

# Optimization of the CMB-galaxy cross-correlation signal for studying the Integrated Sachs-Wolfe effect

Arthur Diniz Meirelles

Advisor: Edivaldo Moura Santos

Coadvisor: Ronaldo Carloto Batista

Institute of Physics  
Universidade de São Paulo

15/08/2024



- 1 Theoretical Aspects
- 2 Optimized Galaxy Survey
- 3 Data Processing
- 4 Analysis and Results
- 5 Conclusions

## 1 Theoretical Aspects

## 2 Optimized Galaxy Survey

## 3 Data Processing

## 4 Analysis and Results

## 5 Conclusions

# Introduction

- The integrated Sachs-Wolfe (ISW) effect occurs due to time changes in gravitational potentials;
- During periods of transition from or to matter-dominated eras of the Universe, potentials change more rapidly;
- Naturally, a statistical correlation between CMB temperature maps and gravitational potentials is expected.

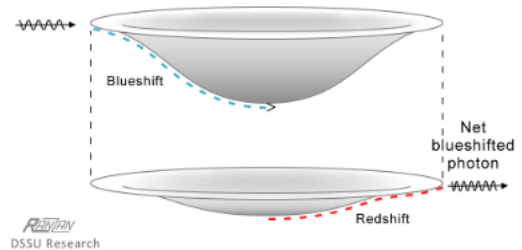


Figure 2: Illustration of the ISW effect. In this case, the gravity wells flattened, leaving a net blueshift.

# The $\Lambda$ CDM Model

To build the theoretical framework used, it is assumed

- The Universe is homogeneous and isotropic at large scales;
- Initial conditions are obtained by assuming an inflationary period when the Universe was very compact and energetic;
- The Universe is composed of baryonic matter, radiation, neutrinos, cold dark matter and a dark energy component described by a cosmological constant  $\Lambda$ ;

The homogeneous and isotropic background is first set up, and linear perturbations are added to it.

# The Perturbed Metric

We express the perturbed metric with two functions  $\Psi(\mathbf{x}, t)$  and  $\Phi(\mathbf{x}, t)$

$$\begin{cases} g_{00} = -1 - 2\Psi(\mathbf{x}, t), \\ g_{0i} = g_{i0} = 0, \\ g_{ij} = a^2(t)\delta_{ij}[1 + 2\Phi(\mathbf{x}, t)]. \end{cases} \quad (1)$$

# The $\Lambda$ CDM Model

The  $\Lambda$ CDM model explains some important observations, such as

- The CMB and its properties;
- The accelerating expansion of the Universe;
- The Universe's large scale structure;

And made some important predictions, like

- Baryon accoustic oscillations (BAO);
- The CMB polarization and its properties.

# The Cosmic Microwave Background

The temperature of the Cosmic Microwave Background (CMB) can be expressed by

$$T(\mathbf{x}, \hat{\mathbf{p}}, t) = \bar{T}(t)[1 + \Theta(\mathbf{x}, \hat{\mathbf{p}}, t)]. \quad (2)$$

The temperature perturbation  $\Theta$  can be expressed in Fourier space according to

$$\Theta(\hat{\mathbf{k}}, \mu) = \sum_{\ell=0}^{\infty} (2\ell + 1)(-i)^{\ell} \Theta_{\ell}(\hat{\mathbf{k}}) \mathcal{P}_{\ell}(\mu), \quad (3)$$

where  $\mu = \hat{\mathbf{k}} \cdot \hat{\mathbf{p}}$  and  $\mathcal{P}_{\ell}$  are Legendre polynomials.



# Analytical Approximation

Working with first-order equations and the tightly coupled limit, and assuming recombination to be an instantaneous process happening at conformal time  $\eta = \eta_*$ , we can obtain

$$\begin{aligned} \Theta_\ell(k, \eta_0) \approx & [\Theta_0(k, \eta_*) + \Psi(k, \eta_*)] j_\ell[k(\eta_0 - \eta_*)] \\ & + i v_b(k, \eta_*) \left\{ j_\ell[k(\eta_0 - \eta_*)] - (\ell + 1) \frac{j_\ell[k(\eta_0 - \eta_*)]}{k(\eta_0 - \eta_*)} \right\} \\ & + \int_0^{\eta_0} d\eta e^{-\tau} [\Psi'(k, \eta) - \Phi'(k, \eta)] j_\ell[k(\eta_0 - \eta)]. \end{aligned} \quad (4)$$

Here, the third term in the equation describes the Integrated Sachs-Wolfe effect.

# Correlation Functions

To calculate the CMB autocorrelation function  $C_\ell^{tt}$ , we expand  $\Theta$  in spherical harmonics

$$\Theta(\mathbf{x}, \hat{\mathbf{p}}, t) = \sum_{\ell=1}^{\ell_{\max}} \sum_{m=-\ell}^{\ell} a_{\ell m}(\mathbf{x}, t) Y_{\ell m}(\hat{\mathbf{p}}), \quad (5)$$

and calculate the autocorrelation between the  $a_{\ell m}$  terms:

$$\langle a_{\ell m} a_{\ell' m'}^* \rangle = \delta_{\ell \ell'} \delta_{m m'} C_\ell^{tt} \quad (6)$$

Other autocorrelation spectra  $C_\ell^{xx}$  follow the same process. It is common to use

$D_\ell^{XX} = \frac{\ell(\ell+1)}{2\pi} C_\ell^{xx}$  for some spectra for better visualization.

