Test 2 Ryan English 03/27/21

1. Consider the keys 50, 283, 198, 15, 211, 332, 124, 17, 100 into hash table $m=11$ $h'(k)=k$

Linear probing $h(k,i) = (k+i) \bmod 11$

0	198	
1	100	
2	211	*
3	332	*
4	15	*
5	124	
6	50	*
7	17	
8	283	
9		
10		

$h(50,0) = (50+0) \bmod 11 = 6$
$h(283,0) = (283+0) \bmod 11 = 8$
$h(198,0) = (198+0) \bmod 11 = 0$
$h(15,0) = (15+0) \bmod 11 = 4$
$h(211,0) = (211+0) \bmod 11 = 2$
$h(332,0) = (332+0) \bmod 11 = 2$
$h(332,1) = (332+1) \bmod 11 = 3$
$h(124,0) = (124+0) \bmod 11 = 3$
$h(124,1) = (124+1) \bmod 11 = 4$
$h(124,2) = (124+2) \bmod 11 = 5$
$h(17,0) = (17+0) \bmod 11 = 6$
$h(17,1) = (17+1) \bmod 11 = 7$
$h(100,0) = (100+0) \bmod 11 = 1$

50, 283, 198, 15, 211, 332, 124, 17, 100

Quadratic $h(k,i) = (k + i + 3i^2) \bmod 11$

0	198		$h(50,0) = (50+0+0) \bmod 11 = 6$
1	332	*	$h(283,0) = (283+0+0) \bmod 11 = 8$
2	211	*	$h(198,0) = (198+0+0) \bmod 11 = 0$
3	124		$h(15,0) = (15+0+0) \bmod 11 = 4$
4	15		$h(211,0) = (211+0+0) \bmod 11 = 2$
5	100		$h(332,0) = (332+0+0) \bmod 11 = 2$
6	50	*	$h(332,1) = (332+1+3^2) \bmod 11 = 1$
7			$h(124,0) = (124+0+0) \bmod 11 = 3$
8	283		$h(17,0) = (17+0+0) \bmod 11 = 6$
9			$h(17,1) = (17+1+3) \bmod 11 = 10$
10	21		$h(100,0) = (100+0+0) \bmod 11 = 1$
			$h(100,1) = (100+1+3) \bmod 11 =$

50, 283, 198, 15, 211, 332, 124, 17, 100

Double Hashing $h(k,i) = (k + i(1 + k \bmod m-1)) \bmod 11$

0	198	✗
1	100	
2	211	✗ ✗
3	124	✗
4	15	
5	332	✗
6	50	
7		
8	283	✗
9		
10	17	

$h(50,0) = (50+0) \bmod 11 = 6$

$h(283,0) = (283+0) \bmod 11 = 8$

$h(198,0) = (198+0) \bmod 11 = 0$

$h(15,0) = (15+0) \bmod 11 = 4$

$h(211,0) = (211+0) \bmod 11 = 2$

$h(332,0) = (332+0) \bmod 11 = 2$

$h(332,1) = (332 + (1 + 332 \bmod 10))$

$(332 + 1 + 2) \bmod 11 = 5$

$h(124,0) = (124+0) \bmod 11 = 3$

$h(17,0) = (17+0) \bmod 11 = 6$

$h(17,1) = (17 + (1 + 17 \bmod 10)$

$(17+8) \bmod 11 = 3$

$h(17,2) = (17 + 2(1 + 17 \bmod 10)$

$(17 + 2(8)) = (17+16) \bmod 11 = 0$

$h(17,3) = (17 + 3(8)) = 41 \bmod 11 = 8$

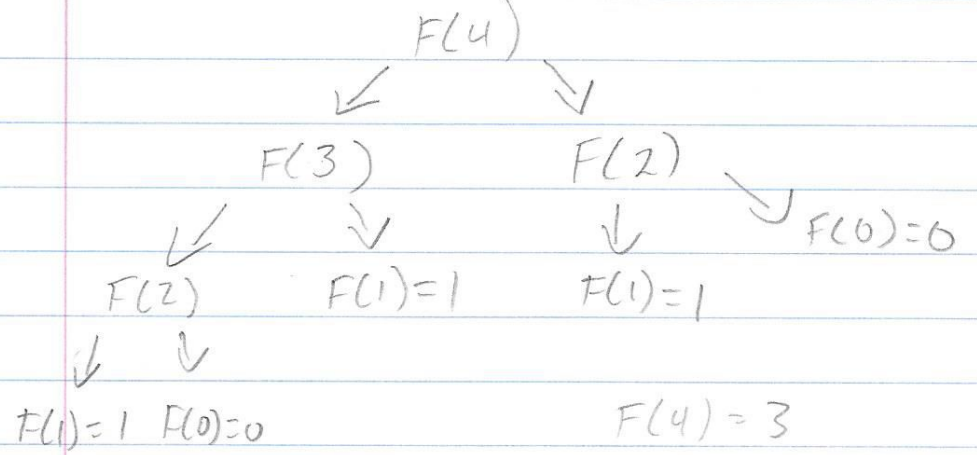
$h(17,4) = (17 + 4(8)) = 49 \bmod 11 = 5$

