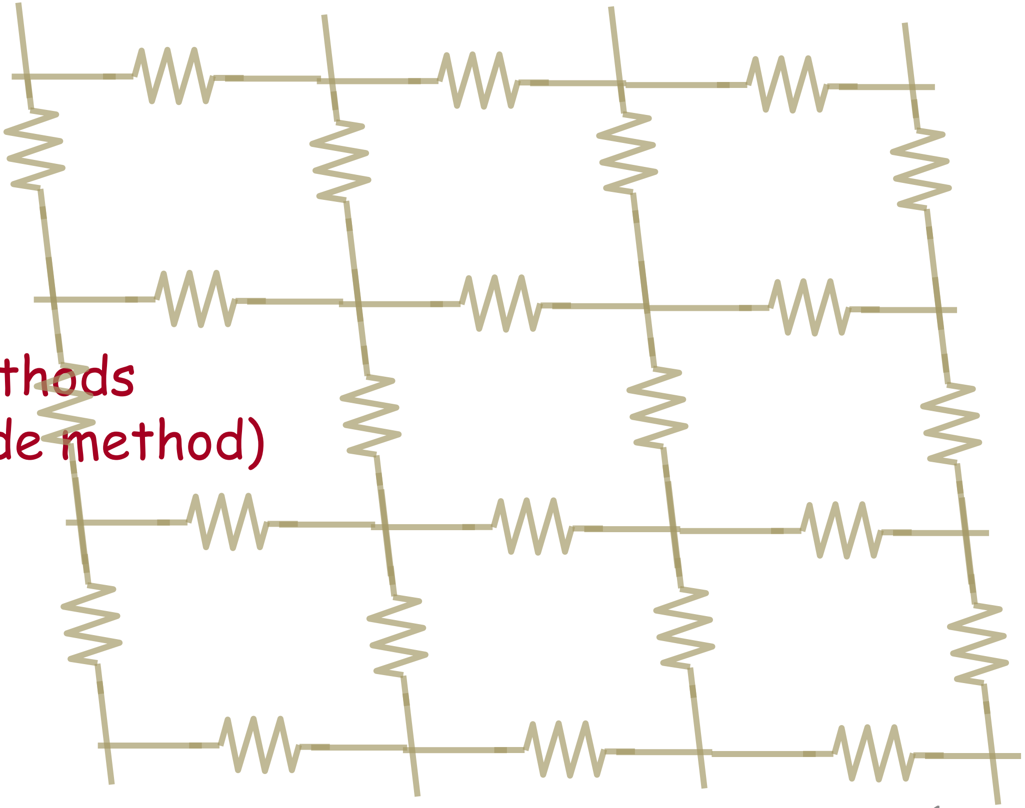
6.002x

CIRCUITS AND ELECTRONICS

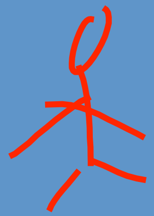


Basic Circuit Analysis Methods
(KVL and KCL method, Node method)

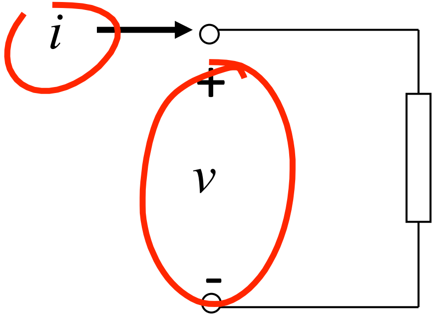
Review

Remember, our EECS playground

Observe the lumped matter discipline LMD



Review

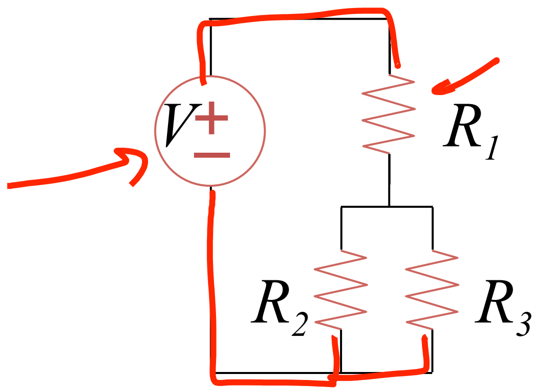


Lumped circuit element

power consumed by element = vi

Review

LMD allows us to create the lumped circuit abstraction



Review

Maxwell's equations simplify to algebraic KVL and KCL under LMD!