# Cosmic Variance

The low number of  $a_{\ell m}$  coefficients for lower multipoles  $\ell$  leads to a high uncertainty in this region called cosmic variance

$$\left( \frac{\Delta C_{\ell}^{XX}}{C_{\ell}^{XX}} \right)_{CV} = \sqrt{\frac{2}{2\ell + 1}} \quad (7)$$



# The Matter Power Spectrum

To calculate a cross-correlation function, we need the 3D matter power spectrum  $P(k, z)$ , defined by

$$\langle \delta(\mathbf{k}, z) \delta^*(\mathbf{k}', z) \rangle = (2\pi)^3 P(k, z) \delta_D(\mathbf{k} - \mathbf{k}')$$

We have used CAMB's implementation of the HALOFIT model to calculate the matter power spectrum.

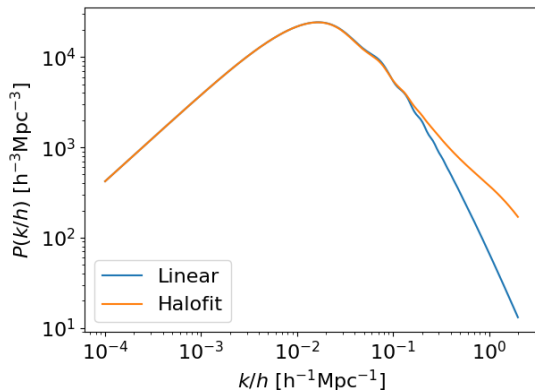


Figure 4: Matter power spectrum calculated using both a linear approximation and the HALOFIT model.

# The Cross-correlation Spectrum

To trace the matter density anisotropies, we used galaxy contrast maps, which can be calculated using

$$\delta_g(\mathbf{k}) = \frac{\rho_g(\mathbf{k}) - \bar{\rho}_g}{\bar{\rho}_g} \quad (8)$$

It is assumed that  $\delta_g = b_g \delta$ , where  $b_g$  is the bias factor, which we assume to be a slowly varying function of redshift.

The cross-correlation function  $C_\ell^{tg}$  can then be calculated with

$$\langle a_{\ell m}^t a_{\ell' m'}^g \rangle = C_\ell^{tg} \delta_{\ell \ell'} \delta_{mm'} \quad (9)$$

# Analytical Formula

Given fields  $x$  and  $y$ , representing either the ISW contribution to the CMB temperature ( $x, y = t$ ) or the galaxy contrast ( $x, y = g$ ), we can calculate the associated auto- or cross-correlation spectra using [1]

$$C_{\ell}^{xy} = \frac{2}{\pi} \int dk k^2 W_{\ell}^x(k) W_{\ell}^y(k) P(k), \quad (10)$$

with

$$W_{\ell}^t = -3\Omega_m \left( \frac{H_0}{k} \right)^2 \int dz \frac{d[(1+z)D(z)]}{dz} j_{\ell}[k\chi(z)] \quad (11)$$

$$W_{\ell}^g = \int dz b_g(z) \frac{dN}{dz} D(z) j_{\ell}[k\chi(z)] \quad (12)$$

# ISW Contribution: Temperature Autocorrelation

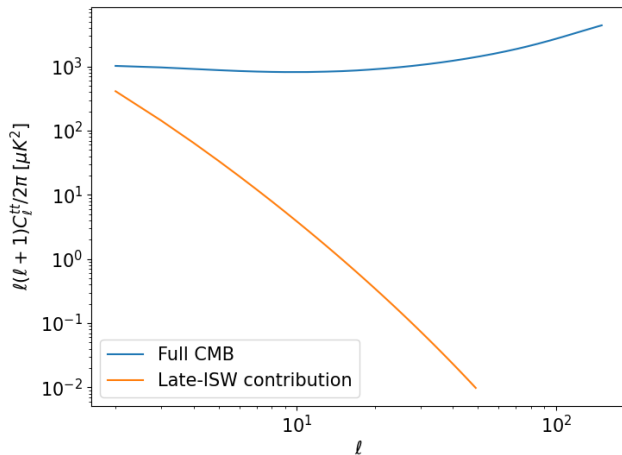


Figure 5: CMB autocorrelation comparison between full spectrum and ISW contribution.



