AEC Project

Final-Evaluations

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Analog Electronic Circuits

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What's a Quadrature Downconverter?

- Quadrature down converters (QDCs) are a type of mixer used in radio receivers to convert a high-frequency signal to a lower-frequency signal. QDCs are used in a variety of applications, including satellite communications, cellular communications, and radar.

- The Down-conversion uses two mixers, one fed with a **cosine** wave and the other with **sine wave** to produce in-phase (v_{IF_I}) and quadrature-phase (v_{IF_Q}) intermediate frequency (IF) signals, respectively, which are 90° apart in phase.

Block Diagram of the QDC

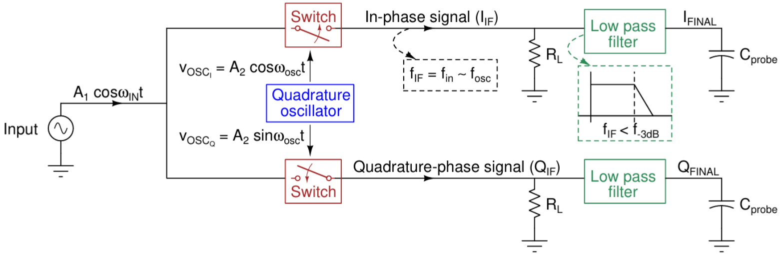

Figure: The Quadrature Downconverter

Components involved in a QDC

As can be seen in the figure above, the QDC can be simply broken into 3 parts:

- The **Oscillator** generates sinusoidal signals of a certain target frequency (for us, 100 kHz), and with some topologies we can also have control over the amplitude ($1V_{pp}$ in our case). These sinusoidal signals (which as mentioned above have a 90° phase difference), are then fed into mixers. They are called *quadrature* oscillators as, in mathematics, "quadrature" describes things that are at right angles (90°) to each other.

- A **Mixer** works on the principle of the switching action of MOSFETs, mixing of two signals is equivalent to their multiplication. We have implemented the mixer using a single gate MOSFET. The input signal $v_{in} = A_1 cos(\omega_{IN} t)$ is mixed with $v_{OSC_I} = A_2 cos(\omega_{OSC} t)$ and $v_{OSC_Q} = A_2 cos(\omega_{OSC} t)$ to produce the in-phase (v_{IF_I}) and quadrature-phase (i.e., with a phase offset of one-quarter cycle (90 degrees)) (v_{IF_Q}) intermediate frequency (IF) signals, respectively.

- The Low-Pass Filter is given the mixed signal output to pass only the IF signals with frequency $\omega_{IF} = (\omega_{IN} - \omega_{OSC})$, which can be a sufficiently low value for sufficiently high values of ω_{IN} and ω_{OSC}.

- So essentially, the input signal is brought into it's baseband (kind of like a de-modulation of passband) through the QDC, as the oscillator generates sinusoids of frequency 100kHz, which when mixed with the input, gives us the baseband, and some other frequency components, which are filtered out using the LPF, thus obtaining the important low frequency message signal.

An example is of the Narrow-band signal model as given below:

In an angle modulation application, with carrier frequency f, φ is also a time-variant function, giving:

$$\begin{aligned} A(t)\cdot\cos[2\pi ft+\varphi(t)] &= \cos(2\pi ft)\cdot A(t)\cos[\varphi(t)] \;+\; \cos\left(2\pi ft+\tfrac{\pi}{2}\right)\cdot A(t)\sin[\varphi(t)] \\ &= \underbrace{\cos(2\pi ft)\cdot A(t)\cos[\varphi(t)]}_{\text{in-phase}} \;-\; \underbrace{\sin(2\pi ft)\cdot A(t)\sin[\varphi(t)]}_{\text{quadrature}}. \end{aligned}$$

Figure: Narrowband Signal Model [Source: Wikipedia]

How does an Oscillator select a particular frequency?

- Background Noise: Electronic circuits always contain a degree of random noise spanning a wide range of frequencies. This noise provides the initial "seed" for oscillations.
- Now, Oscillators contain a resonant circuit (like in our case, an RC circuit) that has a preferred frequency of oscillation. This circuit acts like a filter, favoring a specific frequency while suppressing others.
- The op-amp takes a portion of the filtered signal and feeds it back to the resonant circuit's input. If the loop gain (amplification within this cycle) is greater than one, the oscillation grows with each pass.
- For sustained oscillations, two conditions must be met, known as the **Barkhausen criterion**:
  - Loop Gain: The total gain around the feedback loop must be equal to or slightly greater than one.
  - Phase Shift: The total phase shift around the feedback loop must be 0 degrees or an integer multiple of 360 degrees, ensuring the feedback signal reinforces itself.