KVL:

$$\sum_j v_j = 0$$

For all loops

$$\oint E \cdot dl = -\frac{\partial \phi_B}{\partial t}$$

KCL:

$$\sum_j i_j = 0$$

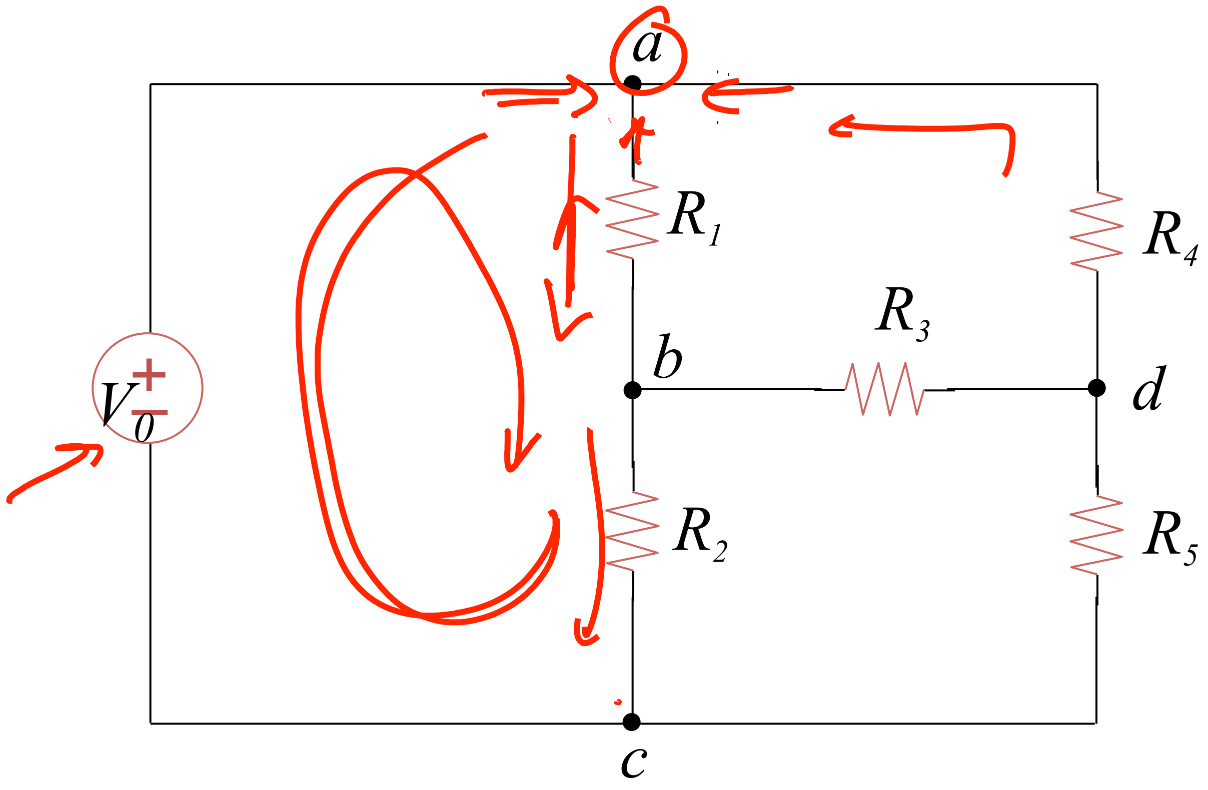
For all nodes

Review

$i_{ca} + i_{da} + i_{ba} = 0$

KCL

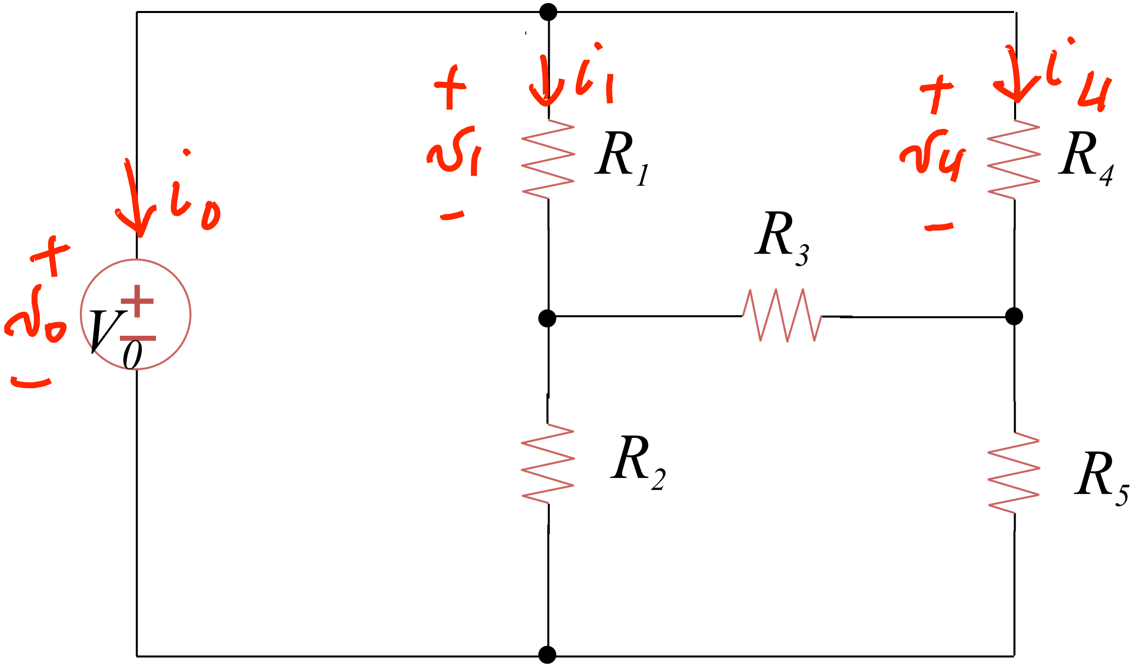
DEMO



KVL

$v_{ca} + v_{ab} + v_{bc} = 0$

Let's Begin by Building a Toolchest of Analysis Techniques



Analyzing a circuit means:

Find all the element v's and i's

Method 1: Basic KVL, KCL method of Circuit analysis

Goal: Find all element v's and i's

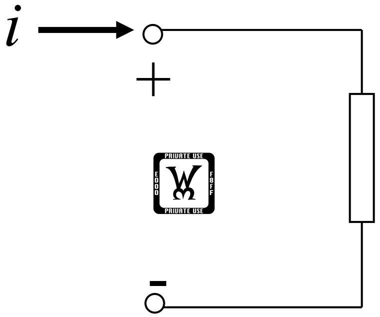
1. write element v-i relationships
   (from lumped circuit abstraction)
2. write KCL for all nodes
3. write KVL for all loops

lots of unknowns
lots of equations
lots of fun
solve

Method 1: Basic KVL, KCL method of Circuit analysis

Goal: Find all element v's and i's

Labeling element v's and i's



Element e

This convention is called: Associated variables discipline

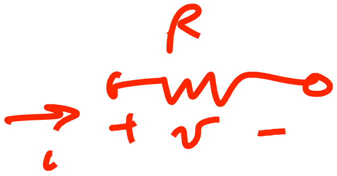
Current is taken to be positive going into the positive voltage terminal

Then power consumed by element e $\Big\}$ $= vi$ is positive

Method 1: Basic KVL, KCL method of Circuit analysis

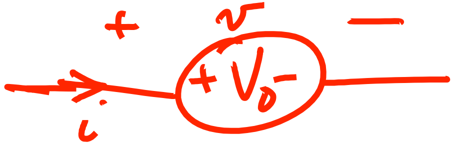
You will need this for step 1: Element Relationships