# ISW Contribution: Galaxy Autocorrelation and Cross-correlation

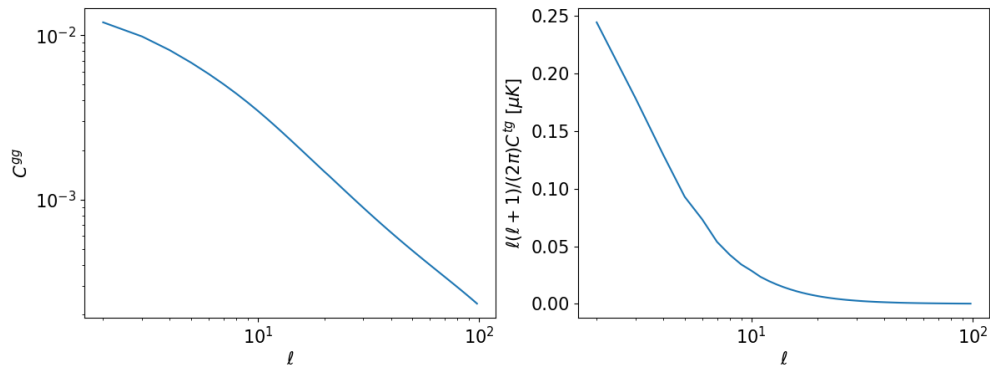


Figure 6: Galaxy autocorrelation spectrum (left) and late-ISW contribution to the galaxy-CMB cross-correlation (right).

- 1 Theoretical Aspects
- 2 **Optimized Galaxy Survey**
- 3 Data Processing
- 4 Analysis and Results
- 5 Conclusions

# Selection Function Parametrization

The function  $\frac{dN}{dz}$  in equation (12) is called the selection function. We are assuming its parametrization to be [2]

$$\frac{dN}{dz}(z|\lambda, \beta, z_0) dz = \frac{\beta}{\Gamma(\lambda)} \left(\frac{z}{z_0}\right)^{\beta\lambda-1} \exp\left[-\left(\frac{z}{z_0}\right)^\beta\right] d\left(\frac{z}{z_0}\right) \quad (13)$$

We have explored how to maximize the cross-correlation signal using an idealized selection function.

# 2MASS Bands Comparison

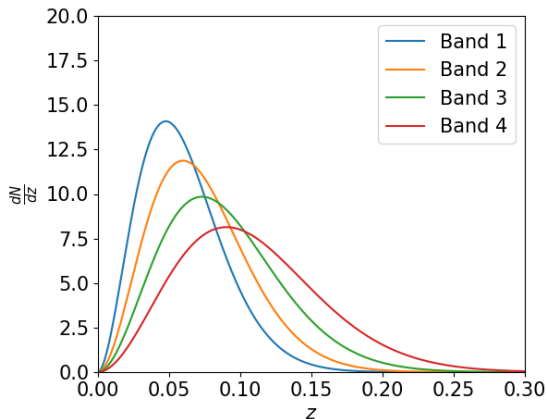


Figure 7: Selection function calculated for the 4 bands of the 2MASS catalog.

Band	$z_0$	$\beta$	$\lambda$
1	0.043	1.825	1.524
2	0.054	1.800	1.600
3	0.067	1.765	1.636
4	0.084	1.723	1.684

Table 2: Parameter values for the 4 bands of the 2MASS catalog.

# Exploring the Parameter Space

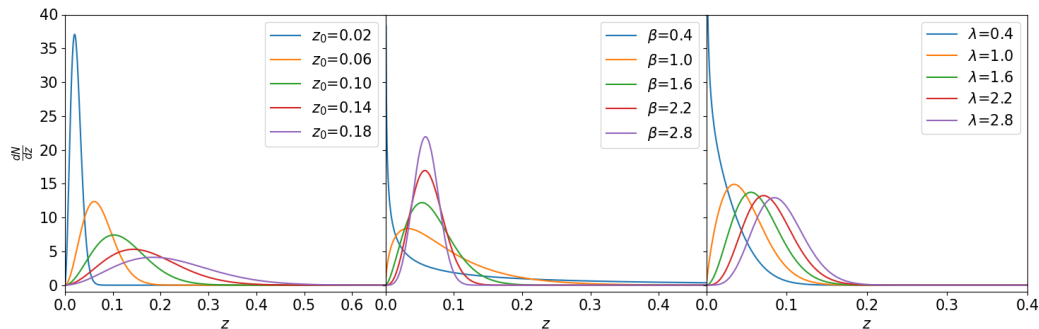


Figure 8: Selection function calculated for various parameter values.

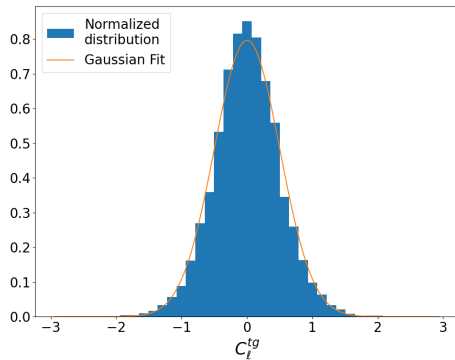
# Null Hypothesis

For the process of finding a galaxy survey with an idealized selection function, we first needed an estimation for the probability of  $C_\ell^{tg}$  being 0, which would be our null hypothesis. The following process was used:

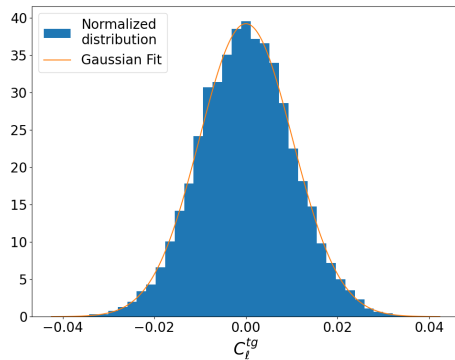
- Synthesize multiple uncorrelated CMB temperature and galaxy contrast maps using HEALPix;
- Calculate the cross-correlation  $C_\ell^{tg}$  for each pair of uncorrelated maps at each multipole;
- For each multipole  $\ell$ , a histogram of  $C_\ell^{tg}$  was produced;
- Each histogram was fit with Gaussian distributions with average  $\mu = \mu_\ell \approx 0$  and  $\sigma^2 = \sigma_\ell^2$ .

