```
Student information
       Name: Hoang Tuan Tu
       ID: 21000709
In [ ]: # Importing library
        import numpy as np
       import pandas as pd
       from sklearn.linear_model import LinearRegression
In [ ]: # Read data
        data = []
       with open("vidu3_lin_reg.txt") as f:
           data = f.readlines()
       # Remove header row
       data = data[1:]
        # slit and cast data to float
       data = [list(map(float, x.strip().split())) for x in data]
       # Cast data to numpy array
       data = np.array(data)
       # Remove index row
       data = data[:, 1:]
       print(data)
                                           1.95]
      [[ 56.
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In [ ]: # Slit data to input data set and label
       label = data[:, len(data[0]) - 1:]
        data = data[:, :-1]
       print(data[:5])
      [[ 56. 21. 160.
                                    6. ]
                            14.
       [ 76. 18. 150.
                           12.
                                    4.97]
       [ 63. 16. 160.
                           4.4 6.39]
       [ 78. 20. 100.
                            4. 7. ]
       [ 87. 20. 110.
                            4.6 4.1 ]]
In [ ]: # Slit data to test and train
        ## Train dataset
       X_{train} = data[:80]
       Y_{train} = label[:80]
       ## Test dataset
       X_{test} = data[80:]
       Y_{test} = label[80:]
In [ ]: # Train data
       print(X_train[:5])
       print(Y_train[:5])
      [[ 56. 21. 160.
                                    6. ]
       [ 76. 18. 150.
                                    4.97]
       [ 63. 16. 160.
                                   6.39]
                            4.4
                            4. 7.]
       [ 78. 20. 100.
       [ 87. 20. 110.
                            4.6 4.1 ]]
       [[1.95]
       [1.33]
       [0.83]
       [2.]
       [1.3]]
In [ ]: # Test data
       print(X_test[:5])
       print(Y_test[:5])
      [[ 49. 24. 140.
                              4.3 5.5 ]
       [ 36. 23. 140.
                             4.3
                                    4.2
       [ 74. 21. 140.
                                    3.3 ]
                           17.
                            5.6 5.9]
       [ 53. 21. 140.
                            4.1 4.73]]
       [ 56. 19. 140.
       [[0.8]]
       [0.7]
       [1. ]
       [0.8]
       [0.89]]
In [ ]: # Define function
       def qr_householder(A):
           M = A.shape[0]
           N = A.shape[1]
           # set Q to the identity matrix
           Q = np.identity(M)
           # set R to zero matrix
           R = np.copy(A)
           for n in range(N):
           # vector to transform
               x = A[n:, n]
               k = x.shape[0]
               # compute ro=-sign(x0)||x||
               ro = -np.sign(x[0]) * np.linalg.norm(x)
               # compute the householder vector v
               e = np.zeros(k)
               e[0] = 1
               v = (1 / (x[0] - ro)) * (x - (ro * e))
               # apply v to each column of A to find R
               for i in range(N):
                   R[n:, i] = R[n:, i] - (2 / (v@v)) * ((np.outer(v, v)) @ R[n:, i])
               # apply v to each column of Q
               for i in range(M):
                   Q[n:, i] = Q[n:, i] - (2 / (v@v)) * ((np.outer(v, v)) @ Q[n:, i])
           return Q.transpose(), R
       def linear_regression(x_data, y_data):
           # add column 1
           x_bars = np.concatenate((np.ones((x_data.shape[0], 1)), x_data), axis=1)
           Q, R = qr_householder(x_bars) # QR decomposition
           R_pinv = np.linalg.pinv(R) # calculate inverse matrix of R
           A = np.dot(R_pinv, Q.T) # apply formula
           return np.dot(A, y_data)
In [ ]: # Solve
       w = linear_regression(X_train, Y_train)
       w = w.T.tolist()
       intercept = w[0][0]
       coef = w[0][1:]
       print('Intercept:', intercept)
       print("Coefficient: ", coef)
      Intercept: 0.04306436410329317
      Coefficient: [0.008989196889296797, -0.00047742422185270694, 0.002602179867555783, 0.008086342231978141, 0.007085352341923808]
In [ ]: # Predict with test data
       x = np.array(X_test)
       y_pred = np.array([intercept] * len(x))
        for i in range(len(x)):
           for j in range(len(x[0])):
               y_pred[i] += coef[j] * x[i, j]
       print(y_pred)
       [0.91012272 0.78452963 1.22339369 0.96085817 0.96836123 1.05009561
       1.19694736 1.0521678 0.9936207 0.68503543 0.78429172 0.77109845
       1.19748204 1.06844174 0.99721921 1.03380704 1.14558205 1.19228584
       1.29236178 1.26252546]
In [ ]: # Skit learn solution
       model = LinearRegression()
        model.fit(X_train, Y_train)
       print(model.coef_[0])
       print(model.intercept_[0])
       [ 0.0089892 -0.00047742 0.00260218 0.00808634 0.00708535]
      0.04306436410329706
In [ ]: # Predict with sklearn solution
       pred = model.predict(X_test)
       print(pred.T[0])
       [0.91012272 0.78452963 1.22339369 0.96085817 0.96836123 1.05009561
       1.19694736 1.0521678 0.9936207 0.68503543 0.78429172 0.77109845
       1.19748204 1.06844174 0.99721921 1.03380704 1.14558205 1.19228584
       1.29236178 1.26252546]
In [ ]: df = pd.DataFrame({"My Solution" : y_pred, "Sklearn Solution" : pred[:, 0], "Label": Y_test[:, 0]})
       print(df)
          My Solution Sklearn Solution Label
             0.910123
                              0.910123 0.80
             0.784530
                              0.784530 0.70
             1.223394
      2
                              1.223394 1.00
                              0.960858 0.80
      3
             0.960858
                              0.968361 0.89
             0.968361
                              1.050096 0.60
             1.050096
                              1.196947 1.50
             1.196947
             1.052168
                              1.052168 0.70
                              0.993621 1.10
      8
             0.993621
                              0.685035 0.81
      9
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      10
             0.784292
                              0.784292 0.70
                              0.771098 0.71
      11
             0.771098
                              1.197482 2.70
             1.197482
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16

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18

19

1.068442 0.997219

1.033807

1.145582

1.192286

1.292362

1.262525

print(mse)

6.5084895842612145

In []: # Caculate Mean Squared Error

1.068442 1.13

0.997219 1.70

1.033807 0.90 1.145582 1.16

1.192286 1.00

1.292362 0.97

1.262525 2.30

 $mse = np.sum((Y_test - y_pred) ** 2) / len(y_pred)$

In []: # caculate Variance of the Error Term
 var = np.sum((Y_test - y_pred - mse) ** 2) / len(y_pred)
 nrint(var)

print(var) 832.8930197387374