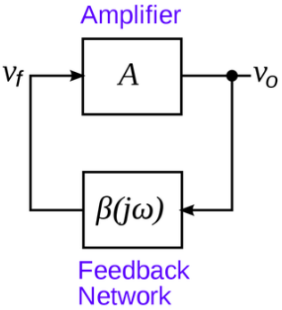

Figure: Block diagram for Barkhausen criterion, $A\beta \geq 1$ for Oscillation

- Only the (unique) frequency which satisfies the Barkhausen criterion is selected by the oscillator, which is the frequency given out as output by the oscillator.

The Quadrature Oscillator: Abstract

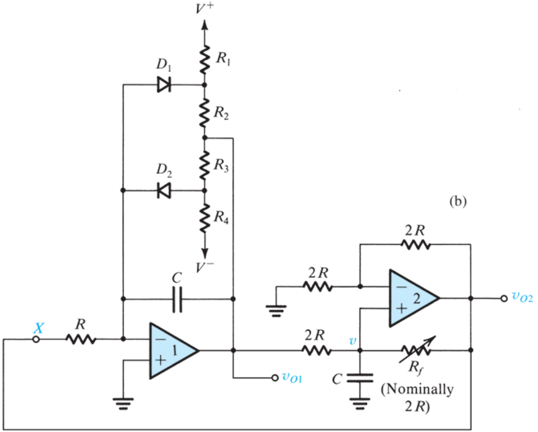

Figure: Quadrature Oscillator

- Our quadrature oscillator circuit from above, has 3 essential components, the **inverting integrator** (circuit related to op-amp 1 in above figure), the two diodes and the resistances used for the **limiter circuit** for amplitude control of the output, and the **non-inverting integrator** (related to op-amp 2 in the figure above), also called the deboo-integrator.

- The outputs of the inverting integrator and the deboo-integrator are sinusoids with 90 degrees phase difference.

- Reason for choosing this Oscillator: We are using a quadrature oscillator to produce a sine wave and then a 90 degree shifted cosine wave. Since sine and cosine integrate into one another, it is intuitive to use a sequence of integrator circuits to realize the oscillator. As we will see, the two phase shifts correspond to the transfer functions j and -j, and the product of these is equal to 1 which perfectly fits the oscillation criteria.

Parts of Quadrature Oscillator - 1) Inverting Integrator

Let us now understand the Quadrature Oscillator part by part, with correspondence to our LTSpice simulations.

- For LTSpice, we assume no offset for our op-amp. Thus, our inverting integrator configuration will be as follows:

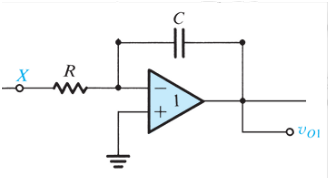

Figure: Inverting Integrator Circuit

In negative feedback we can employ the virtual ground concept (assuming it is in linear mode, and since we have negative feedback), to assume that the voltage at the inverting terminal is grounded too. Now, we can equate the currents flowing in/out of the inverting terminal.

Once we write this equation $\frac{-V_{in}}{R} = C\frac{dV_{out}}{dt}$, which means in the equation, $V_{out} = \frac{-1}{RC}\int v_{\text{in}}\, dt$ we take the laplace transform on both sides, and get the relation $\frac{V_{out}(s)}{V_{in}(s)} = \frac{-1}{j\omega RC}$ $(s = j\omega)$.

Now, we have tuned the circuit such that $\omega = \frac{1}{RC}$, giving us, $\frac{V_{out}(s)}{V_{in}(s)} = j$.

Hence, we achieve a 90° phase shift!

Non-Inverting Amplifier

In this step, we want to shift the phase by -90° to obtain a $1V_{pp}$ cosine wave. For this, we can use a non-inverting amplifier, which uses the configuration as below:

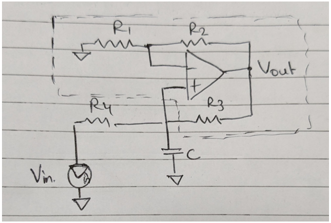

Figure: Non-Inverting Integrator

To analyze the integrating nature of the circuit, and derive the transfer function, we find the equivalent circuit of the section enclosed in the dotted lines above.