The null hypothesis is compatible with  $\Omega_m = 1$ .

# Synthesized Maps' Histograms



(a)  $\ell = 4$



(b)  $\ell = 30$

Figure 9: Distribution of cross-correlation values on different multipoles for  $10^4$  maps synthesized with null cross-correlation.

# Null Hypothesis

For  $f_\ell$  corresponding to the Gaussian fit made for the multipole  $\ell$

$$f_\ell(C_\ell^{tg}) = \frac{1}{\sqrt{2\pi\sigma_\ell^2}} \exp \left[ -\frac{1}{2} \left( \frac{C_\ell^{tg} - \mu_\ell}{\sigma_\ell} \right)^2 \right], \quad (14)$$

we have defined the null hypothesis probability distribution to be

$$P_{\text{null}} = \prod_{\ell=2}^{\ell_{\text{max}}} f_\ell(C_\ell^{tg}) \quad (15)$$



# Exploring the Parameter Space

For various points  $(\beta, z_0, \lambda)$  in the parameter space, we have calculated the ratio  $P_{\text{null}}(\beta, z_0, \lambda) / P_{\text{null}}^{\text{2MASS}}$  and produced the heat maps in Figure 10.

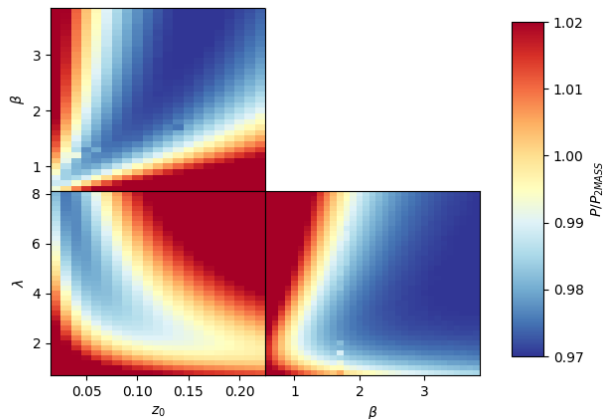


Figure 10: Heat maps used to explore the parameter space.

# Minimizer

The heat maps provided initial points to run an algorithm that minimizes  $P_{\text{null}}(z_0, \beta, \lambda)$ . The point found that minimizes the null hypothesis probability in that region of the parameter space was

$$(\beta, z_0, \lambda) = (3.088, 0.1508, 4.9401) \quad (16)$$

# Properties of the Minimum

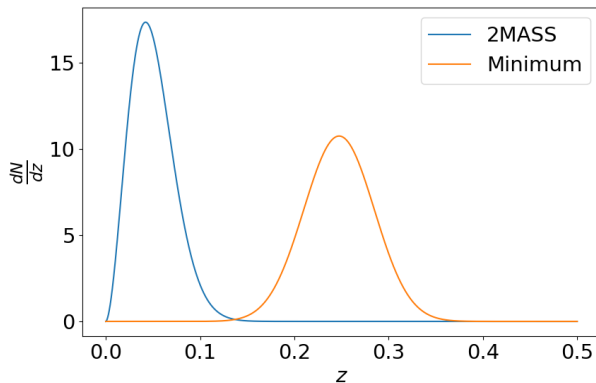


Figure 11: Selection functions comparison.

The selection function found is deeper than that of 2MASS band 1. It does not favor galaxies at  $z^* = 0.63$ , the estimated redshift at which the accelerated expansion started.

# Properties of the Minimum

The selection function found reduces the maximum value of the cross-correlation function, but its peak is at higher redshifts ( $\ell \approx 10$ ), reducing the influence of cosmic variance in the signal.

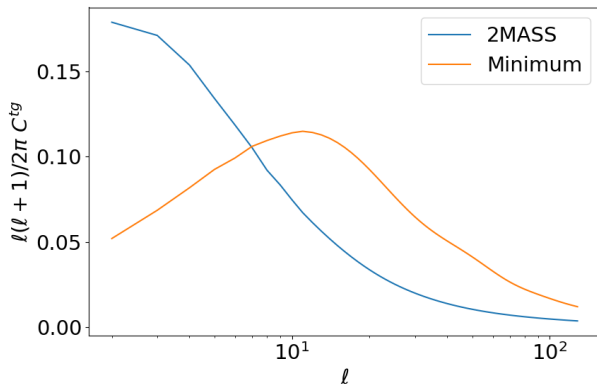


Figure 12: Theoretical cross-correlation spectrum comparison.

# Discussion

- The ratio  $P_{\text{null}}(z_0, \beta, \lambda) / P_{\text{null}}^{2\text{MASS}} = 0.971$  for the minimum;
- Despite the small statistical gain, this optimal selection function yields reasonably better results for constraints on  $\Omega_m$ , as will be discussed;
- A similar work has reported a small preference towards the non-null signal with changes to the depth of the survey and fainter magnitude limits [3].

