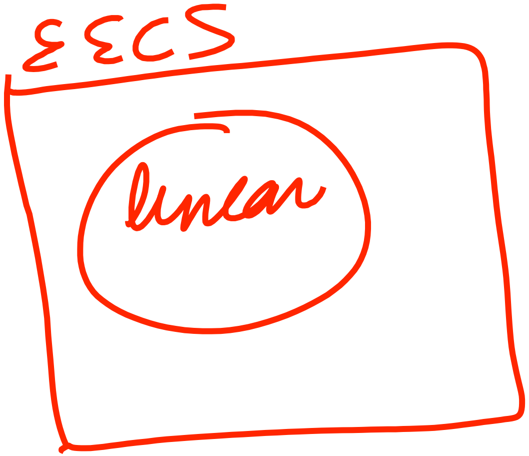
6.002 CIRCUITS AND ELECTRONICS

Superposition, Thévenin and Norton

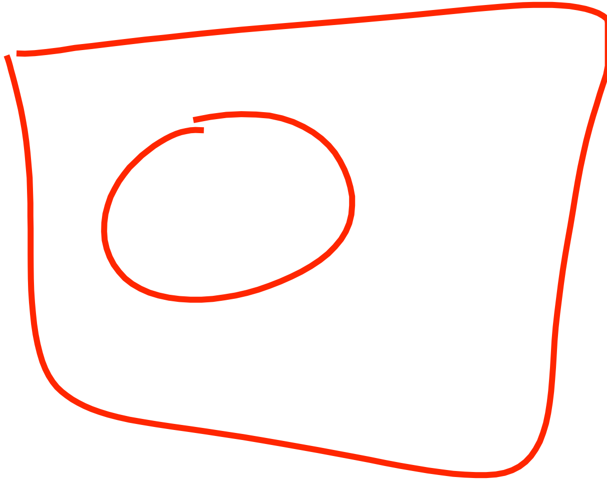


Reading: Chapter 3 of A&L

Review

Circuit Analysis Methods

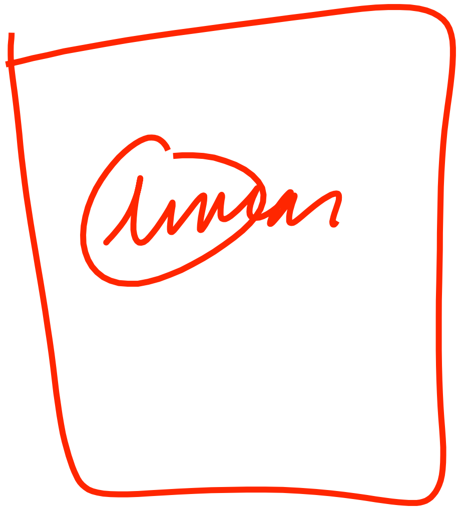
- KVL: $\sum_{loop} V_i = 0$ KCL: $\sum_{node} I_i = 0$ VI



- Circuit composition rules
- Node method – the workhorse of 6.002
  KCL at nodes using V's referenced from ground
  KVL implicit in pattern "$(e_i - e_j)\frac{1}{R}$"

Overview

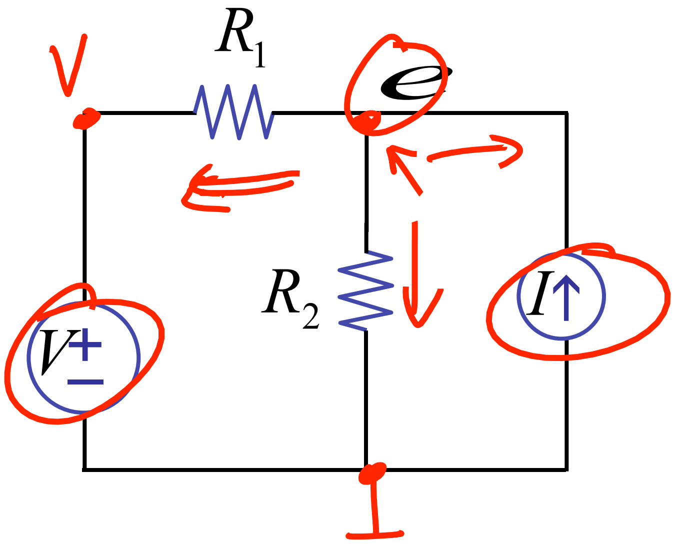
- Introduction to linear circuits
- Properties of linearity
- The superposition tool for your toolkit
- The Thévenin method
- The Norton method



Let’s start by introducing linearity

Linearity

Consider

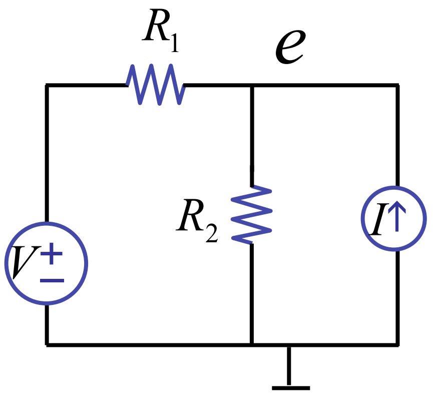


Write node equations –

$$\frac{e-V}{R_1} + \frac{e}{R_2} - I = 0$$

Notice:
linear in
e, V, I
No terms
eV, V^2, VI . . .

Linearity



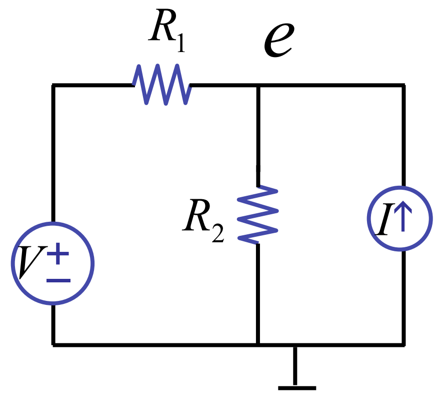
Write node equations --

$$\frac{e-V}{R_1}+\frac{e}{R_2}-I=0 \quad \text{linear in } e, V, I$$

Rearrange --

$$\left[\frac{1}{R_1}+\frac{1}{R_2}\right]e=\frac{V}{R_1}+I$$

Linearity



$$\frac{e-V}{R_1}+\frac{e}{R_2}-I=0$$

$$\frac{R_2+R_1}{R_1R_2} \leftarrow \left[\frac{1}{R_1}+\frac{1}{R_2}\right]e = \frac{V}{R_1}+I \leftarrow$$