For R



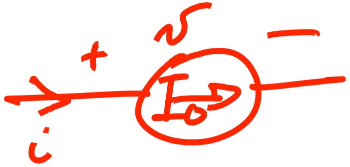
$v = iR$

For voltage source



$v = V_0$

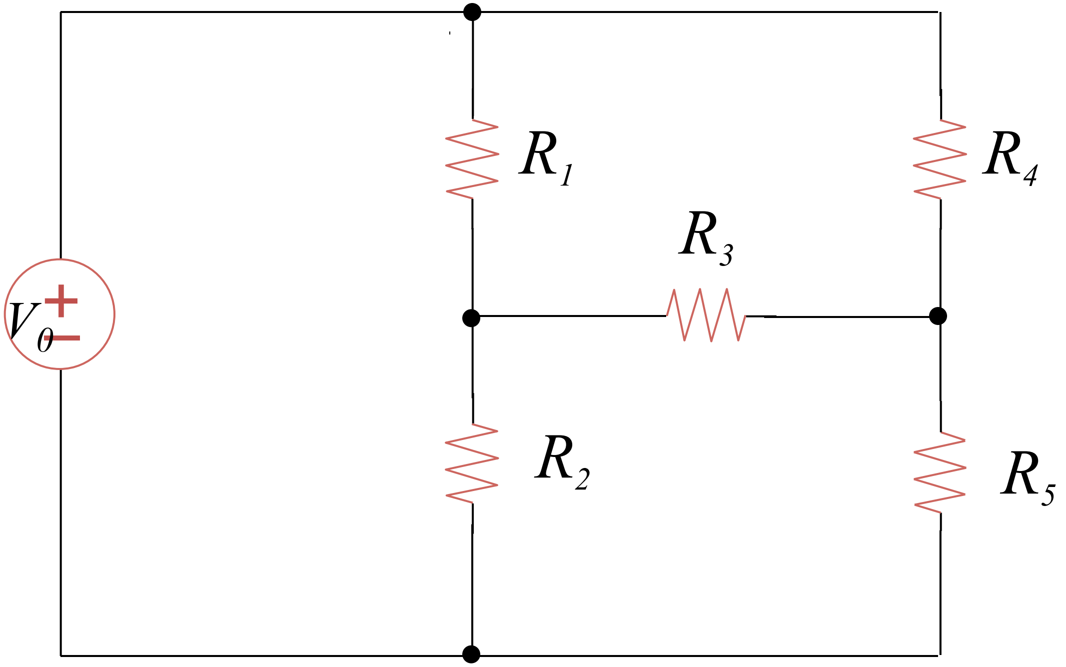
For current source



$i = I_0$

Let's Apply KVL, KCL Method to this Example

Goal: Find all element v's and i's

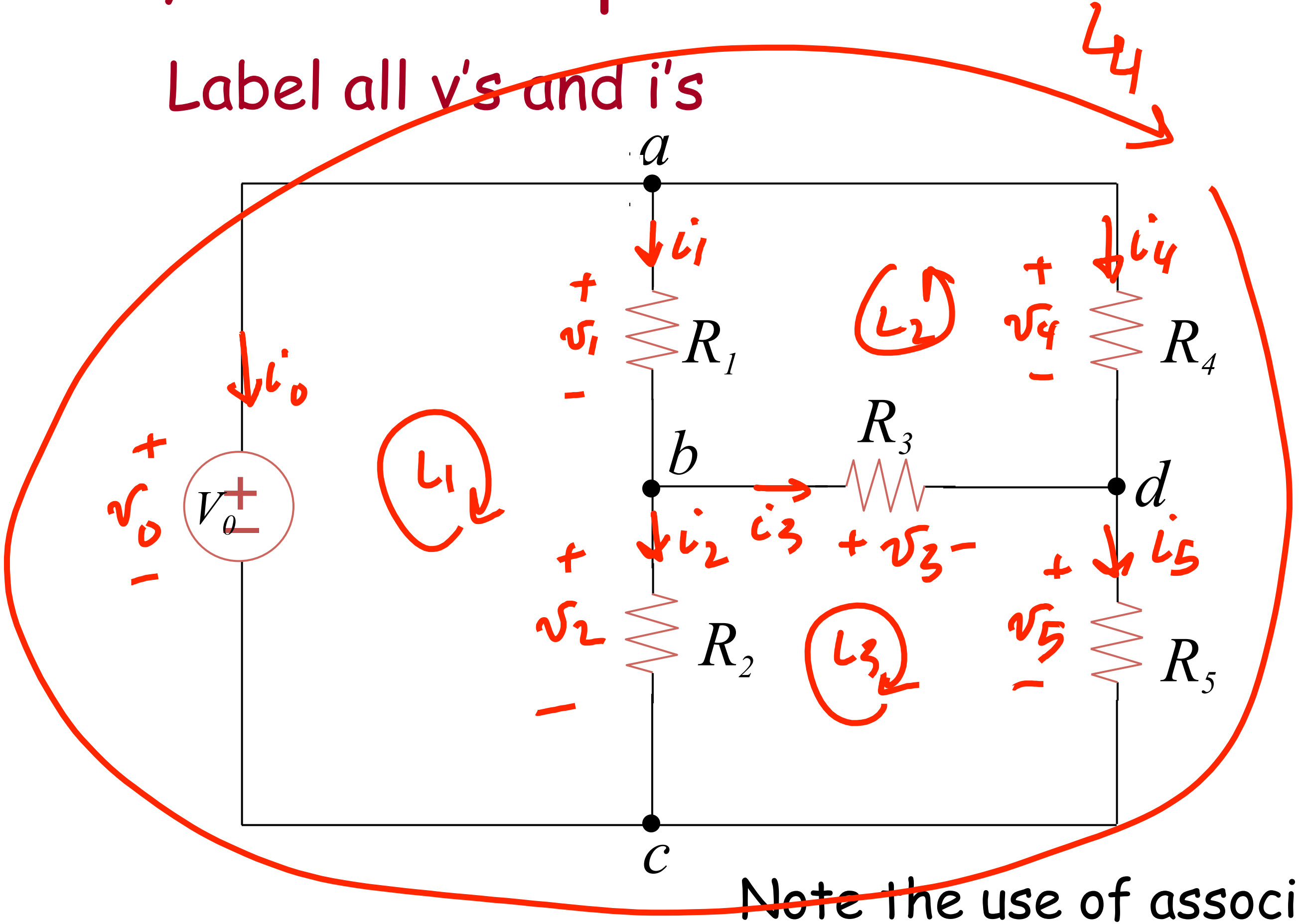


The Demo Circuit

KVL, KCL Example

Label all v's and i's

Goal: Find all element v's and i's



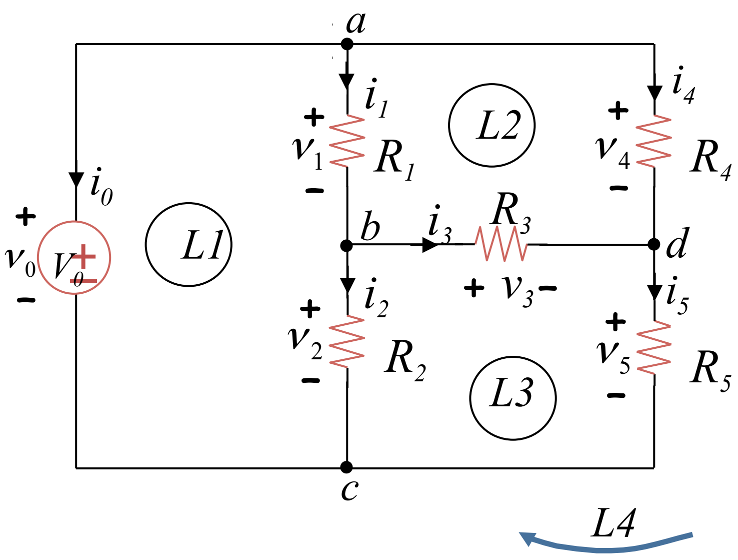
12 unknowns

$v_0 \ldots v_5, i_0 \ldots i_5$

Note the use of associated variables…

Step 1 of KVL, KCL Method

$v_0 \ldots v_5, i_0 \ldots i_5$ 12 unknowns



1. Element relationships (v, i)

$v_0 = V_0$ ← given

$v_1 = i_1 R_1$

$v_2 = i_2 R_2$

$v_3 = i_3 R_3$

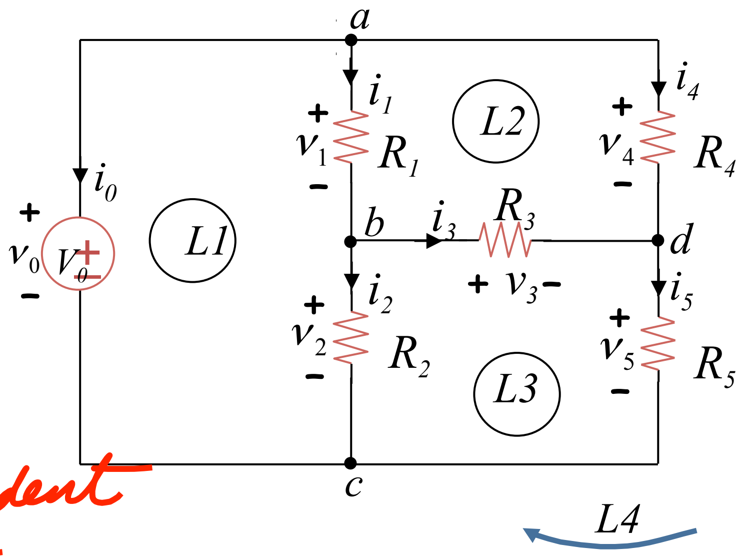
$v_4 = i_4 R_4$

$v_5 = i_5 R_5$

6 equations

Step 2 of KVL, KCL Method

$v_0 \ldots v_5, i_0 \ldots i_5$ 12 unknowns