- 1 Theoretical Aspects
- 2 Optimized Galaxy Survey
- 3 Data Processing**
- 4 Analysis and Results
- 5 Conclusions

# WMAP Data

The CMB temperature maps used in this project are the ones provided by WMAP9 [4].

- Three frequency bands were used, which are named bands Q (40GHz), V (60GHz) and W (90GHz);
- The temperature intensity maps' noise power can be well modeled as uncorrelated Gaussian fluctuations;
- Despite having a lower resolution ( $0.3^\circ$ ) compared to Planck's ( $0.083^\circ$ ), this project focuses on lower multipoles;
- We have combined both maps' masks and used it for analysing each one, resulting in a fraction  $f_{\text{sky}} = 0.7$ .

## WMAP Data

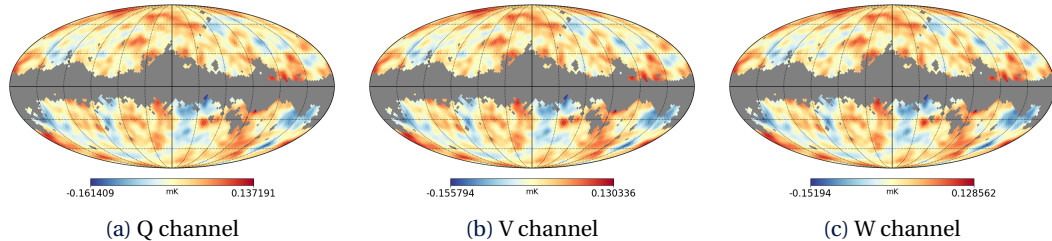


Figure 13: Mollweide projection of three (Q, V, W) WMAP9 CMB temperature (in mK) maps in galactic coordinates with an  $f_{\text{sky}} = 0.70$  mask applied

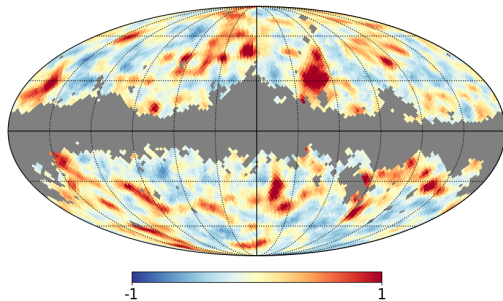


# 2MASS Catalog

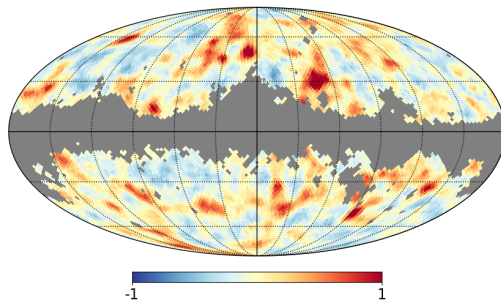
Wide sky coverage is a very influential aspect of studying cross-correlation spectra, and the 2MASS catalog contains raw imaging data covering 99.998% of the sky [5], which was the dataset used.

- We have used the  $K_s$  ( $2.16\mu\text{m}$ ) band of the Extended Source Catalog (XSC);
- The data of the  $K_s$  band obtained from a 20 mag aperture ( $K_{20}$ ) were corrected for galactic extinction, and the remaining data was further divided into 4 bands: Bands 1 ( $12.0 < K'_{20} < 12.5$ ), 2 ( $12.5 < K'_{20} < 13.0$ ), 3 ( $13.0 < K'_{20} < 13.5$ ) and 4 ( $13.5 < K'_{20} < 14.0$ );
- Each band has a different selection function, as shown in Figure 7.

# 2MASS Catalog



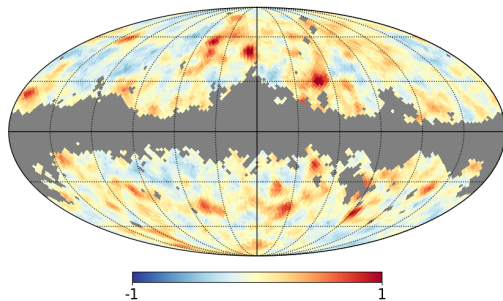
(a) Band 1



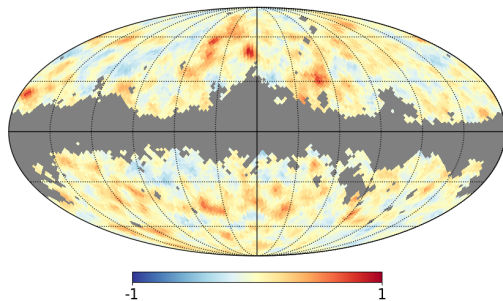
(b) Band 2

Figure 14: Mollweide projection of the 2MASS-XSC galaxy contrast maps in galactic coordinates with the combined 2MASS+WMAP mask applied ( $f_{sky} = 0.70$ ).

# 2MASS Catalog



(a) Band 3



(b) Band 4

Figure 15: Mollweide projection of the 2MASS-XSC galaxy contrast maps in galactic coordinates with the combined 2MASS+WMAP mask applied ( $f_{sky} = 0.70$ ).

