

## Function Sheet 2

### Definition 3.9- Negative Binomial Probability Distribution

$$p(y) = \binom{y-1}{r-1} p^r q^{y-r}$$

### Theorem 3.9- Mean and Standard Deviation for Negative Binomial Distribution

$$\mu = E(Y) = \frac{r}{p} \text{ and } \sigma^2 = V(Y) = \frac{r(1-p)}{p^2}$$

### Definition 3.10- Hypergeometric Probability Distribution

$$p(y) = \frac{\binom{r}{y} \binom{N-r}{r-y}}{\binom{N}{n}}$$

### Theorem 3.10- Mean and Standard Deviation for Hypergeometric Distribution

$$\mu = E(Y) = \frac{nr}{N} \text{ and } \sigma^2 = V(Y) = n \left( \frac{r}{N} \right) \left( \frac{N-r}{N} \right) \left( \frac{N-n}{N-1} \right)$$

### Definition 3.11- Poisson Probability Distribution

$$p(y) = \frac{\lambda^y}{y!} e^{-\lambda}, \quad y = 0, 1, 2, \dots, \lambda > 0$$

### Theorem 3.14- Tchebysheff's Theorem

$$P(|Y - \mu| < k\sigma) \geq 1 - \frac{1}{k^2} \text{ or } P(|Y - \mu| \geq k\sigma) \leq \frac{1}{k^2}$$

### Definition 4.1- Cumulative Distribution Function

Let  $Y$  denote any random variable. The distribution function of  $Y$ , denoted by  $F(y)$ , is such that  $F(y) = P(Y \leq y)$  for  $-\infty < y < \infty$ .

### Definition 4.3- Probability Density Function

$$f(y) = \frac{dF(y)}{dy} = F'(y)$$

**Definition 4.6- Uniform Probability Distribution**

$$f(y) = \begin{cases} \frac{1}{\Theta_2 - \Theta_1}, & \Theta_1 \leq y \leq \Theta_2 \\ 0, & \text{elsewhere} \end{cases}$$

**Definition 4.9- Gamma Probability Distribution**

$$f(y) = \begin{cases} \frac{y^{\alpha-1} e^{-y/\beta}}{\beta^\alpha \Gamma(\alpha)}, & 0 \leq y < \infty \\ 0, & \text{elsewhere} \end{cases}$$

where

$$\Gamma(\alpha) = \int_0^\infty y^{\alpha-1} e^{-y} dy$$

**Definition 4.10- Chi-Square Distribution**

*Let  $\nu$  be a positive integer. A random variable  $Y$  is said to have a chi-square distribution with  $\nu$  degrees of freedom if and only if  $Y$  is a gamma-distributed random variable with parameters*

$$\alpha = \nu/2 \quad \text{and} \quad \beta = 2$$

**Theorem 4.9- Mean and Standard Deviation for Chi-Square Distribution**

$$\mu = E(Y) = \nu \quad \text{and} \quad \sigma^2 = V(Y) = 2\nu$$

**Definition 4.11- Exponential Distribution**

$$f(y) = \begin{cases} \frac{1}{\beta} e^{-y/\beta}, & 0 \leq y < \infty \\ 0, & \text{elsewhere} \end{cases}$$

**Theorem 4.10- Mean and Standard Deviation for Exponential Distribution**

$$\mu = E(Y) = \beta \quad \text{and} \quad \sigma^2 = V(Y) = \beta^2$$