2. KCL at the nodes

3 independent equations

a: $i_0 + i_1 + i_4 = 0$

b: $i_2 + i_3 - i_1 = 0$

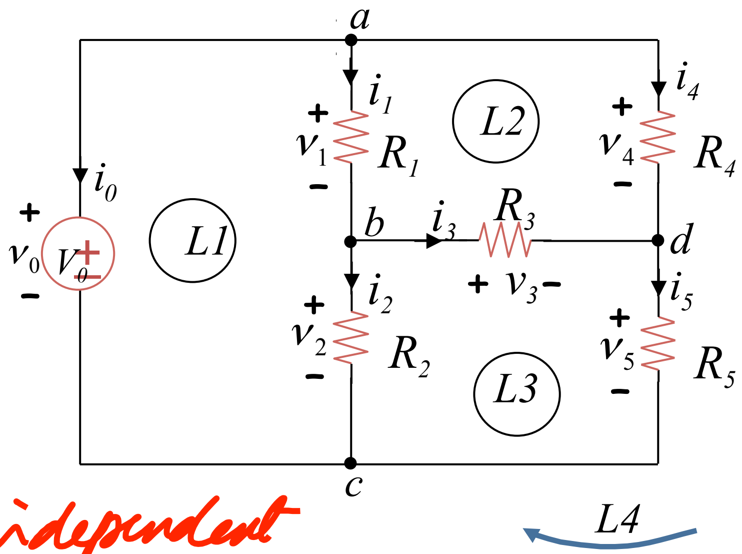
d: $i_5 - i_3 - i_4 = 0$

c: $-i_0 - i_2 - i_5 = 0$ redundant

(use convention, e.g., sum currents leaving the node)

Step 3 of KVL, KCL Method

$v_0 \ldots v_5, i_0 \ldots i_5$ 12 unknowns



3. KVL for loops

$L1: -v_0 + v_1 + v_2 = 0$

$L2: +v_1 + v_3 - v_4 = 0$

$L3: v_3 + v_5 - v_2 = 0$

$L4: -v_0 + v_4 + v_5 = 0$ redundant

3 independent equations

(use convention, e.g., as you go around loop, assign first encountered sign to each voltage)

KVL, KCL Method

1. Element v, i relationships

$$v_0 = V_0 \qquad v_3 = i_3 R_3$$
$$v_1 = i_1 R_1 \qquad v_4 = i_4 R_4$$
$$v_2 = i_2 R_2 \qquad v_5 = i_5 R_5$$

6

2. KCL at the nodes

$$a: \quad i_0 + i_1 + i_4 = 0$$
$$b: \quad i_2 + i_3 - i_1 = 0$$
$$d: \quad i_5 - i_3 - i_4 = 0$$