conductance matrix G node voltage e linear sum of sources S

$$G e = S$$

$$[\,\vdots\,][\,\vdots\,] = [\,\vdots\,]$$

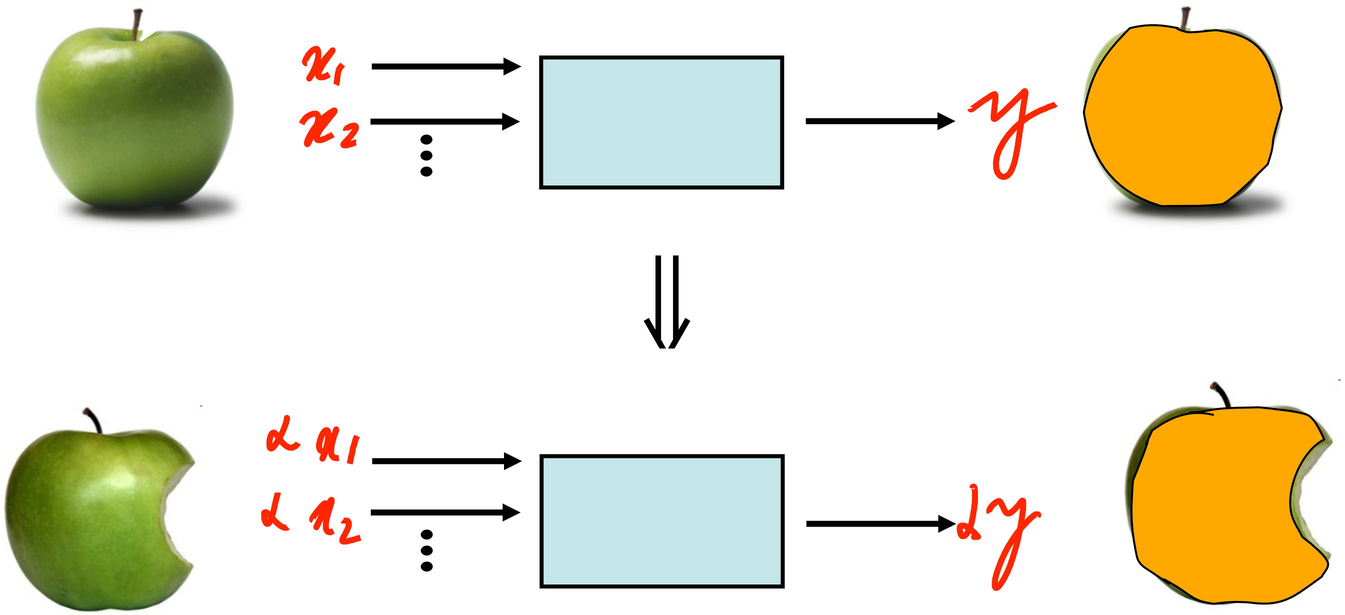
$$\rightarrow \quad e = \frac{R_2 V}{R_1+R_2} + \frac{R_1R_2 I}{R_1+R_2}$$

linear $\rightarrow$ $\boxed{e = a_1V_1 + a_2V_2 + \cdots + b_1I_1 + b_2I_2 + \cdots}$

Linearity $\implies$ Homogeneity
Superposition

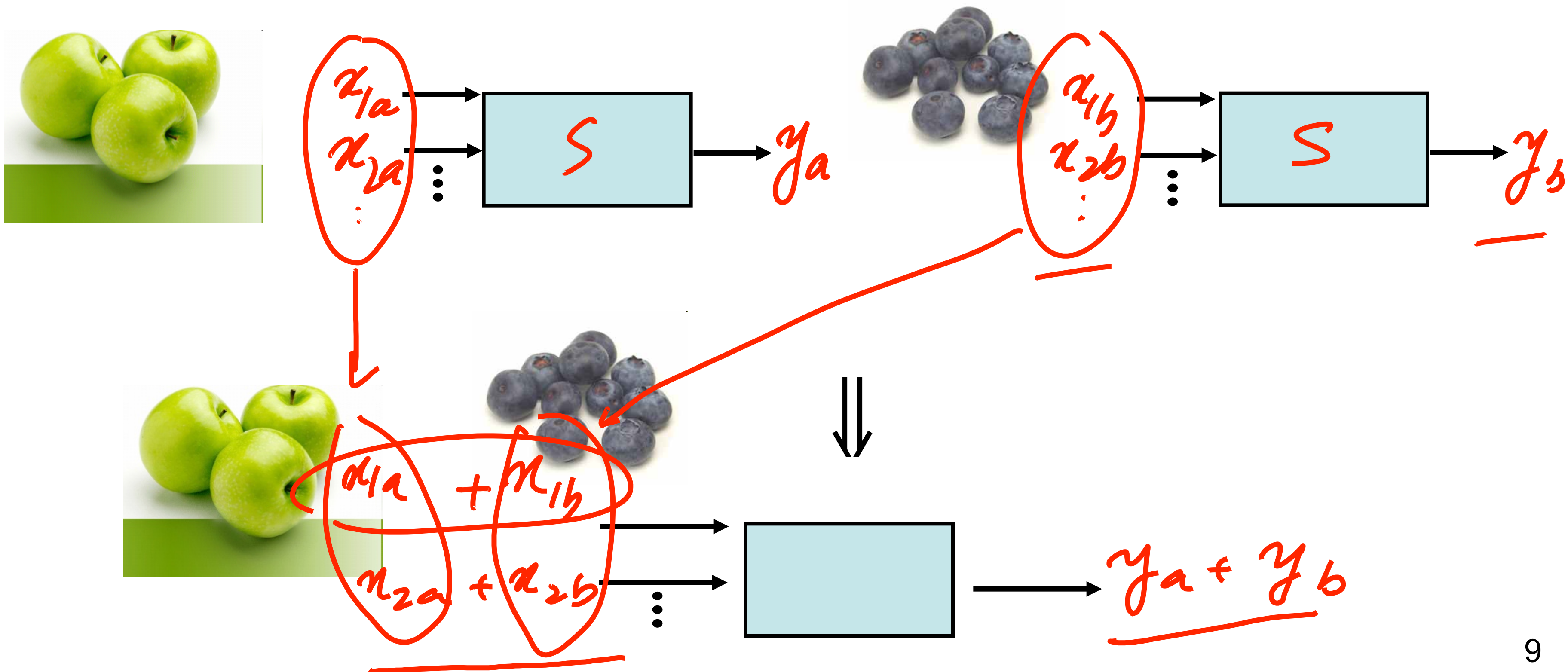
Linearity $\Longrightarrow$ Homogeneity, Superposition

Homogeneity

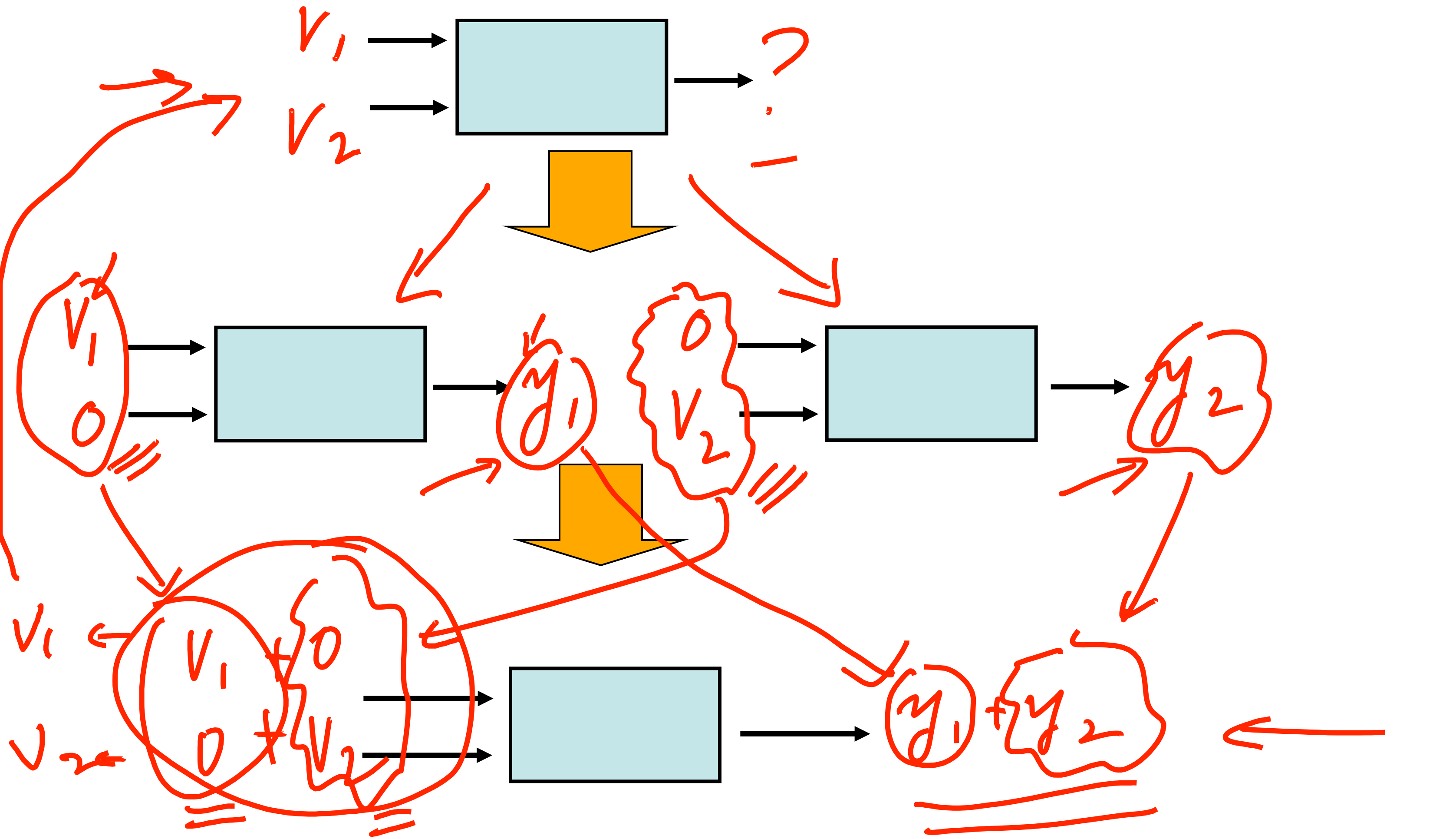


Linearity $\Longrightarrow$ Homogeneity
Superposition

Superposition



Specific superposition example:



Method 4: Superposition method

EECS

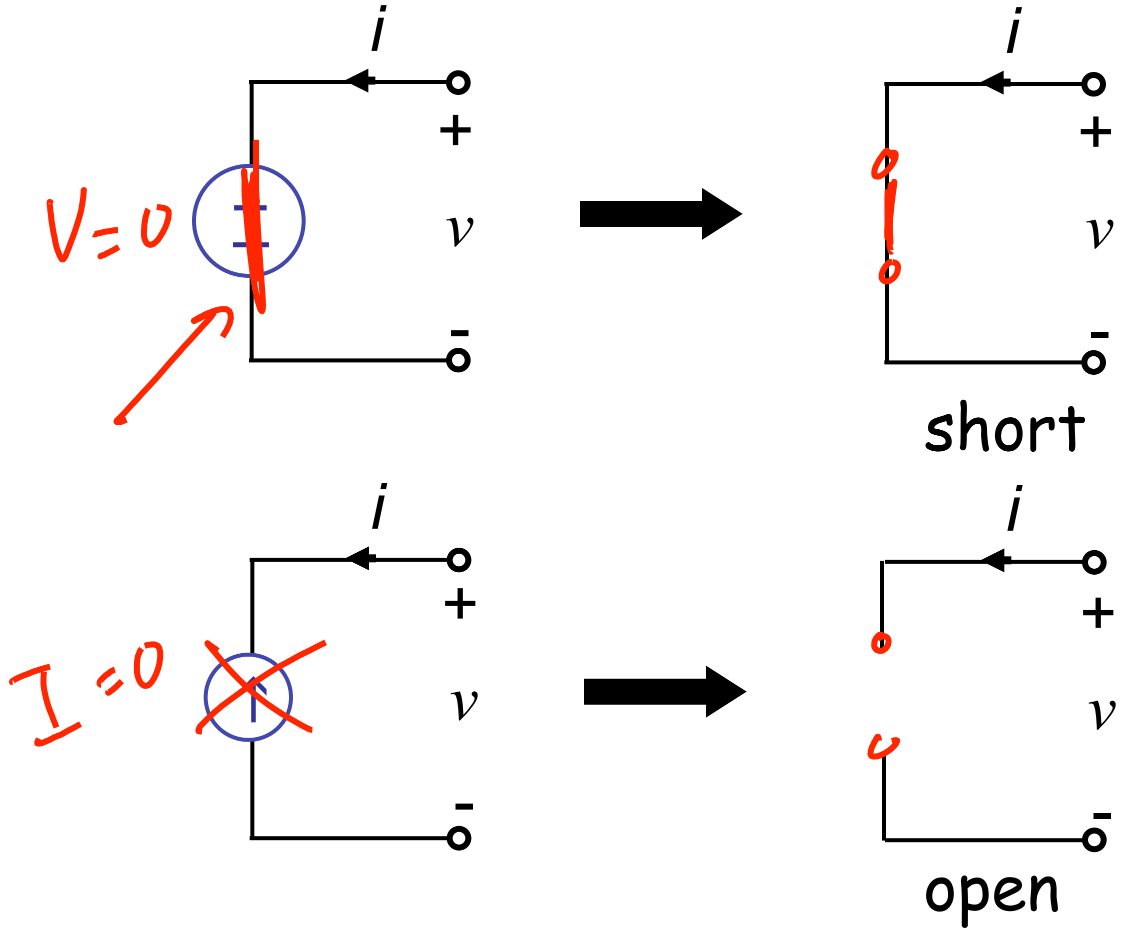
model

Linear

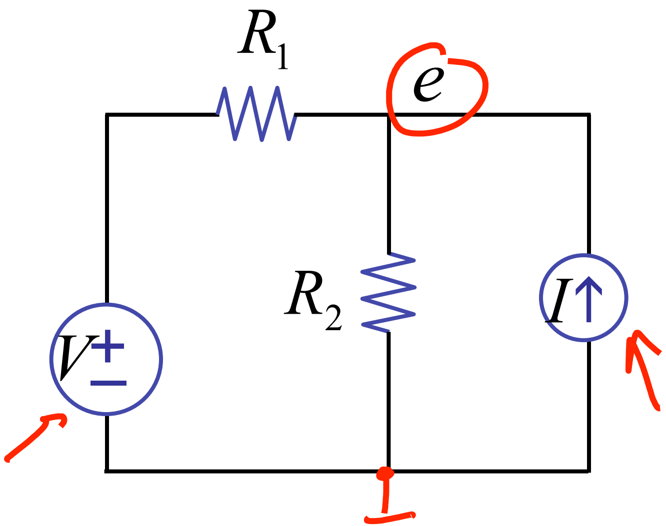
1. Find the responses of the circuit to each source acting alone
2. Sum the individual respones

Independent sources

Each source acting alone means this



Back to the example



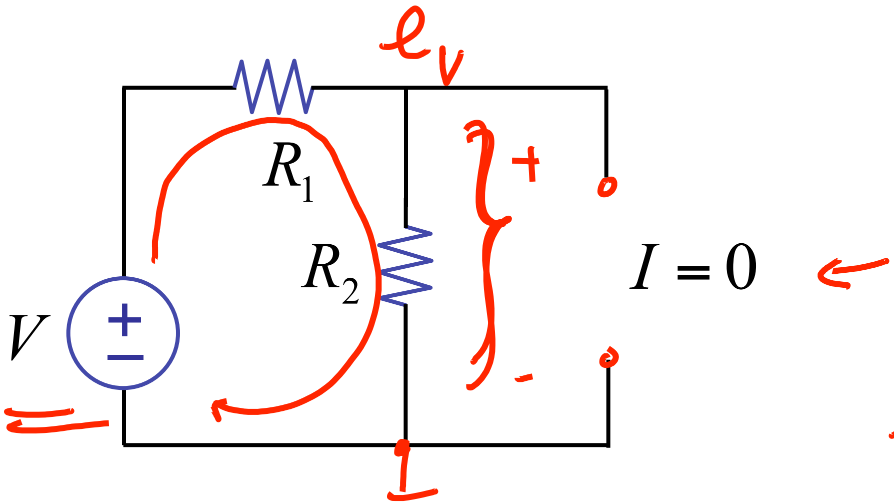
Use superposition method

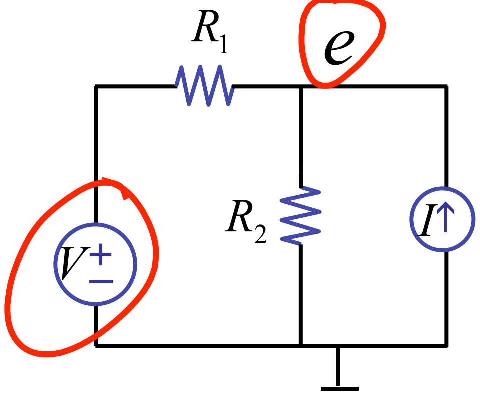
Back to the example

Use superposition method

1.