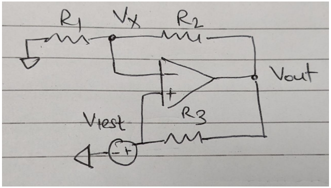

Figure: Analyzing the transfer function

To find the equivalent impedance of the circuit, we apply a test source and measure the test current. No current flows through the terminals of the op-amp, thus we have, $i_{test} = \frac{V_{test} - V_{out}}{R_3}$. By voltage division, we get $V_X = \frac{R_1}{R_1+R_2} V_{out}$. So, we get $V_{out} = \frac{R_1+R_2}{R_1} V_X$.

Thus we have $i_{test} = \frac{V_{test} - \frac{R_1+R_2}{R_1} V_X}{R_3}$ We realize that for the configuration to be stable, we must assume virtual shorting (negative feedback! and ofcourse, assumption of linear mode :)). Hence,
$V_+ = V_- \implies V_{test} = V_X$

Thus we have,

$$i_{test} = V_{test}\left(\frac{1}{R_3} - \frac{R_1 + R_2}{R_3 R_1}\right)$$

$$\therefore \frac{V_{test}}{i_{test}} = -\frac{R_1 R_3}{R_2}$$

We obtain a negative impedance! This denotes an opposite direction of the current. Now substituting this in the circuit:

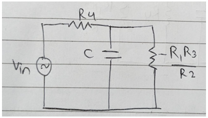

Figure: The circuit we now get

We can convert the series voltage source R_4 into a current source, parallel to R_4,

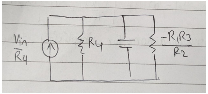

Figure: voltage source to current source conversion

Note that the configuration below is an integrator

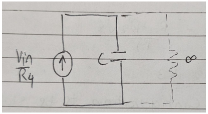

Figure: Integrator

..from the equation: $i = C\frac{dV_c}{dt}$

Hence, $\boxed{V_c = \frac{1}{R_4 C}\int V_{\text{in}}\, dt}$

Thus, this is equivalent to an ideal current source (infinite parallel resistance!), So we take:

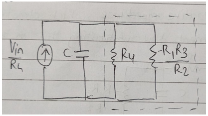

Figure: Equivalent Model

$$\frac{1}{R_{parallel}} = \frac{1}{R_4} + \frac{1}{\left(\frac{-R_1 R_3}{R_2}\right)}$$

We can obtain $R_{parallel} = \infty$, if

$$\boxed{\frac{R_1 R_3}{R_2} = R_4}$$

and thus we have found the integrator condition. For simplicity, take $R_1 = R_2 = R_3 = R_4 = R'$ Now we can compute the transfer function. We know: $\frac{V_{in(s)}}{sR'C} = V_c(s)$

Since we have assumed inverting and non-inverting terminals are at the same voltage, and the resistors have the same values: $\frac{V_{in}(s)}{j\omega R'C} = \frac{V_{out}(s)}{2}$ Thus,

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{2}{\omega R'C} \cdot -j$$

As we recall from the inverting integrator, $\omega = \frac{1}{RC}$. We will use the same capacitance for our non-inverting integrator.

$$\frac{V_{out}(s)}{V_{in}(s) = \frac{2}{(\frac{1}{RC})R'C} = (\frac{2R}{R'}) \cdot -j}$$

Since we want to achieve a transfer of -j, (-90° phase shift), we must set $\boxed{R' = 2R}$

Thus, the overall loop gain $= H(s)_1 \cdot H(s)_2 = j \cdot -j = 1$, which satisfies the criterion for oscillation.

The Mixer as a Switch - Explained

- The mixer is the component of the down converter that returns the modulated signal to the base-band by re-multiplying it with the carrier signals.
- To implement the mixer, we use a MOSFET as a switch. It is essential to provide a bias voltage at the gate of the MOSFET that is close to the threshold voltage.
- The key working principle is as follows: We utilize the oscillator signal to switch the MOSFET on and off, since oscillator can swing the bias point by $\pm 500mV$. Thus, the MOSFET will be continuously switching between ON(linear) and OFF(cutoff) depending on which part of the cycle the oscillator signal is in.

Mixer, Continued

- As the MOSFET rapidly switches on and off, we are essentially **sampling** the input voltage: when I_{DS} is non-zero in linear, so we will have an output that reflects the change in the source voltage, where we are providing the input to the mixer.

- Sampling is analogous to multiplication with a train of impulses in the time domain. This translates to **convolution** in the frequency domain with a train of impulses. This convolution is what results in the production of many harmonics, the strongest of which are the frequency of the input + the fundamental frequency of the oscillator. This is why we observe strong peaks at $\omega_{IN} - \omega_{OSC}$ and $\omega_{IN} + \omega_{OSC}$!
- Since the signal is a real signal, we know the Fourier Transform is symmetric about the y-axis.