3

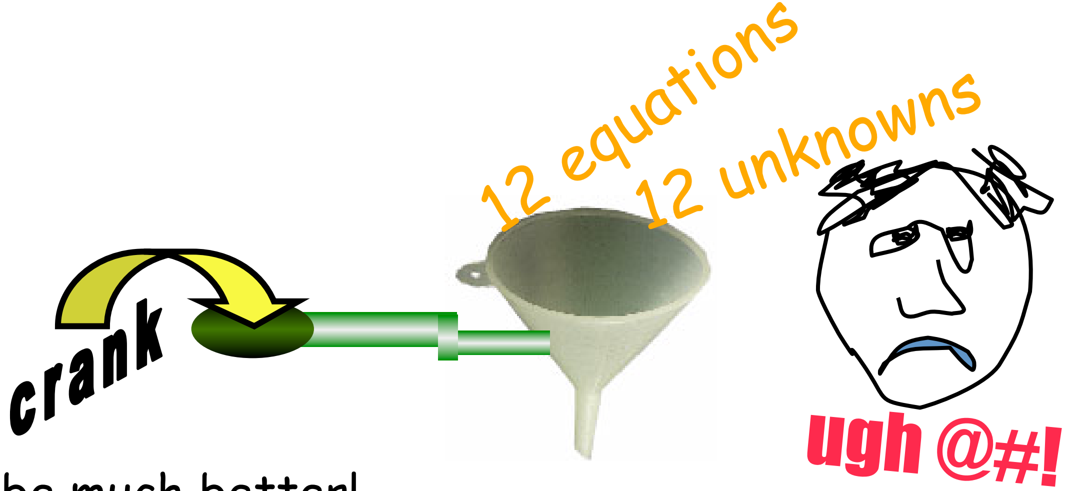
$$c: \quad -i_0 - i_2 - i_5 = 0$$ redundant

3. KVL for loops

$$L1: \quad -v_0 + v_1 + v_2 = 0$$
$$L2: \quad v_1 + v_3 - v_4 = 0$$
$$L3: \quad v_3 + v_5 - v_2 = 0$$

3

$$L4: \quad -v_0 + v_4 + v_5 = 0$$ redundant

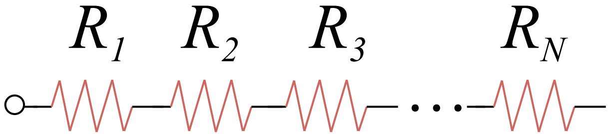


Method 3 – the node method will be much better!

Other Analysis Methods

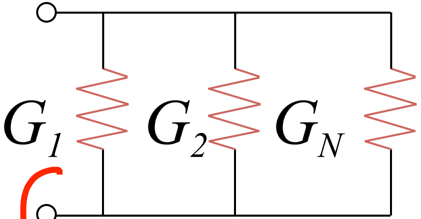
Method 2— Apply element combination rules

