}$$

We want $\dot{\mathrm{x}}^{0} = \mathbf{0}$ for any $\dot{\mathbf{q}}_{null}$

If $\mathbf{N} = \mathbf{I} - \mathbf{J}^{\dagger}\mathbf{J}$, then $\dot{\mathrm{x}}^{0} = \mathbf{0}$

Question: What do we want?

Kinematics

$$\dot{\mathbf{q}}_{total} = \dot{\mathbf{q}}_{task} + \dot{\mathbf{q}}_{null}$$

$$\dot{\mathbf{q}}_{task} = \mathbf{J}^{\dagger}\mathbf{x}$$

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We want $\dot{\mathbf{x}}^{0} = \mathbf{0}$ for any $\dot{\mathbf{q}}_{null}$

If $\mathbf{N} = \mathbf{I} - \mathbf{J}^{\dagger}\mathbf{J}$, then $\dot{\mathbf{x}}^{0} = \mathbf{0}$

Dynamics

$$\tau_{total} = \tau_{task} + \tau_{null}$$

$$\tau_{task} = \mathbf{J}^{\top}(\mathbf{\Lambda}\ddot{\mathbf{x}} + \eta)$$

$$\tau_{null} = \mathbf{P}\tau^{0}$$

We want $\ddot{\mathbf{x}}^{0} = \mathbf{0}$ for any τ_{null}

What is $\mathbf{P}$?

Inverse Dynamics with Redundancy Resolution

$$\tau_{total} = \tau_{task} + \tau_{null}$$

$$\tau_{task} = \mathbf{J}^\top(\mathbf{\Lambda}\ddot{\mathbf{x}}^* + \eta)$$

$$\tau_{null} = \mathbf{P}\tau^0$$

$$\mathbf{P} = \mathbf{I} - \mathbf{J}^\top(\mathbf{J}\mathbf{M}^{-1}\mathbf{J}^\top)^{-1}\mathbf{J}\mathbf{M}^{-1}$$

[0] Proof available in Khatib, O (1995) Inertial properties in robotic manipulation: an object-level framework. The International Journal of Robotics Research 14(1): 19–36.

Inverse Dynamics with Redundancy Resolution

$$\tau_{total} = \tau_{task} + \tau_{null}$$
$$\tau_{task} = \mathbf{J}^\top(\mathbf{\Lambda}\ddot{\mathbf{x}}^* + \eta)$$
$$\tau_{null} = \mathbf{P}\tau^0$$
$$\mathbf{P} = \mathbf{I} - \mathbf{J}^\top(\mathbf{J}\mathbf{M}^{-1}\mathbf{J}^\top)^{-1}\mathbf{J}\mathbf{M}^{-1}$$

Dynamically consistent projection matrix

τ_{null} does not generate accelerations in the task space since

$$\mathbf{J}\mathbf{M}^{-1}\mathbf{P} = \mathbf{0}$$

[0] Proof available in Khatib, O (1995) Inertial properties in robotic manipulation: an object-level framework. The International Journal of Robotics Research 14(1): 19–36.

Question: How do we choose τ_{null} so we can go to the default position?

Let $\mathbf{q}^{ref} = \mathbf{q}^0, \dot{\mathbf{q}}^{ref} = 0, \ddot{\mathbf{q}}^{ref} = 0$

$$\begin{aligned}
\ddot{\mathbf{q}}^* &= \ddot{\mathbf{q}}^{ref} + \mathbf{D}(\dot{\mathbf{q}}^{ref} - \dot{\mathbf{q}}) + \mathbf{K}(\mathbf{q}^{ref} - \mathbf{q}) \\
\tau_0 &= \mathbf{M}\ddot{\mathbf{q}}^* + \mathbf{h} \\
\tau_{null} &= \mathbf{P}\tau_0
\end{aligned}$$

Outline

Error Dynamics

Task-space Dynamics

Redundancy Resolution in Dynamics

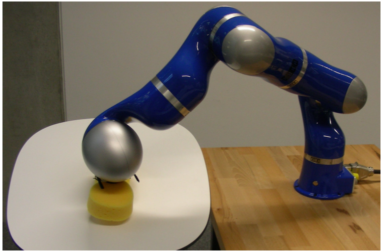
Force Control

Previously,...

$$\tau = \mathbf{M}\ddot{\mathbf{q}}^* + \mathbf{h}$$

What if my task has nothing to do with position error?

Example



$\dot{\mathbf{q}}, \ddot{\mathbf{q}} = \mathbf{0}$ since the hand is fixed at the same position

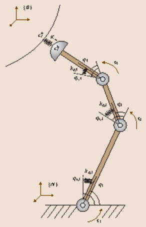
Force Control

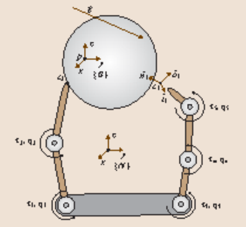
$\tau = \mathbf{h} + \mathbf{J}^\top \mathbf{f}^*$ where $\mathbf{f}^*$ is the desired force

Force Control

$$\tau = \mathbf{h} + \mathbf{J}^\top \mathbf{f}^*$$ where $\mathbf{f}^*$ is the desired force

Example

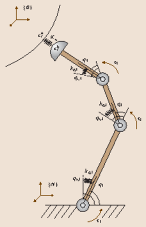


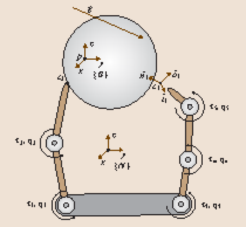


Force Control

$$\tau = \mathbf{h} + \mathbf{J}^\top \mathbf{f}^* \text{ where } \mathbf{f}^* \text{ is the desired force}$$

Example





More details when we discuss contacts and grasping

Comparisons

Question: We have learnt a few classic controllers now. Which one to use?

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Configuration level?
Task level?
Force level?

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Should I control with kinematics or dynamics?

Comparisons

	inverse kinematics	inverse dynamics
Input	$\dot{\mathbf{x}}^*$	$\ddot{\mathbf{q}}^*$, $\ddot{\mathbf{x}}^*$, or $\mathbf{f}^*$
Output	$\dot{\mathbf{q}}$ or $\mathbf{q}$	τ
Parameters	None	K, D
Advantages	Simple	Compliance Natural way to control
Disadvantages	Potentially Dangerous Cannot handle force	No garantee of convergence Harder to implement, more computations

Next time

Motion planning
Trajectory Generation
Potential Field