Mixer in LTSpice with Simulations

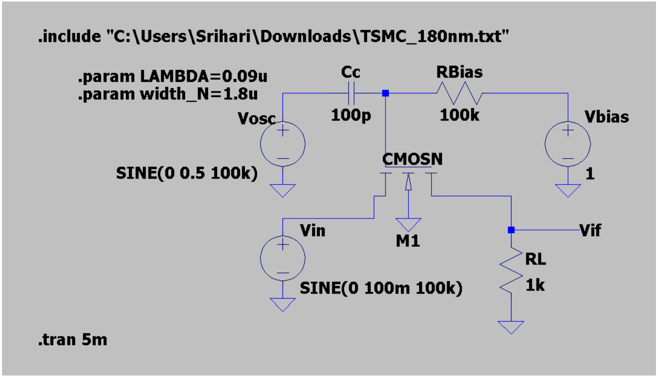

Figure: Mixer(Switch) Circuit

Above is the image of the Mixer circuit (single gate MOSFET) in LTSpice. Now below are the transient plots for different frequencies of input.

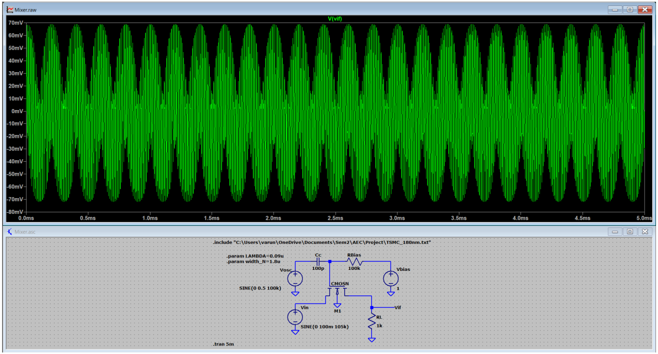

Figure: Mixer(Switch) Circuit Output Waveforms

Here is a sample image of how the output waveforms looks like. Upon zooming in, we find that the signal is the product of 2 sines (or a sine and cosine, depending on the input, we have taken 2 sines without loss of generality).