A

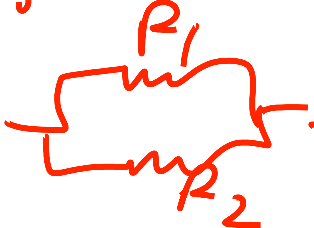


$R = R_1 + R_2 + \dots R_N$

B



$G_1 + G_2 + G_3 + \dots G_N$



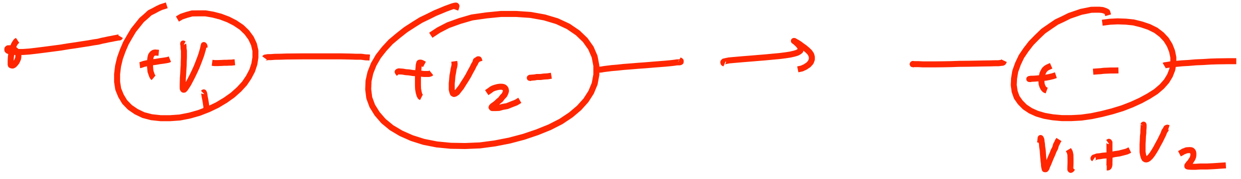
$\frac{R_1 R_2}{R_1 + R_2}$

$G_1 = \frac{1}{R_1}$ conductance

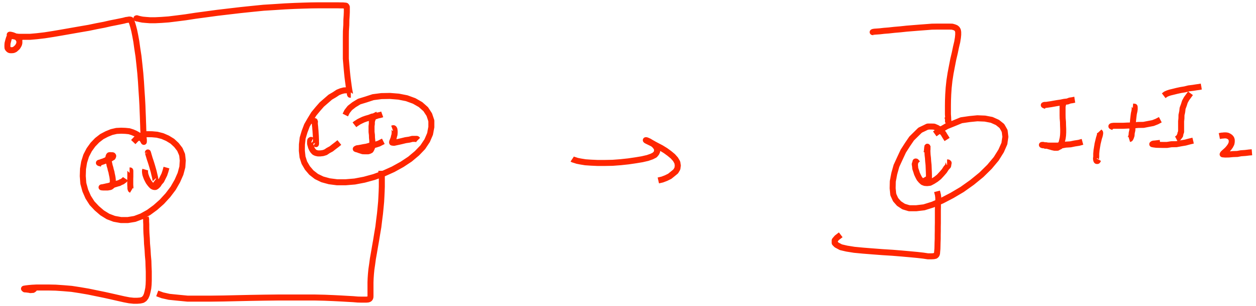
Reading: Chapter 2 of A & L

Method 2 — Apply element combination rules

C

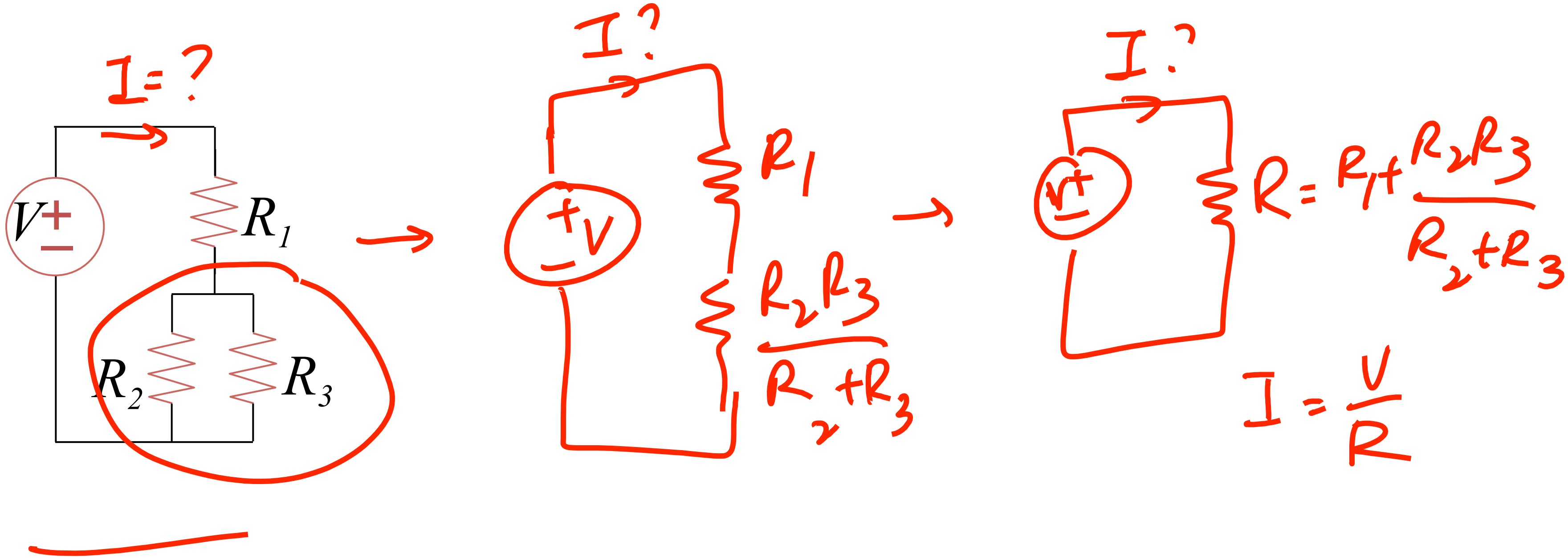


D



Method 2— Apply element combination rules

Example



$$R = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

$$I = \frac{V}{R}$$

Method 3 — Node analysis

Particular application of KVL, KCL method

1. Select reference node (⊥ ground) from which voltages are measured.

2. Label voltages of remaining nodes with respect to ground. These are the primary unknowns.

3. Write KCL for all but the ground node, substituting device laws and KVL.

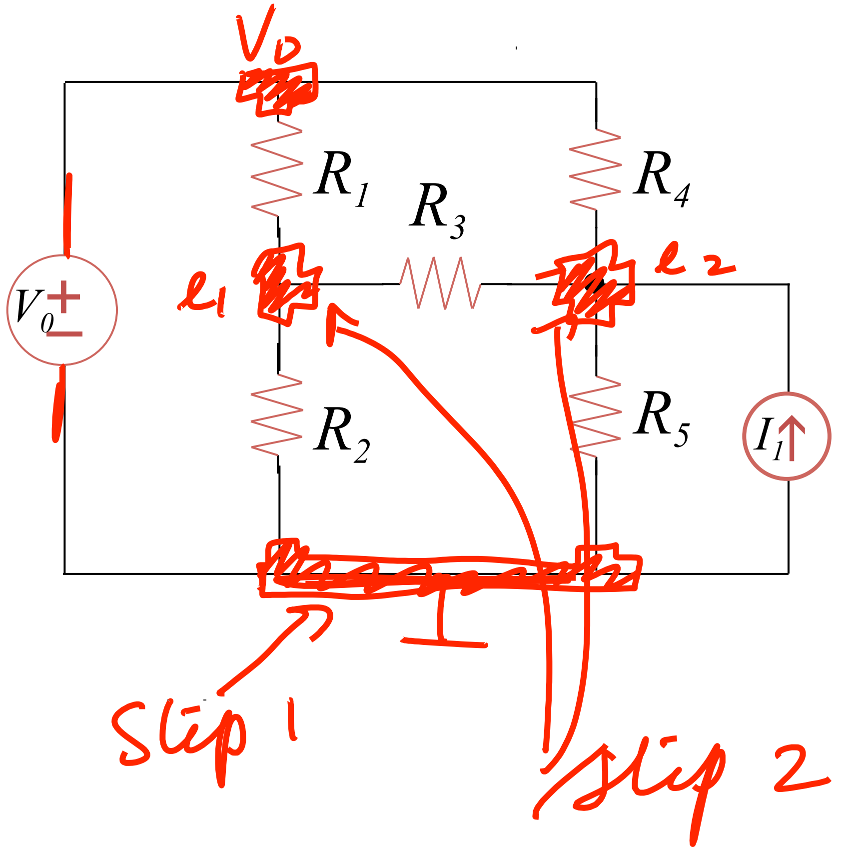
4. Solve for node voltages.