V acting alone





Voltage divider pattern

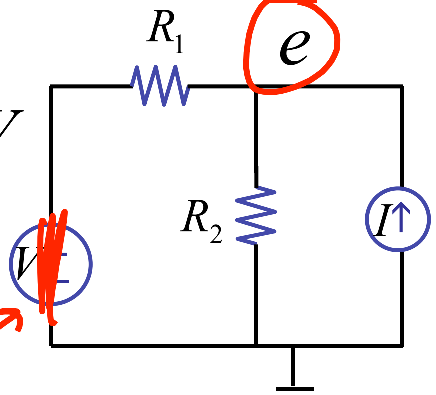
$$e_V = \frac{R_2}{R_1+R_2} V$$

Back to the example

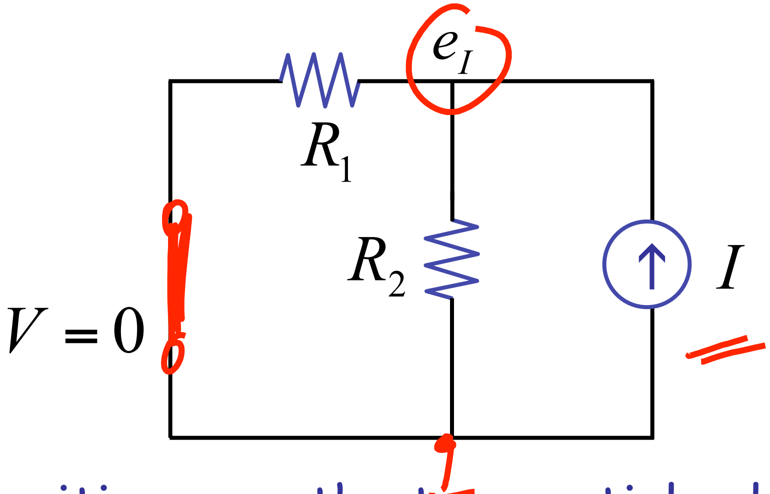
Use superposition method

1.

$$e_V = \frac{R_2}{R_1 + R_2} V$$



***I* acting alone**



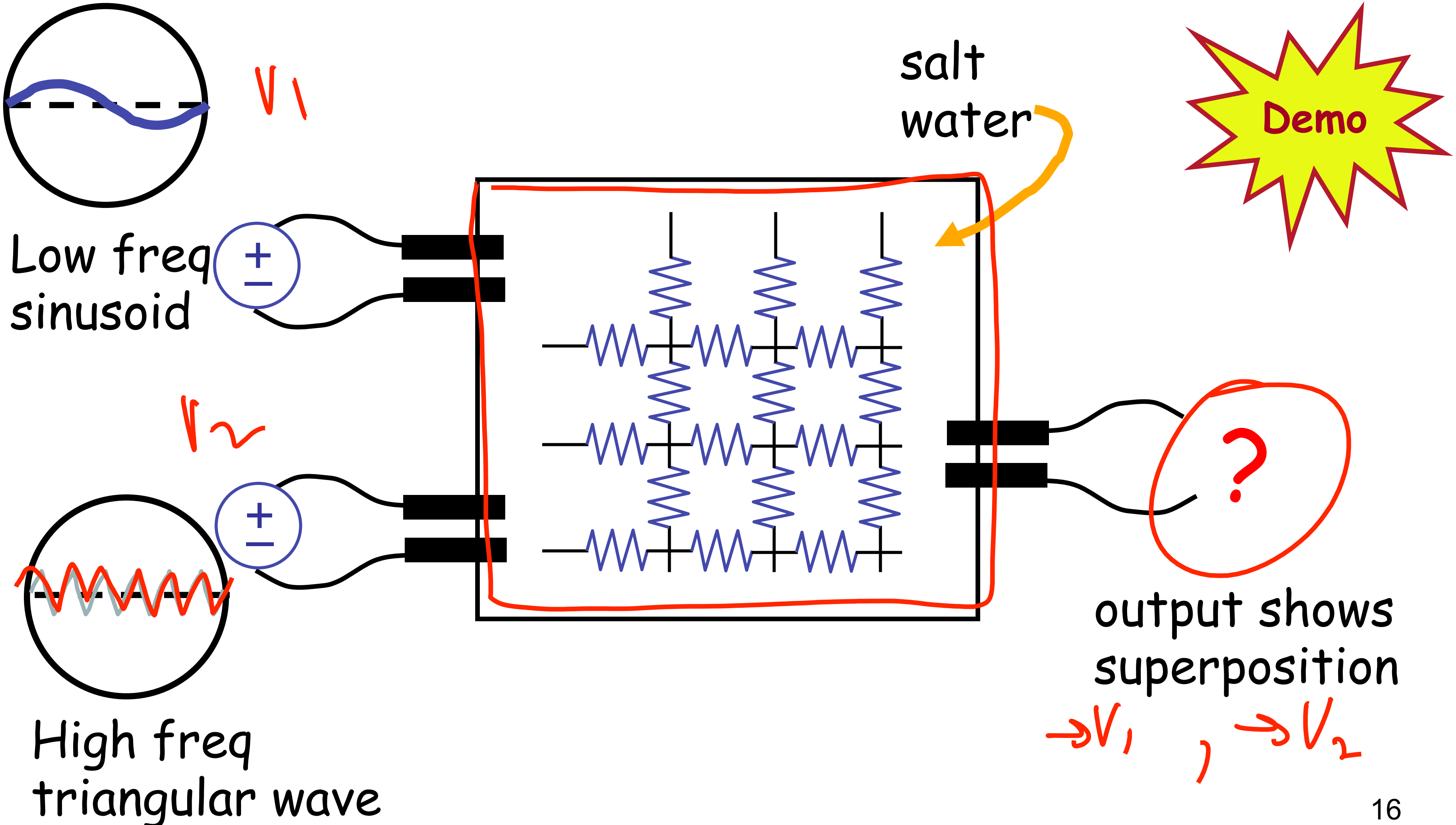
Current flowing into a resistor pair in parallel

$$e_I = \frac{R_1 R_2}{R_1 + R_2} \cdot I$$

2. By superposition, sum the two partial voltages

$$e = e_V + e_I = \frac{R_2}{R_1 + R_2} V + \frac{R_1 R_2}{R_1 + R_2} \cdot I$$

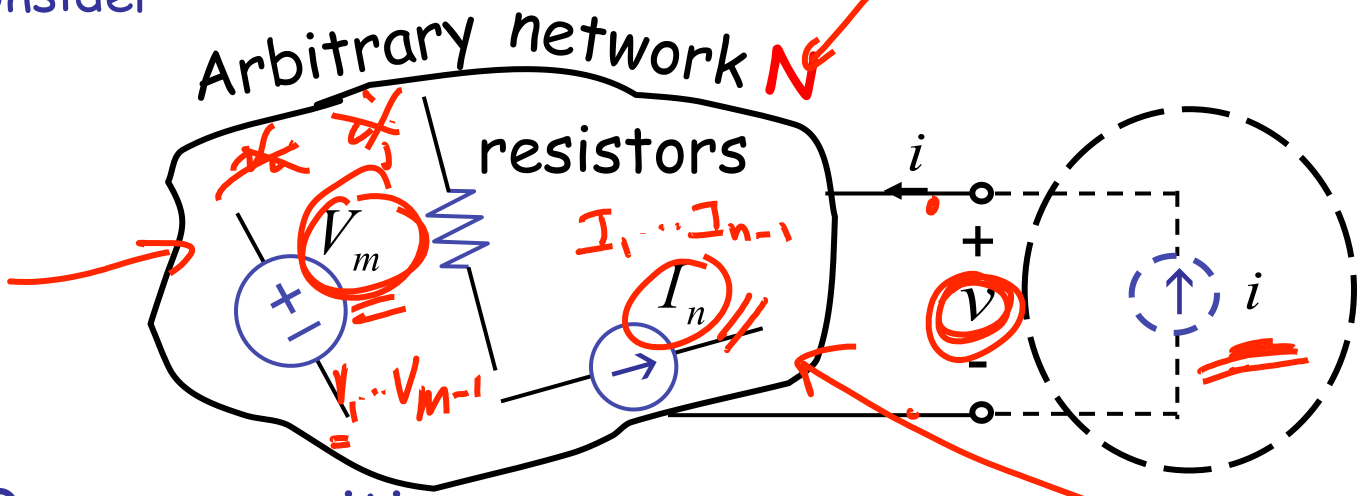
Voilà!