Circuit for $f_{IN} = 95kHz$

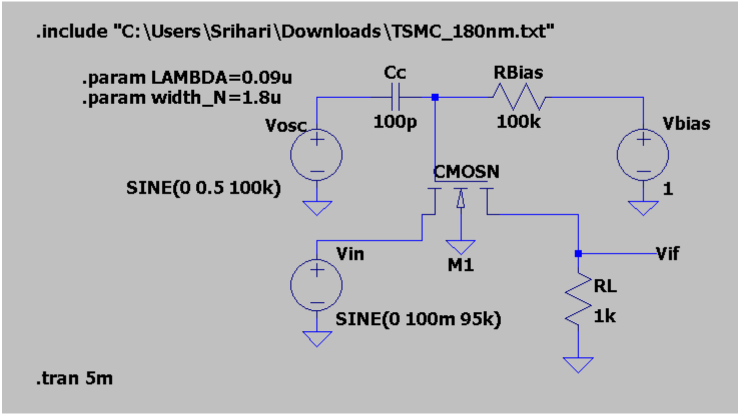

Figure: Mixer(Switch) Circuit for $f_{IN} = 95k$

Transient Plot for $f_{IN} = 95kHz$

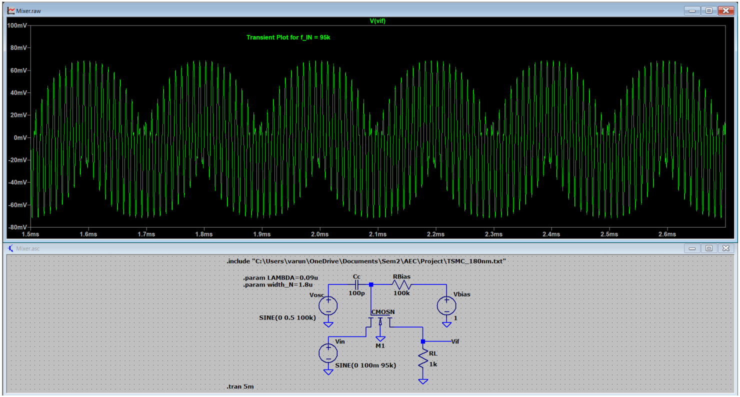

Figure: Mixer(Switch) Circuit Output Waveforms for $f_{IN} = 95k$

FFT Plot for $f_{IN} = 95kHz$

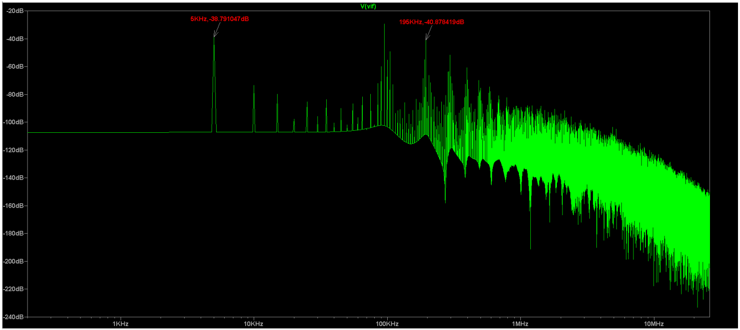

Figure: Mixer(Switch) FFT Waveform for $f_{IN} = 95k$

We see harmonics at 5KHz and 195kHz as expected by theory.

Circuit for $f_{IN} = 98kHz$

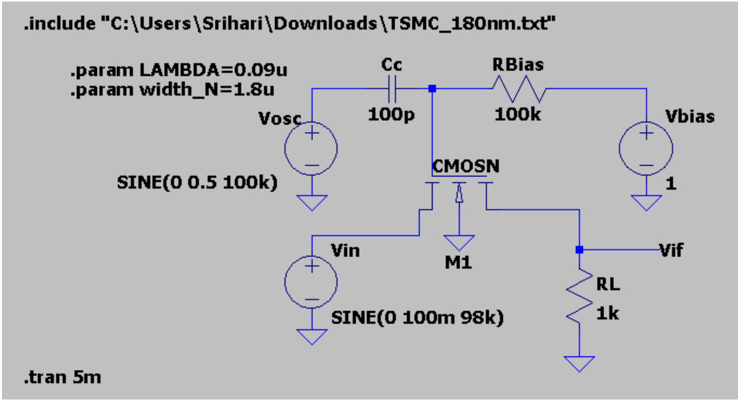

Figure: Mixer(Switch) Circuit for $f_{IN} = 98k$

Transient Plot for $f_{IN} = 98kHz$

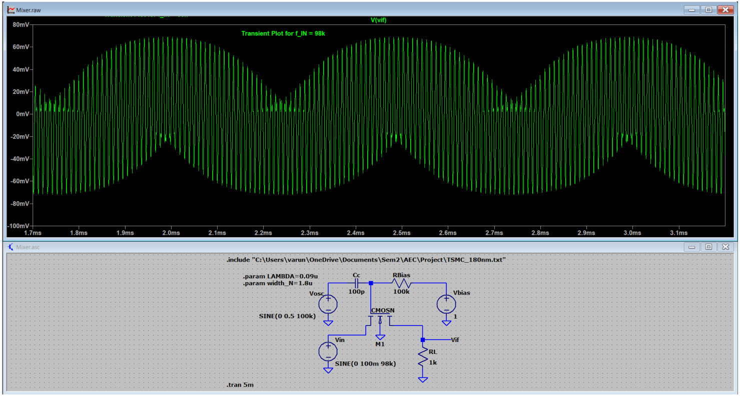

Figure: Mixer(Switch) Circuit Output Waveforms for $f_{IN} = 98k$

FFT Plot for $f_{IN} = 98kHz$

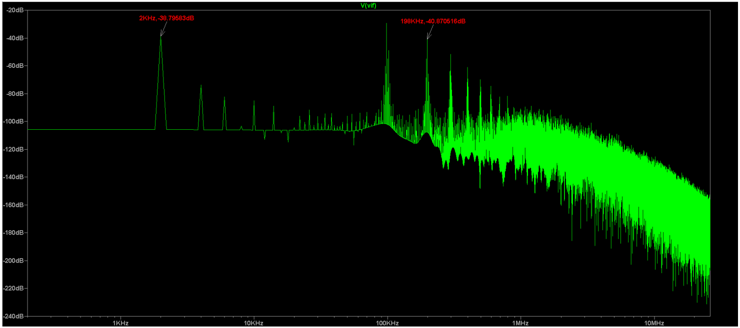

Figure: Mixer(Switch) FFT Waveform for $f_{IN} = 98k$

We see harmonics at 2KHz and 198kHz as expected by theory.