# Correlation Spectra Estimator

To estimate the correlation spectra that describe the pixelized maps  $d$  with primordial signal  $s$  and noise  $n$  (with  $\mathbf{S}$  and  $\mathbf{N}$  being the corresponding covariance matrices), we could use the following likelihood function.

$$\mathcal{L} = P(\mathbf{d}|\mathbf{C}) = \frac{1}{(2\pi)^{n_{\text{dim}}/2} |\mathbf{C}|^{1/2}} \exp\left(-\frac{1}{2} \mathbf{d}^T \mathbf{C}^{-1} \mathbf{d}\right). \quad (17)$$

With a prior  $\pi(\mathbf{S})$  we can use Bayes' Theorem to obtain

$$P(\mathbf{C}|\mathbf{d}) \propto \pi(\mathbf{S}) P(\mathbf{d}|\mathbf{C}) \quad (18)$$

# Correlation Spectra Estimator

If we can sample from  $P(\mathbf{C}|\mathbf{s}, d)$  and  $P(\mathbf{s}|\mathbf{C}, d)$ . Then we iterate

$$\mathbf{s}^{i+1} \leftarrow P(\mathbf{s}|\mathbf{C}, d) \quad (19)$$

$$\mathbf{C}^{i+1} \leftarrow P(\mathbf{C}|\mathbf{s}, d), \quad (20)$$

to obtain a sample of  $\{(\mathbf{s}^i, \mathbf{C}^i)\}$ .

# The Blackwell-Rao Estimator

We can then use the Blackwell-Rao estimator to obtain an approximation for  $P(C_\ell|\mathbf{d})$

$$P(C_\ell|\mathbf{d}) \approx \frac{1}{N_G} \sum_{i=1}^{N_G} P(C_\ell|\sigma_\ell^i), \quad (21)$$

where

$$\sigma_\ell^i = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} \mathbf{s}_{\ell m} \mathbf{s}_{\ell m}^\dagger. \quad (22)$$

We then maximize the probability  $P(C_\ell|\mathbf{d})$  to obtain the best-fit  $C_\ell$ .

# Matrices Used

In this work,  $\mathbf{s}$  and  $\mathbf{S}$  are defined by

$$\mathbf{s}^T = (s_{00}^{tg}, s_{01}^{tg}, s_{11}^{tg}, \dots, s_{\ell_{\max}0}^{tg}, \dots, s_{\ell_{\max}\ell_{\max}}^{tg}), \quad (23)$$

$$\mathbf{S} = \text{diag}(S_0^{tg}, S_1^{tg}, S_1^{tg}, \dots, S_{\ell_{\max}}^{tg}, \dots, S_{\ell_{\max}}^{tg}), \quad (24)$$

where

$$\mathbf{s}_{\ell m}^{tg} = \begin{pmatrix} a_{\ell m}^t \\ a_{\ell m}^g \end{pmatrix}, \quad \mathbf{S}_{\ell}^{tg} = \begin{pmatrix} C_{\ell}^{tt} & C_{\ell}^{tg} \\ C_{\ell}^{tg} & C_{\ell}^{gg} \end{pmatrix}. \quad (25)$$

# Correlation Spectra Obtained

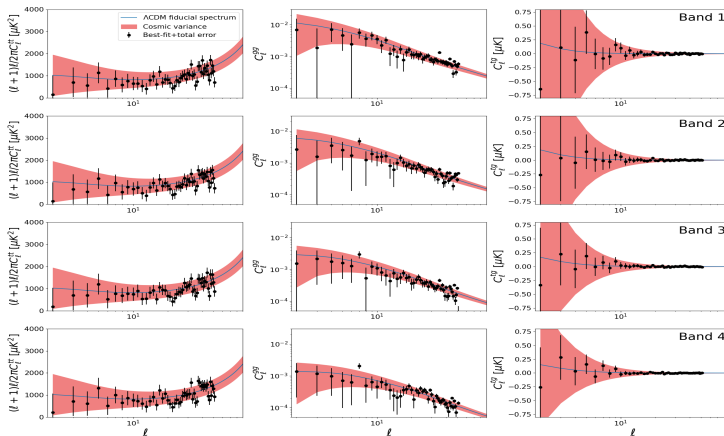


Figure 16: Comparison between theoretical correlation spectra and the ones estimated from WMAP and 2MASS.



- 1 Theoretical Aspects
- 2 Optimized Galaxy Survey
- 3 Data Processing
- 4 Analysis and Results**
- 5 Conclusions

# Monte Carlo Markov Chains

By defining the likelihood  $\mathcal{L}(C_\ell|\theta)$  and the prior  $p(\theta)$ , one can use Markov chains to obtain samples of each parameter in  $\theta$  following the posterior

$$P(\theta|C_\ell) = \mathcal{L}(C_\ell|\theta)p(\theta) \quad (26)$$

Joint posteriors can be easily obtained from each parameter sample, strongly simplifying data analysis for multiple parameter models.

