HW #3 Ryan English

11.2-2 11.3-3 11.4-1 11.4-3 11.3-4

11.2-2

Demonstrate what happens when we insert the keys 5, 28, 19, 15, 20, 33, 12, 17, 10 into a hash table with collision resolved by chaining. Let the table have 9 slots and let the hash func be h(k) = k mod 9

h(k) = k  0, 1, 2, 3, 4, 5, 6, 7, 8, ~~9~~

| h(k) | Keys |
|---|---|
| 0 mod 9 | |
| 1 mod 9 | 28 → 19 → 10 |
| 2 mod 9 | 20 |
| 3 mod 9 | 12 |
| 4 mod 9 | |
| 5 mod 9 | 5 |
| 6 mod 9 | 15 → 33 |
| 7 mod 9 | |
| 8 mod 9 | 17 |

5 = 5 mod 9 = 5
28 = 28 mod 9 = 1
19 = 19 mod 9 = 1
15 = 15 mod 9 = 6
20 = 20 mod 9 = 2
33 = 33 mod 9 = 6
12 = 12 mod 9 = 3
17 = 17 mod 9 = 8
10 = 10 mod 9 = 1

11.3-3

Consider a version of the division method in which $h(k) = k \bmod m$ where $m = 2^p - 1$ and $k$ is a character string interpreted in radix $2^p$. Show that if we can derive string $x$ from string $y$ by permuting its characters, then $x$ and $y$ hash to the same value. Give an example

$h(k) = k \bmod m$

$m = 2^p - 1$

anagrams would map to same value

Base case

$x =$ 'a' or any letter

$y =$ 'a' or any letter

~~$h(a) = a \bmod 1 = 0$~~

$h(1) = 1 \bmod 1 = 1$

Becomes a sum of the length of characters

11.4-1

Consider inserting the keys 10, 22, 31, 4, 15, 28, 17, 88, 59 into the hash table $m=11$ using open addressing with the aux hash $h'(k)=k$. Illustrate the results of inserting these keys using linear probing, using quadratic $c_1=1$ $c_2=3$, and using double hashing $h_1(k)=k$ and $h_2(k) = 1+(k \bmod (m-1))$

[10, 22, 31, 4, 15, 28, 17, 88, 59]    $h'(k)=k$    $m=11$

$h(k) = k \bmod 11$

Linear

$h(k,i) = (h'(k)+i) \bmod m = (k+i) \bmod 11$

i = collisions

Probe up to m    T = 8

| | $T_0$ | $T_1$ | $T_2$ | $T_3$ | $T_4$ (i=1) | $T_5$ | $T_6$ (i=1) | $T_7$ (i=1) | $T_8$ (i=4) |
|---|---|---|---|---|---|---|---|---|---|
| 0 mod 11 | | 22 | | | | | | x | |
| 1 mod 11 | | | | | | | | 88 | |
| 2 mod 11 | | | | | | | | | |
| 3 mod 11 | | | | | | | | | |
| 4 mod 11 | | | | 4 | x | | | | x |
| 5 mod 11 | | | | | 15 | | | | x |
| 6 mod 11 | | | | | | 28 | x | | x |
| 7 mod 11 | | | | | | | 17 | | x |
| 8 mod 11 | | | | | | | | | 59 |
| 9 mod 11 | | | 31 | | | | | | |
| 10 mod 11 | 10 | | | | | | | | |

11.4-1 Continued

Quadratic $c_1 = 1$ $c_2 = 3$

$h(k,i) = (k + i + 3i^2) \bmod 11$

| | $T_0$ | $T_1$ | $T_2$ | $T_3$ | $T_4$ | $T_5$ | $T_6$ | $T_7$ | $T_8$ |
|---|---|---|---|---|---|---|---|---|---|
| 0 mod 11 | | 22 | | | | | | * 0+1+3 | |
| 1 mod 11 | | | | | | | | | |
| 2 mod 11 | | | | | | | | | |
| 3 mod 11 | | | | | | | | | |
| 4 mod 11 | | | | 4 | * 4+1+3=8 | | | 88 | * 4+1+3 |
| 5 mod 11 | | | | | | | | | |
| 6 mod 11 | | | | | | 28 | * 6+1+3=10 | | |
| 7 mod 11 | | | | | | | 17 | | |
| 8 mod 11 | | | | 15 | | | | | 59 |
| 9 mod 11 | | 31 | | | | | | | |
| 10 mod 11 | 10 | | | | | | * 10+2+6 | | |

Double Hash $h(k,i) = (k + i(1 + k) \bmod 10)$

| | $T_0$ | $T_1$ | $T_2$ | $T_3$ | $T_4$ | $T_5$ | $T_6$ | $T_7$ | $T_8$ |
|---|---|---|---|---|---|---|---|---|---|
| 0 mod 11 | | 22 | | | | | * | * 0+2(1+mod 11) | |
| 1 mod 11 | | | | | | | | | |
| 2 mod 11 | | | | | | | 17 | * 6+5 | * 6 |
| 3 mod 11 | | | | | | | | | |
| 4 mod 11 | | | | 4 | * 4+1x(5mod10) | | | | |
| 5 mod 11 | | | | | | | | | |
| 6 mod 11 | | | | | | 28 | * 6+1x(5mod10) | | * 6+3 |
| 7 mod 11 | | | | | | | | 88 | * 7+5 |
| 8 mod 11 | | | | | | | | | 59 |
| 9 mod 11 | | 31 | | 15 | | | | | |
| 10 mod 11 | 10 | | | | | | | | |

11.4-3

Consider an open-address hash table with uniform hashing. Give upper bounds on the expected number of probes in an unsuccessful search on the expected number of probes in a successful search when the load factor is 3/4 and when it is 7/8

$\alpha = 1/1-\alpha$

$1/\alpha \ln 1/1-\alpha$

$\alpha = 3/4$

$\alpha = 3/4$

$1/1-3/4 = 4$ probes unsuccessful

$1/3/4 \ln 1/1-\alpha \approx 1.8$ successful

$\alpha = 7/8$

$1/1-7/8 = 8$ unsuccessful

$1/7/8 \ln 1/1-7/8 \approx 2.3$ successful

11.~~4~~3 3-4

Consider a hash table of size $m=1000$ and a corresponding hash function $h(k) = \lfloor m(kA \bmod 1) \rfloor$ for $A = (\sqrt{5}-1)/2$ compute the location to which the keys 61, 62, 63, 64, and 65 are mapped

$h(k) = \lfloor m(kA \bmod 1) \rfloor$ $\quad m = 1000$

$A = (\sqrt{5}-1)/2 \approx .61$

61, 62, 63, 64, 65

$h(61) = \lfloor 1000(61 \cdot .61 \bmod 1) \rfloor = 700$

$h(62) = \lfloor 1000(62 \cdot .61 \bmod 1) \rfloor = 318$

$h(63) = \lfloor 1000(63 \cdot .61 \bmod 1) \rfloor = 934$

$h(64) = \lfloor 1000(64 \cdot .61 \bmod 1) \rfloor = 554$

$h(65) = \lfloor 1000(648 .61 \bmod 1) \rfloor = 172$