Circuit for $f_{IN} = 99kHz$

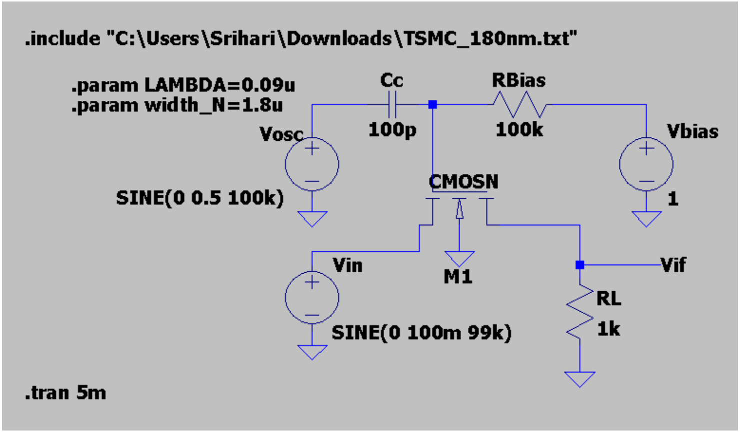

Figure: Mixer(Switch) Circuit for $f_{IN} = 99k$

Transient Plot for $f_{IN} = 99kHz$

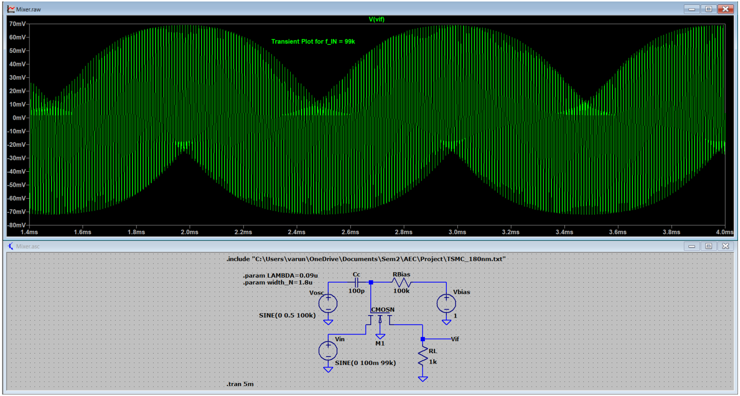

Figure: Mixer(Switch) Circuit Output Waveforms for $f_{IN} = 99k$

FFT Plot for $f_{IN} = 99kHz$

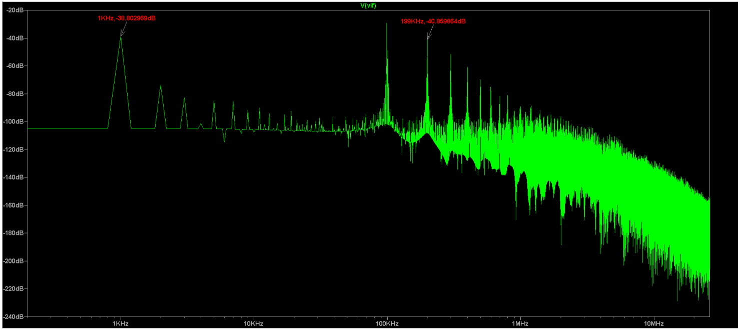

Figure: Mixer(Switch) FFT Waveform for $f_{IN} = 99k$

We see harmonics at 1KHz and 199kHz as expected by theory.