# Planck Likelihoods

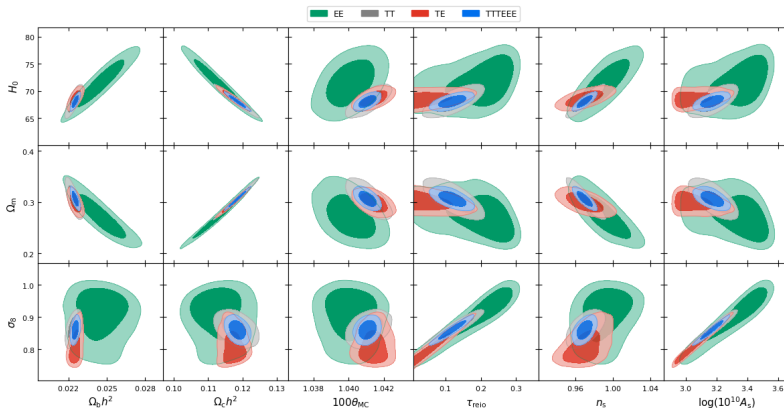


Figure 17: Joint posterior distributions of cosmological parameters using Planck's CMB temperature (T) and polarization (E) data.

# Likelihood Profiling

To analyze the data, we have opted to profile the likelihood functions applied to  $\Omega_m$ . The likelihood of each point in the spectrum was assumed to be Gaussian

$$\mathcal{L}(C_{\ell, \text{theo}}^{xy} | C_{\ell, \text{data}}^{xy}) = \frac{1}{\sigma_\ell \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{C_{\ell, \text{data}}^{xy} - C_{\ell, \text{theo}}^{xy}}{\sigma_\ell} \right)^2 \right]. \quad (27)$$

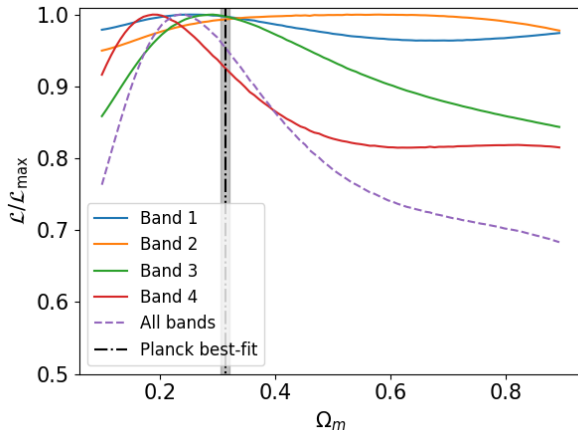
We are varying only  $\Omega_m$ , so  $C_{\ell, \text{theo}}^{xy}$  is a functions of only  $\Omega_m$ . The likelihood of a full spectrum is

$$\mathcal{L}(\Omega_m | C_{\text{data}}^{xy}) = \prod_{\ell=2}^{\ell_{\max}} \mathcal{L}(C_{\ell, \text{theo}}^{xy} | C_{\ell, \text{data}}^{xy}). \quad (28)$$

The  $C^{tg} + C^{gg}$  joint-likelihood is

$$\mathcal{L}(\Omega_m | C_{\text{data}}^{tg}, C_{\text{data}}^{gg}) = \mathcal{L}(\Omega_m | C_{\text{data}}^{tg}) \mathcal{L}(\Omega_m | C_{\text{data}}^{gg}). \quad (29)$$

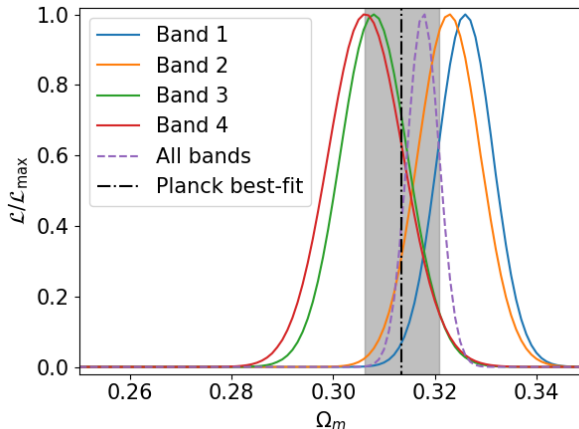
# Results for 2MASS



- All curves are compatible with Planck;
- Not much constraining power on  $\Omega_m$ ;
- Bands 3 and 4 – which have the deepest selection functions – have the most constraining power amongst all 4.

Figure 18:  $C^{tg}$  only likelihood profiles.

# Results for 2MASS



- All likelihoods are compatible with Planck;
- Very high constraining power on  $\Omega_m$  when  $C^{\text{gg}}$  is introduced, comparable to Planck's;
- No significant difference in constraining power between each band.