V1
Low freq
sinusoid
±
V2
±
High freq
triangular wave
salt
water
Demo
?
output shows
superposition
→V1 , →V2

Yet another method...

Consider

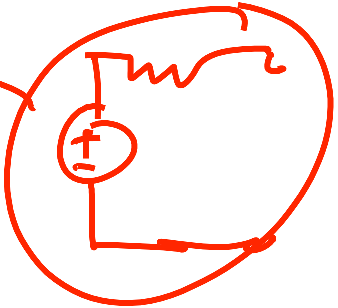


Suppose I want to determine v

By superposition

$$v = \underbrace{\sum_m \alpha_m V_m}_{\forall n\ I_n = 0,\ i = 0} + \underbrace{\sum_n \beta_n I_n}_{\forall m\ V_m = 0,\ i = 0} + \underbrace{R\, i}_{\forall n\ I_n = 0,\ \forall m\ V_m = 0}$$

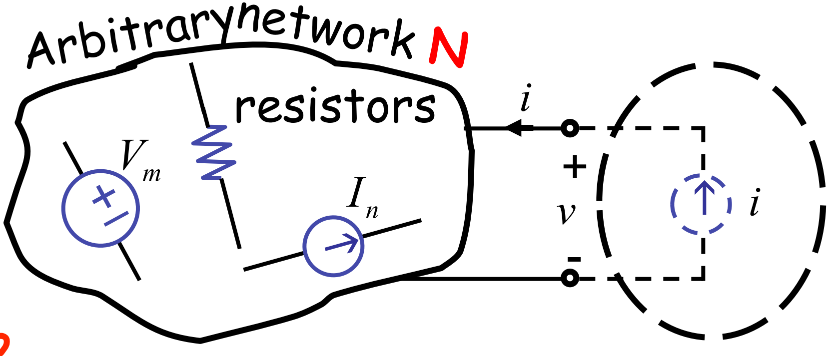
by setting



$v_{TH} + R_{TH}\, i$

$$= \; v = \underbrace{\sum_m \alpha_m V_m + \sum_n \beta_n I_n}_{\text{voltage}} + R i$$

no units   Resistance unit   R



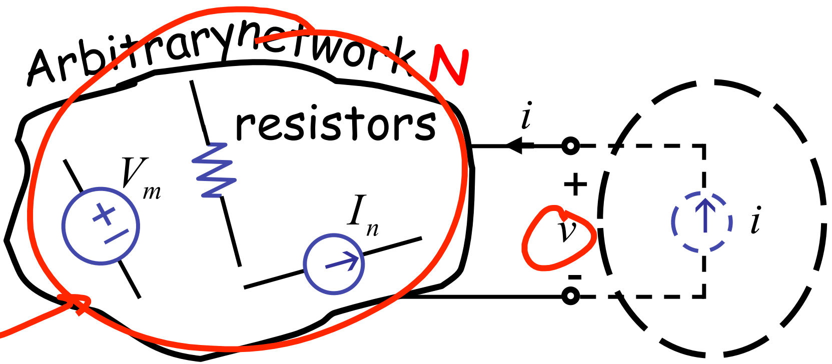
Independent of external excitation and behaves like a voltage.

Let's call it "v_{TH}"

also independent of external excitement and behaves like a resistor.
Let's call it "R_{TH}"

$$v = \sum_m \alpha_m V_m + \sum_n \beta_n I_n + Ri$$

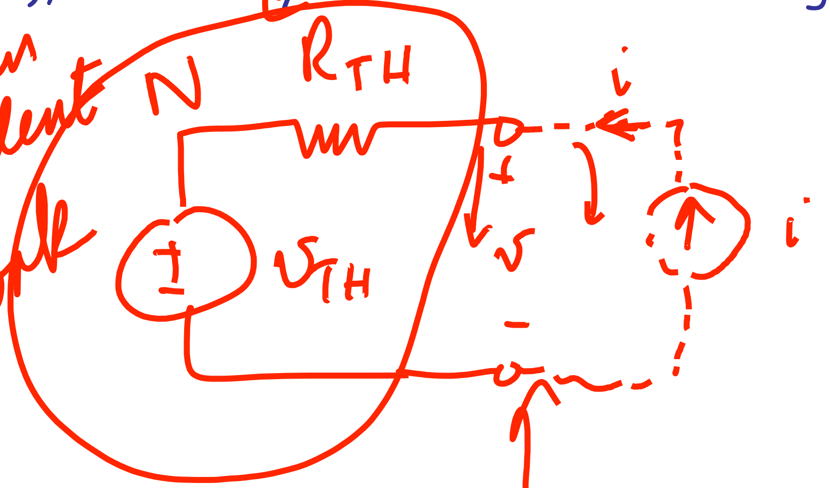
$$v = v_{TH} + R_{TH} \cdot i$$



In other words, as far as the external world is concerned (for the purpose of the i-v relation), "arbitrary network N" is indistinguishable from:

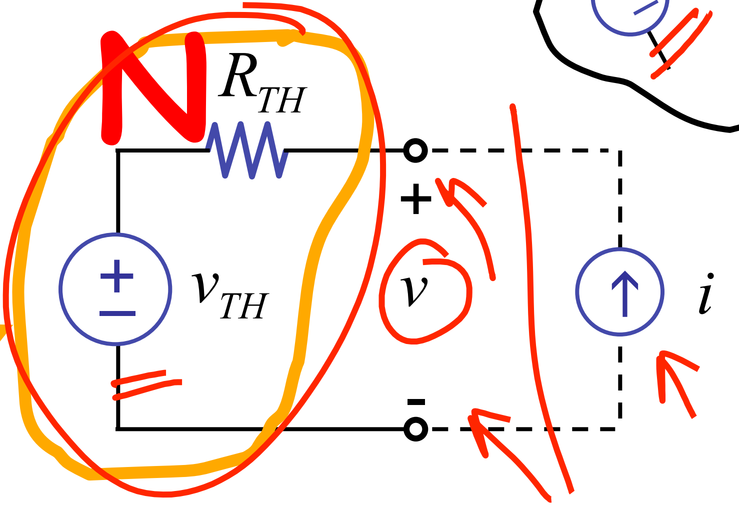
Thevenin pattern

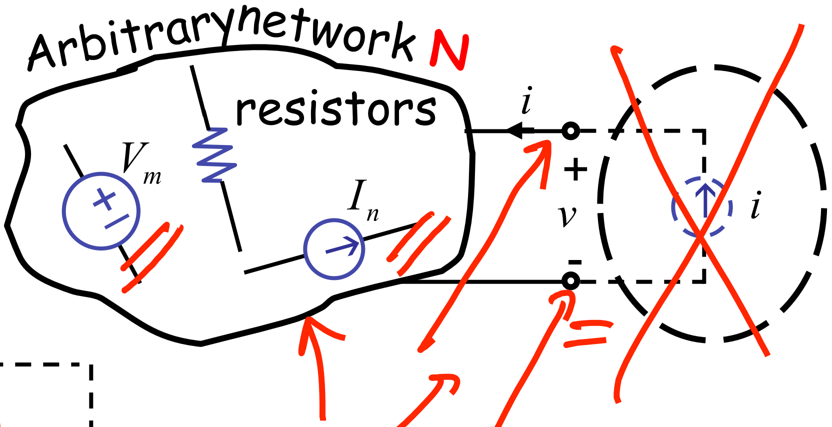
Thevenin equivalent network



$$v = v_{TH} + R_{TH} i$$

Thévenin equivalent network





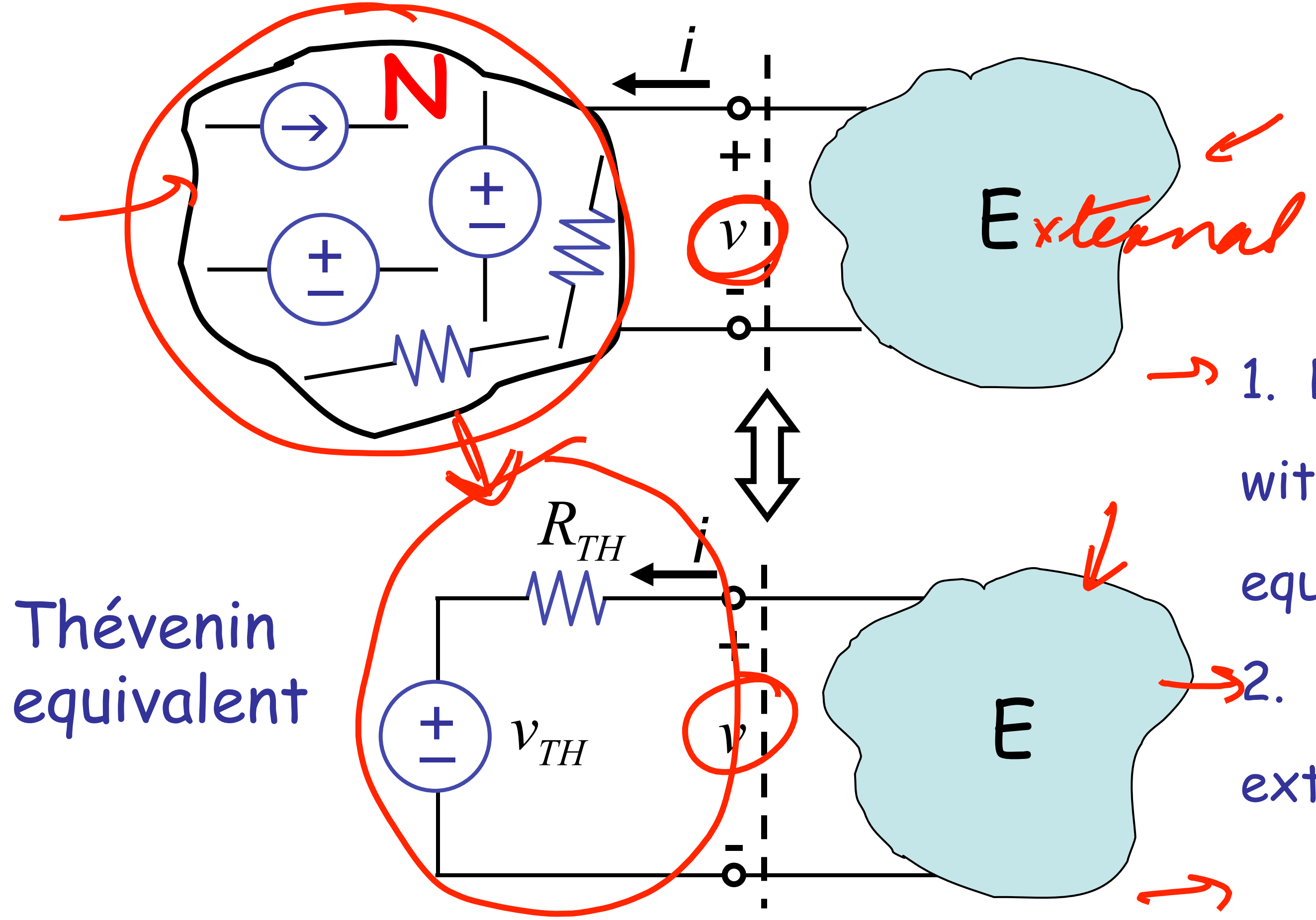
Resistance

v_{TH} ⟶ Open circuit voltage seen at terminal pair (aka port)

R_{TH} ⟶ Resistance of network seen from port

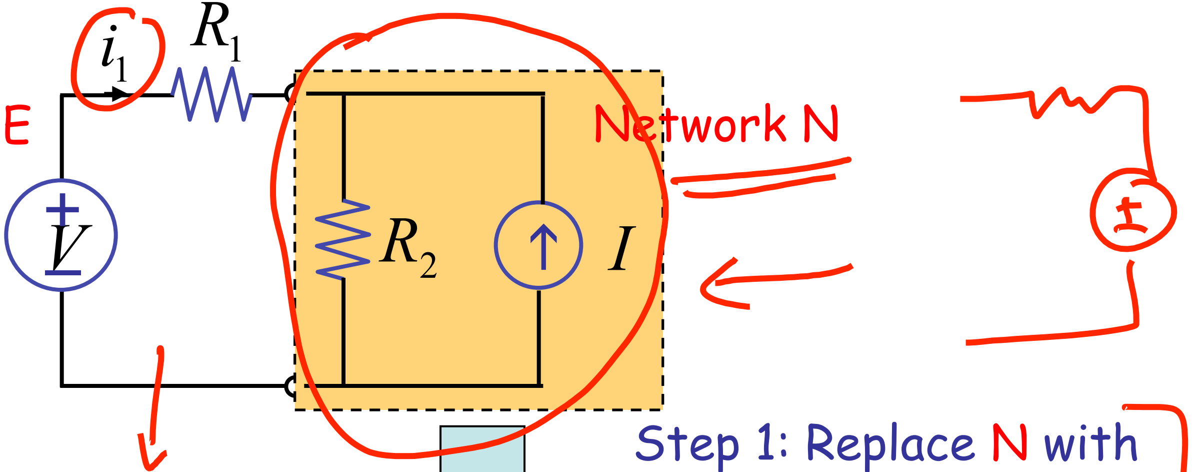
(V_m's, I_n's set to 0)