Circuit for $f_{IN} = 101kHz$

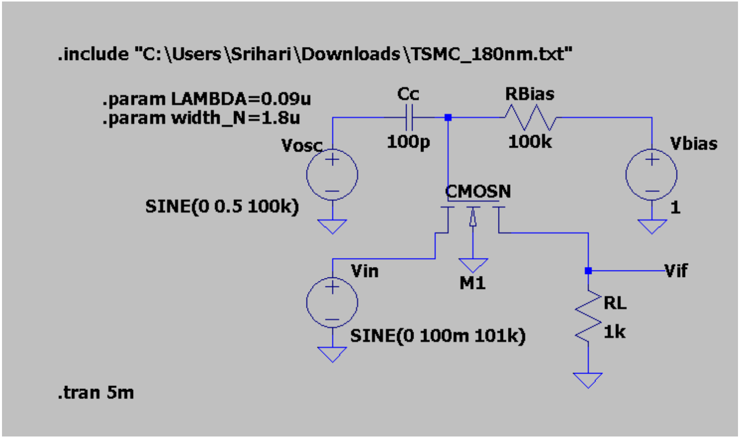

Figure: Mixer(Switch) Circuit for $f_{IN} = 101k$

Transient Plot for $f_{IN} = 101kHz$

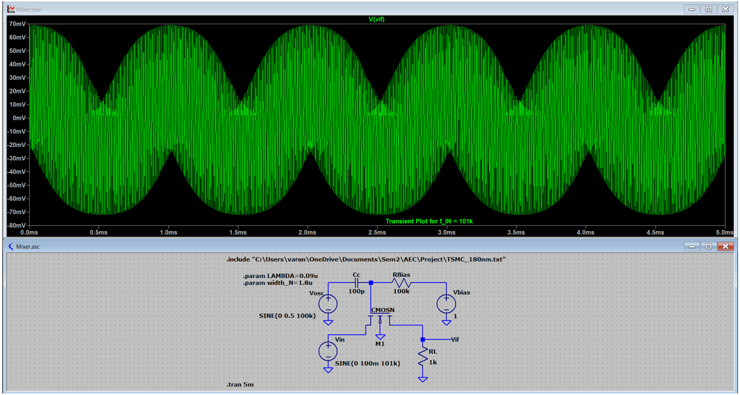

Figure: Mixer(Switch) Circuit Output Waveforms for $f_{IN} = 101k$

FFT Plot for $f_{IN} = 101kHz$

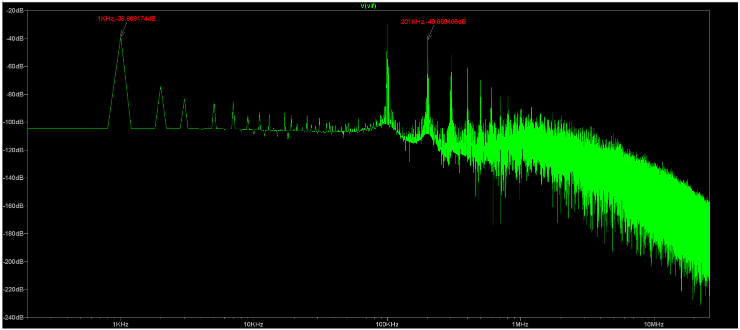

Figure: Mixer(Switch) FFT Waveform for $f_{IN} = 101k$

We see harmonics at 1KHz and 201kHz as expected by theory.