Figure 19: Joint likelihood profiles

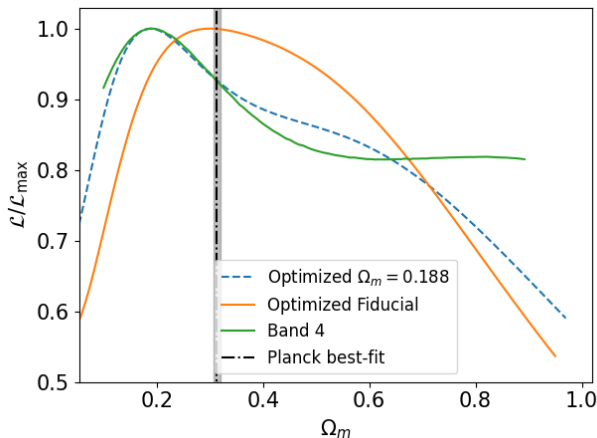
## Forecast for the Optimized Band

To estimate the behavior of a survey that follows our idealized selection function in this analysis, we have produced two synthetic cross-correlation spectra using that selection function. Both use the  $\Lambda$ CDM model with Planck's best-fit parameters, only differing in  $\Omega_m$ :

- One of the datasets was produced using Planck's best-fit of  $\Omega_m = 0.3153$ ;
- The other dataset was produced using  $\Omega_m = 0.188$ , the value that maximizes the likelihood of band 4 for the  $C^{tg}$  only likelihood.

Band 4's errors were used as estimated for both synthetic datasets.

# Forecast for the Optimized Band



- Constraining power still low;
- Reasonable improvement in the constraining power.

Figure 20: Optimized band  $C^{tg}$  only profile.



# Discussion

- The optimized band being deeper than 2MASS means errors on the spectra should be lower, meaning the errors used are overestimated, which indicates the constraints could be better for a real survey following our optimized selection function;
- The optimized band was not found by optimizing the  $\Omega_m$  constraints, but rejecting the null cross-correlation hypothesis.  $\Omega_m = 1$  leads to  $C_\ell^{tg} = 0$ , and the fast decrease in likelihood for higher  $\Omega_m$  is noticeable.

- 1 Theoretical Aspects
- 2 Optimized Galaxy Survey
- 3 Data Processing
- 4 Analysis and Results
- 5 Conclusions**

# Conclusions

- We have obtained the likelihood profiles for  $\Omega_m$  obtained from cross-correlation data by studying the ISW effect;
- The constraints obtained using only  $C^{tg}$  were not strong, but can be used as a complement to other datasets for a combined analysis;
- In the 2MASS catalog, the 2 deepest bands – bands 3 and 4 – yielded better constraining power;
- The Planck collaboration also studied the ISW effect using the cross-correlation spectrum[6], obtaining significantly better results with deeper surveys (SDSS[7] and Planck's Kappa map).

# Conclusions

- An optimized band capable of maximizing the ISW signal was obtained, prioritizing galaxies at higher redshifts and increasing the cross-correlation signal in a region less affected by cosmic variance;
- The likelihoods obtained for artificial data calculated using the optimized band improved the constraints on  $\Omega_m$ ;
- Combining different matter tracers leads to improved results overall.

*Thank You*

## Extra Content

### Discussão de Future prospects

# Extra Content

More extras

# Bibliography I

- [1] E. Moura-Santos et al. “A Bayesian Estimate of the CMB–large-scale Structure Cross-correlation”. In: *The Astrophysical Journal* 826.2 (July 2016), p. 121. DOI: 10.3847/0004-637X/826/2/121. URL: <https://dx.doi.org/10.3847/0004-637X/826/2/121>.
- [2] Niayesh Afshordi, Yeong-Shang Loh, and Michael A. Strauss. “Cross-correlation of the cosmic microwave background with the 2MASS galaxy survey: Signatures of dark energy, hot gas, and point sources”. In: *Phys. Rev. D* 69 (8 Apr. 2004), p. 083524. DOI: 10.1103/PhysRevD.69.083524. URL: <https://link.aps.org/doi/10.1103/PhysRevD.69.083524>.



## Bibliography II

- [3] Caroline L. Francis and John A. Peacock. “Integrated Sachs–Wolfe measurements with photometric redshift surveys: 2MASS results and future prospects”. In: *Monthly Notices of the Royal Astronomical Society* 406.1 (July 2010), pp. 2–13. ISSN: 0035-8711. DOI: 10.1111/j.1365-2966.2010.16278.x. eprint: <https://academic.oup.com/mnras/article-pdf/406/1/2/3673745/mnras0406-0002.pdf>. URL: <https://doi.org/10.1111/j.1365-2966.2010.16278.x>.
- [4] M. R. Greason et al. *Wilkinson Microwave Anisotropy Probe (WMAP): Nine-Year Explanatory Supplement*. Available in electronic form at <http://lambda.gsfc.nasa.gov/>. NASA/GSFC. Greenbelt, MD, 2012.
- [5] M. F. Skrutskie et al. “The Two Micron All Sky Survey (2MASS)”. In: *The Astronomical Journal* 131.2 (Feb. 2006), p. 1163. DOI: 10.1086/498708. URL: <https://dx.doi.org/10.1086/498708>.

# Bibliography III

- [6] Planck Collaboration et al. “Planck 2015 results - XXI. The integrated Sachs-Wolfe effect”. In: *A&A* 594 (2016), A21. DOI: 10.1051/0004-6361/201525831. URL: <https://doi.org/10.1051/0004-6361/201525831>.
- [7] J. J. Condon et al. “The NRAO VLA Sky Survey”. In: *The Astronomical Journal* 115.5 (May 1998), p. 1693. DOI: 10.1086/300337. URL: <https://dx.doi.org/10.1086/300337>.