$h(17,5) = (17 + 5(8)) = 57 \bmod 11 = 2$

$h(17,6) = (17 + 6(8)) = 65 \bmod 11 = 10$

$h(100,0) = (100+0) \bmod 11 = 1$

2. Draw recursive tree F to compute F(4)



$V(n) = 1 + V(n-1) + V(n-2) \quad n > 1$

$V(0) = V(1) = 1$

Draw Table to compute $V(9)$

n	0	1	2	3	4	5	6	7	8	9
V(n)	1	1	3	5	9	15	25	41	67	109

$E(n) = 2 + E(n-1) + E(n-2) \quad n > 1$

$E(0) = E(1) = 0$

Draw table to compute $E(9)$

n	0	1	2	3	4	5	6	7	8	9
E(n)	0	0	2	4	8	14	24	40	66	108

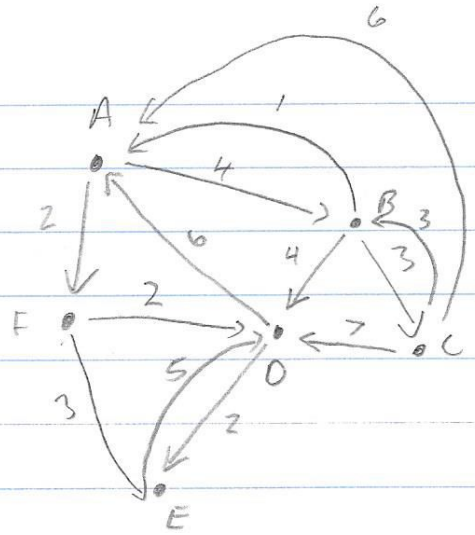
Write a single DP algorithm to compute $F(n), V(n), E(n)$

Running time is same as Fib DP

$T(n) = \Theta(n)$

```
function Fib(n)
{
  f0 = f1 = 1
  V0 = V1 = 1
  E0 = E1 = 0
  Y=0 = V=1 E=0
  if n>1
    for i=2 to n
    {
      Y = f0 + f1
      f0 = f1
      f1 = Y
      E = E0 + E1 + 2
      E0 = E1
      E1 = E
      V = V0 + V1 + 1
      V0 = V1
      V1 = V
```

3.

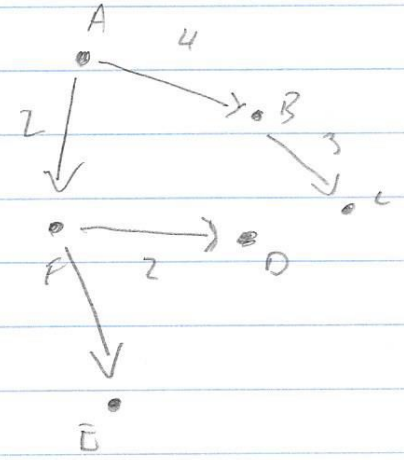


Execute Dijkstra s=A

A	B	C	D	E	F
0	∞	∞	∞	∞	∞
	4	∞	∞	∞	[2]
	[4]	∞	4	5	
		7	[4]	5	
		7		[5]	
		[7]			

A
A, F
A, F, B
A, F, B, D
A, F, B, D, E
A, F, B, D, E, C

TREE



Running Time

$T(n) = O(n^2 \lg(n))$

4. 4 matrices $A_1\ A_2\ A_3\ A_4$

$d = (30, 1, 40, 10, 25)$

Number of rows and columns of

$C = A_1 * A_2 * A_3 * A_4$

$C = 30 \times 25$ matrix

DP of mcm

Using my program for Project two we get

$C = A_1 * ((A_2 * A_3) * A_4)$

# of operations = 1,400

Verify by hand

$M(i,j) = \min[\, M(i,k) + M(k+1,j) + d_{i-1} * d_k * d_j \,]$

$$M = \begin{bmatrix} 0 & 1200 & 700 & 1400 \\ 0 & 0 & 400 & 650 \\ 0 & 0 & 0 & 1000 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$k = 1$

$M(1,2) = M(1,1) + M(2,1) + 30 * 1 * 40$

$M(1,3) = M(1,1) + M(2,3) + 30 * 1 * 40$

$M(1,2) + M(3,3) + 30 * 40 * 10$

$$k = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$M(1,4) =$

Look on next page for calculations

$\min[m(i,k) + m(k+1,j) + d_{i-1} * d_k * d_j]$

$m(1,2) = m(1,1) + m(2,1) + 30 * 1 * 40 = 1200 \quad k=1$
(0 + 0)

$m(1,3) = m(1,1) + m(2,3) + 30 * 1 * 10 = 700 \quad k=1$
(0 + 400)

$m(1,2) + m(3,3) + 30 * 40 * 10 = 13,200$
(1200 + 0)

$m(1,4) = m(1,1) + m(2,4) + 30 * 1 * 25 = 1400 \quad k=1$
(0 + 650)

$m(1,2) + m(3,4) + 30 * 40 * 25 = 41,200$
(1200 + 10,000)

$m(1,3) + m(4,4) + 30 * 10 * 25 = 8200$
(700 + 0)

$m(2,3) = m(2,2) + m(3,3) + 1 * 40 * 10 = 400 \quad k=2$
(0 + 0)

$m(2,4) = m(2,2) + m(3,4) + 1 * 40 * 25 =$
(0 + 0)

$m(2,3) + m(4,4) + 1 * 10 * 25 = 650 \quad k=3$
(400 + 0)

$m(3,4) = m(3,3) + m(4,4) + 40 * 10 * 25 = 10,000 \quad k=3$
(0 + 0)

Running time of DP mcm

$T(n) = O(n^3)$