Abstract

Proof that $\sum_{p=1}^{n} n - 2p + 1 = 0$ where $(n, p) \epsilon \mathbb{N}$.

1 Lemmas

1.1 Lemma 1

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ where $n \epsilon \mathbb{N}$. While this is well known, a proof is included for completeness. **Proof**: The sum of all natural numbers from 1 to n equals the sum from n to 1. Hence, $\sum_{i=1}^{n} i = \sum_{i=1}^{n} n + 1 - i$. Therefore, $\sum_{i=1}^{n} i + \sum_{i=1}^{n} n + 1 - i = 2\sum_{i=1}^{n} i$. Adding the two sums on the left gives $\sum_{i=1}^{n} i + n + 1 - i = \sum_{i=1}^{n} n + 1$, which is simply n sums of $n + 1$, or $n(n + 1)$. This gives $2\sum_{i=1}^{n} i = n(n + 1)$, and therefore $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

2 Proof

$\sum_{p=1}^{n} n - 2p + 1$ may be split into its parts, giving $\sum_{p=1}^{n} n - 2\sum_{p=1}^{n} p + \sum_{p=1}^{n} 1$. The first term is simply n sums of n, or n^2. The second term may be given by Lemma 1, and the third term is simply n sums of 1, or n. Thus, $\sum_{p=1}^{n} n - 2p + 1 = n^2 - 2\frac{n(n+1)}{2} + n = n^2 - n^2 - n + n = 0$.

Q.E.D.