Circuit for $f_{IN} = 102kHz$

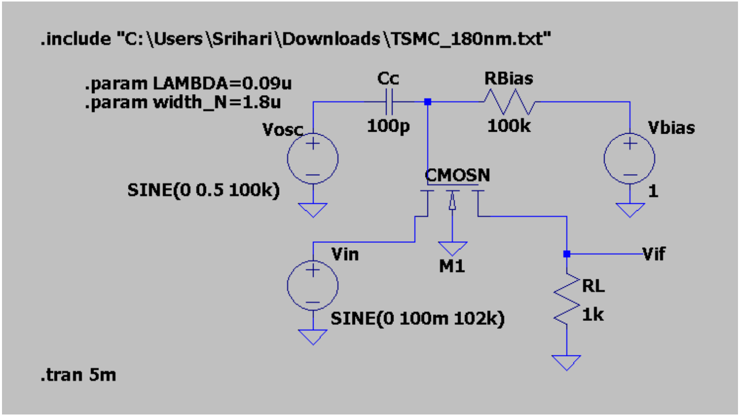

Figure: Mixer(Switch) Circuit for $f_{IN} = 102k$

Transient Plot for $f_{IN} = 102kHz$

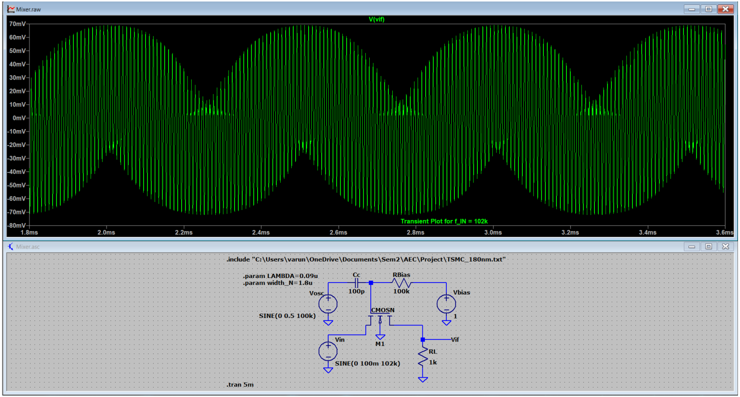

Figure: Mixer(Switch) Circuit Output Waveforms for $f_{IN} = 102k$

FFT Plot for $f_{IN} = 102kHz$

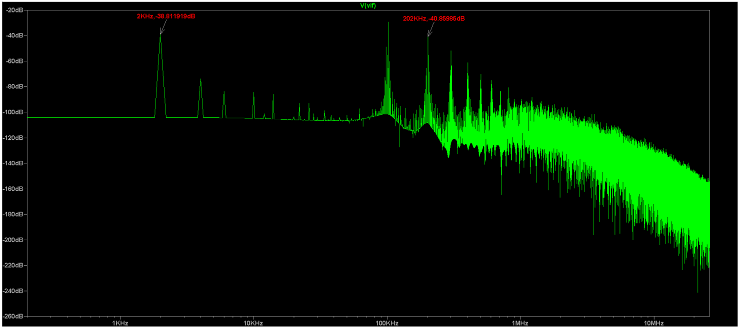

Figure: Mixer(Switch) FFT Waveform for $f_{IN} = 102k$

We see harmonics at 2KHz and 202kHz as expected by theory.

Circuit for $f_{IN} = 105kHz$

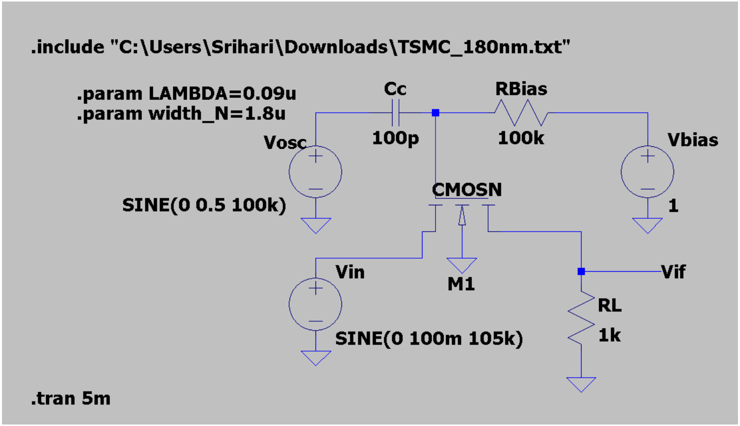

Figure: Mixer(Switch) Circuit for $f_{IN} = 105k$

Transient Plot for $f_{IN} = 105kHz$

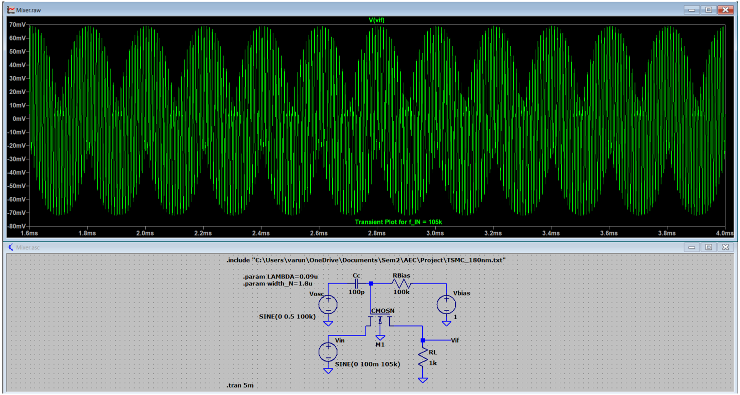

Figure: Mixer(Switch) Circuit Output Waveforms for $f_{IN} = 105k$

FFT Plot for $f_{IN} = 105kHz$

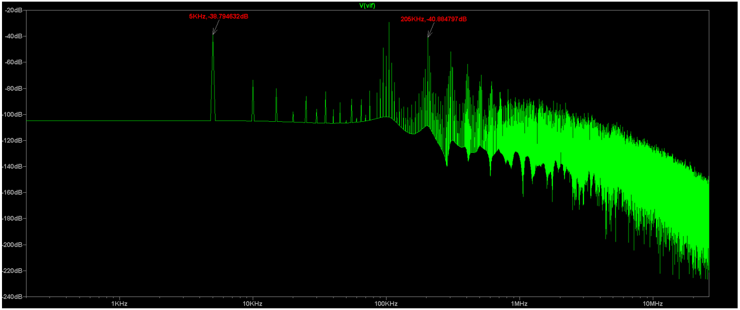

Figure: Mixer(Switch) FFT Waveform for $f_{IN} = 105k$

We see harmonics at 5KHz and 205kHz as expected by theory.