Method 4: The Thévenin Method

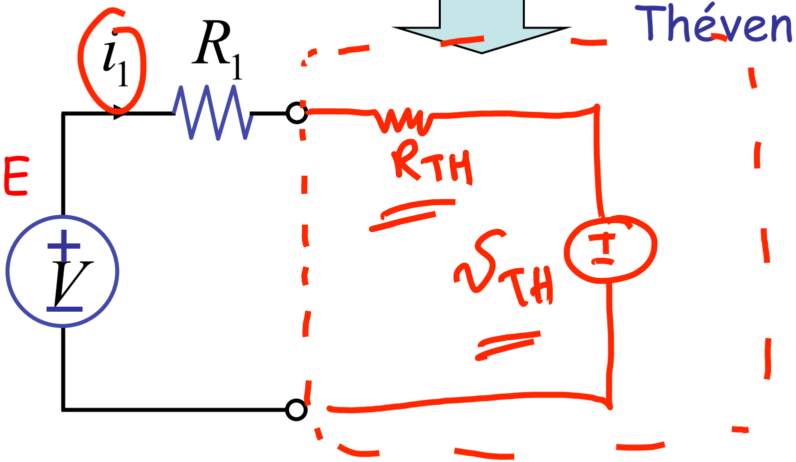


1. Replace network N with its Thévenin equivalent
2. Then solve with external network E.

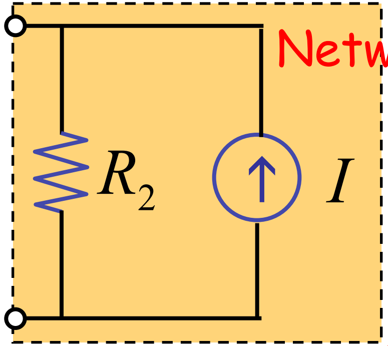
Example: Find i_1



Step 1: Replace N with Thévenin equivalent

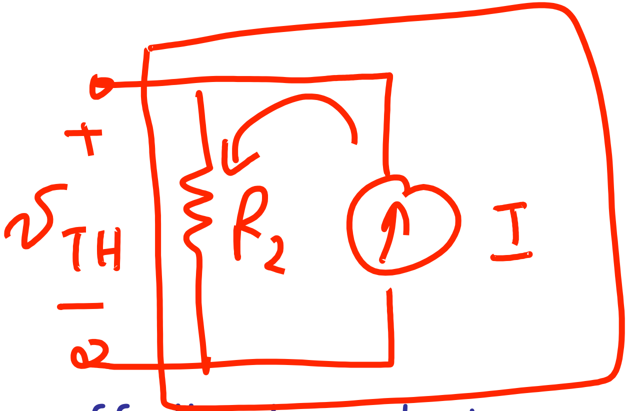


Step 1: Replace with Thévenin equivalent

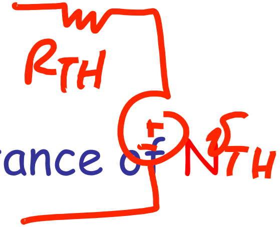


V_{TH} : open circuit voltage of network N

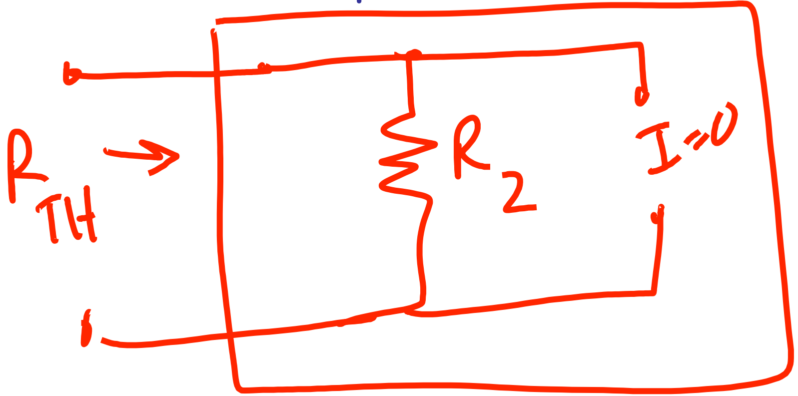
$V_{TH} = IR_2$



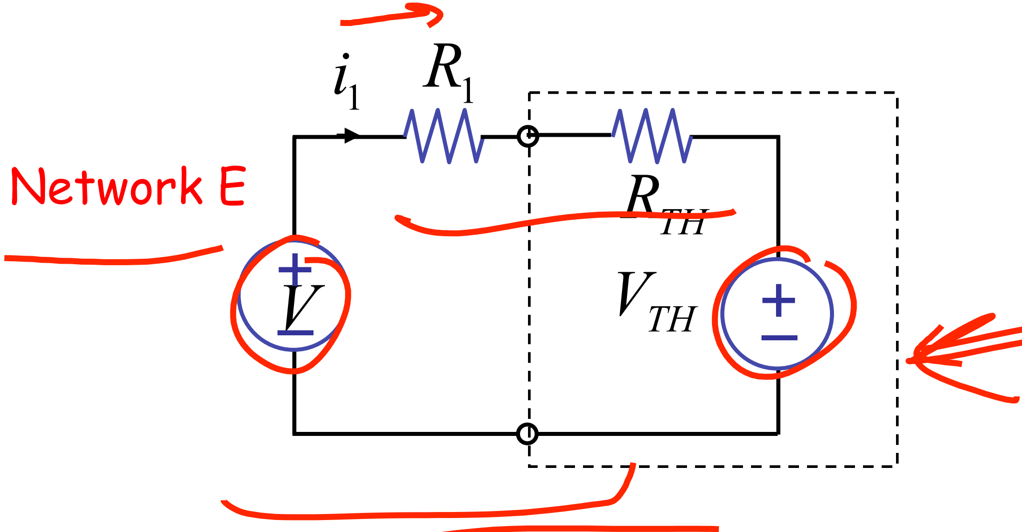
R_{TH} : turn off all independent sources and measure resistance of N

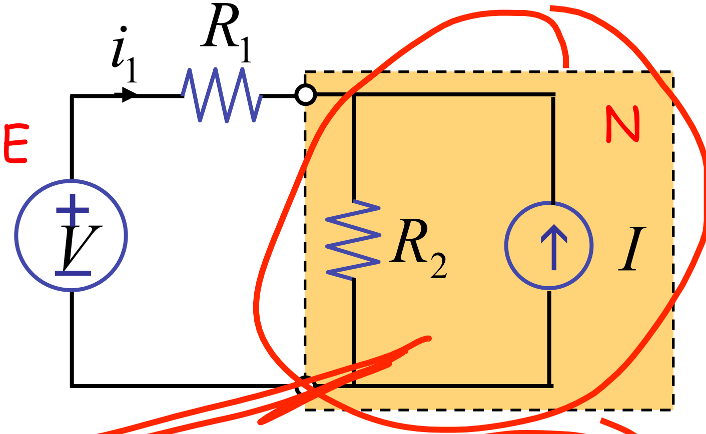


$R_{TH} = R_2$



Step 2: Solve with external network E





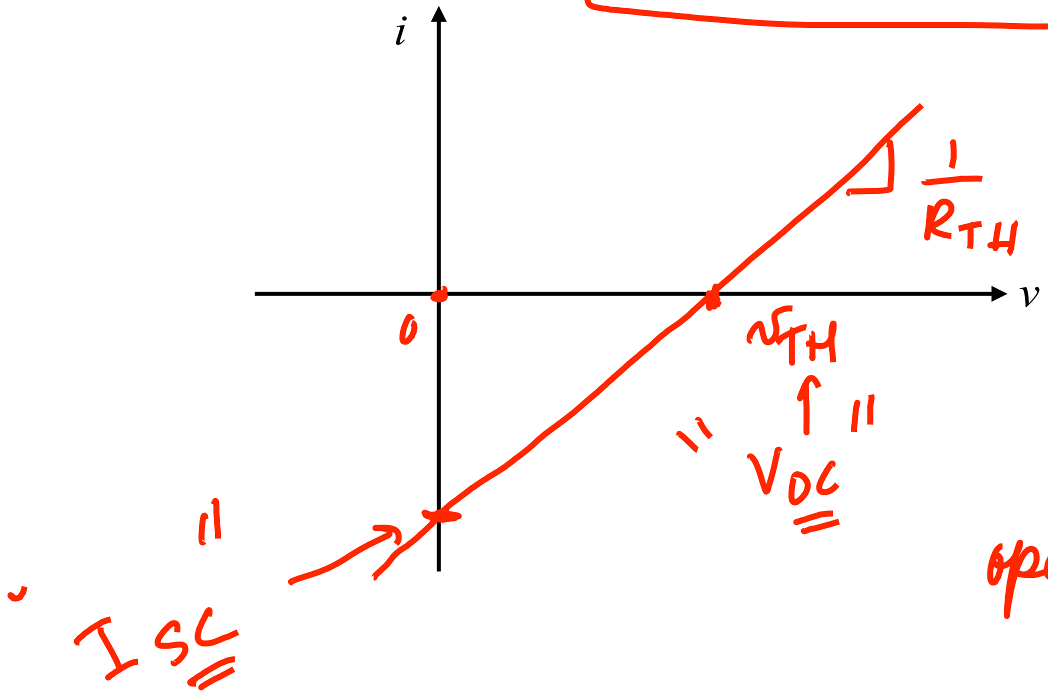
$$V_{TH} = IR_2$$
$$R_{TH} = R_2$$

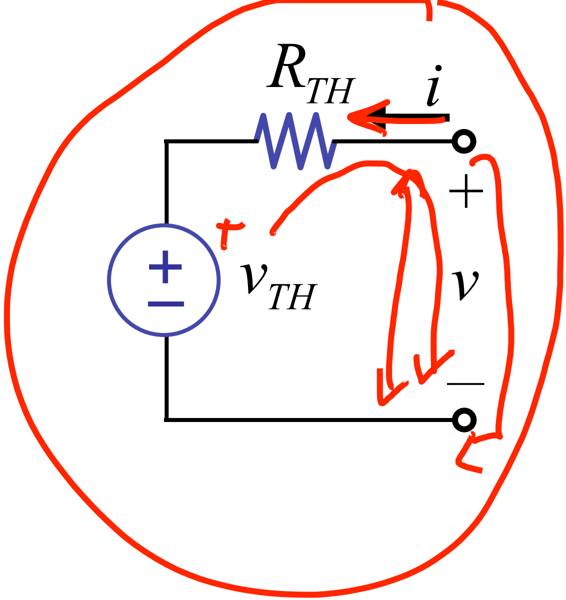
$$i_1 = \frac{V - V_{TH}}{R_1 + R_{TH}}$$

=

Graphically,

$$\boxed{v = v_{TH} + R_{TH} i}$$

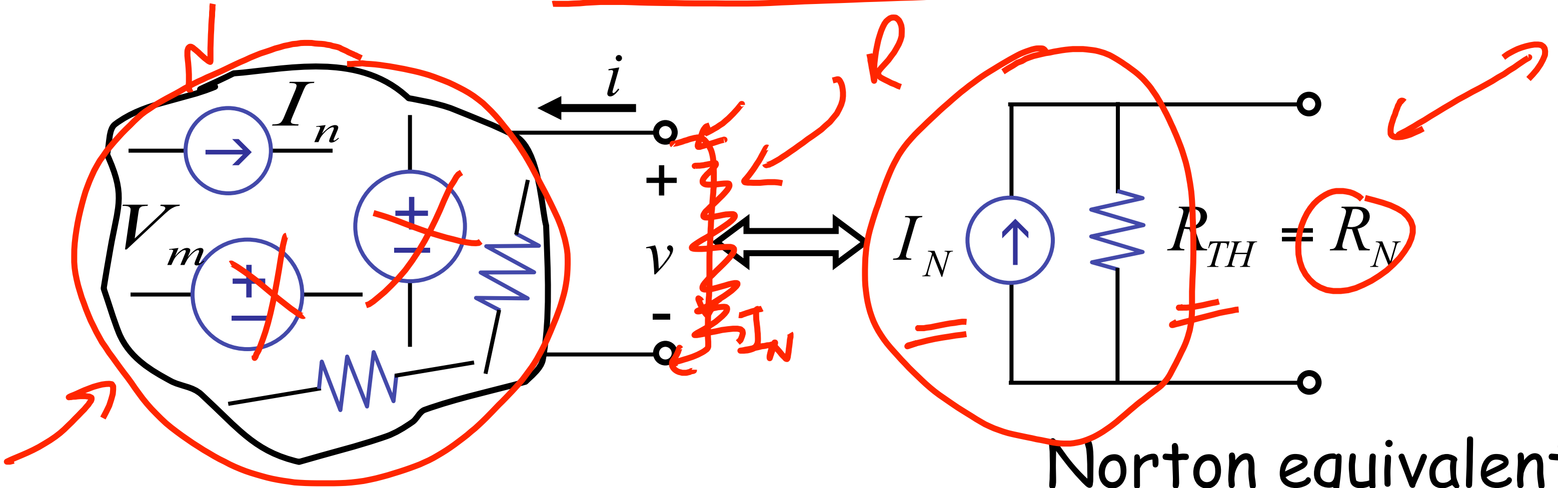




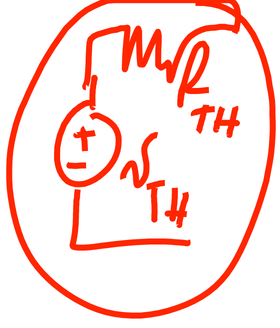
open ckt
$i = 0$
$v = v_{TH}$

short ckt
$v = 0$
$i = -\frac{v_{TH}}{R_{TH}}$

Method 5: The Norton Method



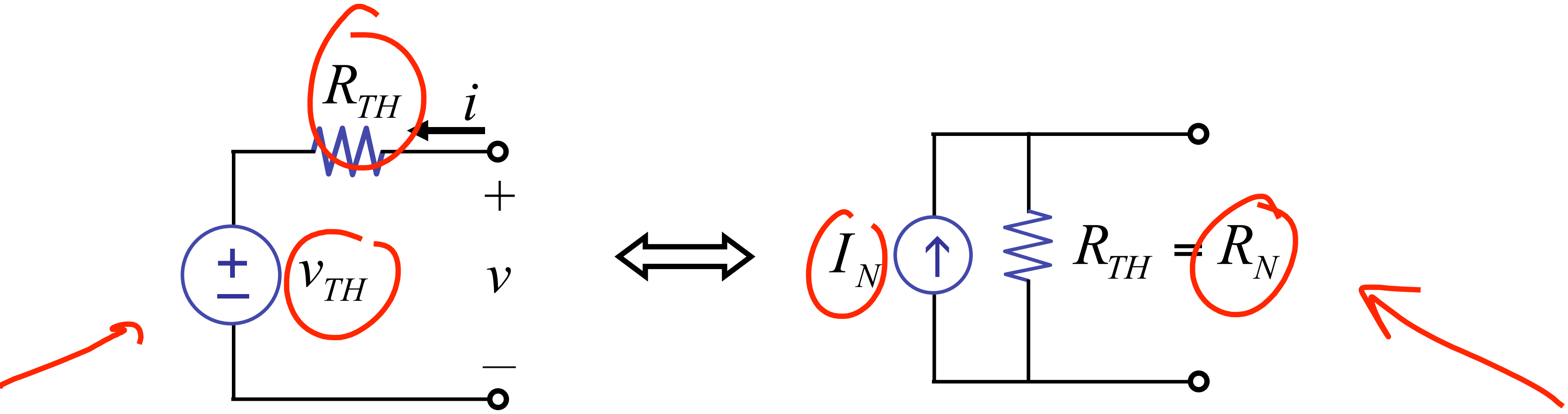
Norton equivalent



I_N ⟶ Short circuit current seen at port

$\cancel{R_{TH}}$ R_N ⟶ Resistance of network seen from port

(V_m's, I_n's set to 0)



Thevenin equivalent

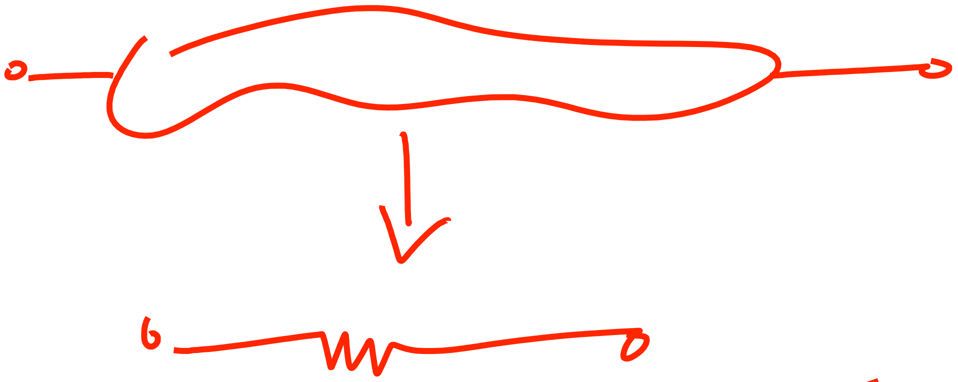
Norton equivalent

$$R_{TH} = R_N$$

$$I_N = \frac{V_{TH}}{R_{TH}}$$

Summary

Discretize matter by agreeing to observe the lumped matter discipline



Lumped circuit abstraction

Summary

Circuit analysis methods

KVL, KCL, I – V – any network
Combination rules –
Node method
Superposition
Thévenin
Norton – linear network

Next – Nonlinear networks