5. Back solve for branch voltages and currents (i.e., the secondary unknowns).

6.002x workhorse!

Method 3 — Node analysis

Example: Old Faithful, plus current source



1. Select reference ground node
2. Label node voltages with respect to ground.

Step 3 of Node Method

For convenience, write $G_i = \dfrac{1}{R_i}$

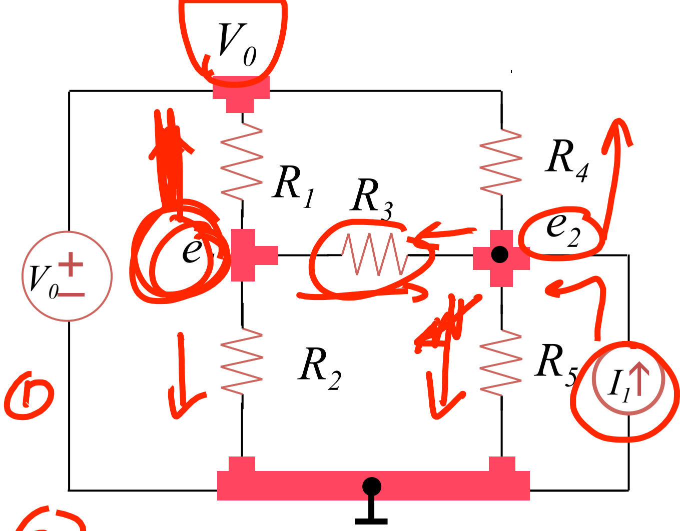
KCL at e_1

$(e_1 - V_0)G_1 + (e_1 - e_3)G_3 + (e_1)G_2 = 0$ ①

KCL at e_2

$(e_2 - e_1)G_3 + (e_2 - V_0)G_4 + e_2 G_5 - I_1 = 0$ ②

e_1 and e_2



3. Write KCL for nodes, substituting device laws and KVL.

To avoid mistakes, use convention –
E.g., always sum the currents leaving a node

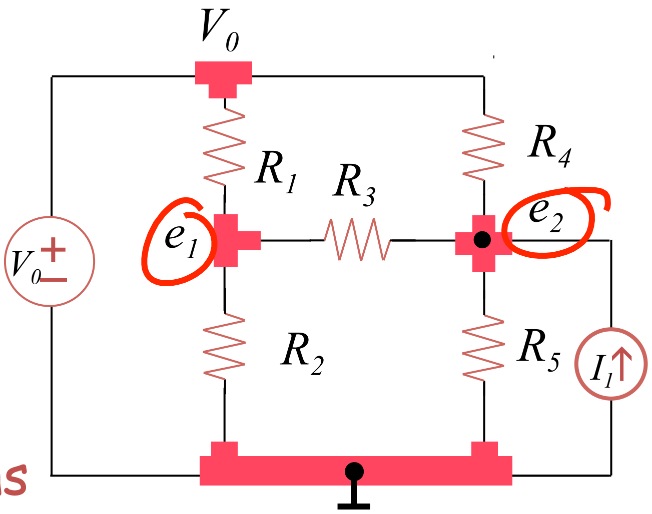
Step 4 of Node Method

KCL at e_1

$$(e_1 - V_0)G_1 + (e_1 - e_2)G_3 + (e_1)G_2 = 0 \quad \text{— ①}$$

KCL at e_2

$$(e_2 - e_1)G_3 + (e_2 - V_0)G_4 + (e_2)G_5 - I_1 = 0 \quad \text{— ②}$$



4. Solve for node voltages

Move constant terms to RHS & collect unknowns

$$e_1(G_1 + G_3 + G_2) + e_2(-G_3) = V_0(G_1)$$

$$e_1(-G_3) + e_2(G_3 + G_4 + G_5) = V_0(G_4) + I_1$$

2 equations, 2 unknowns $\longrightarrow$ Solve for e's

(compare units)

Step 5 of Node Method

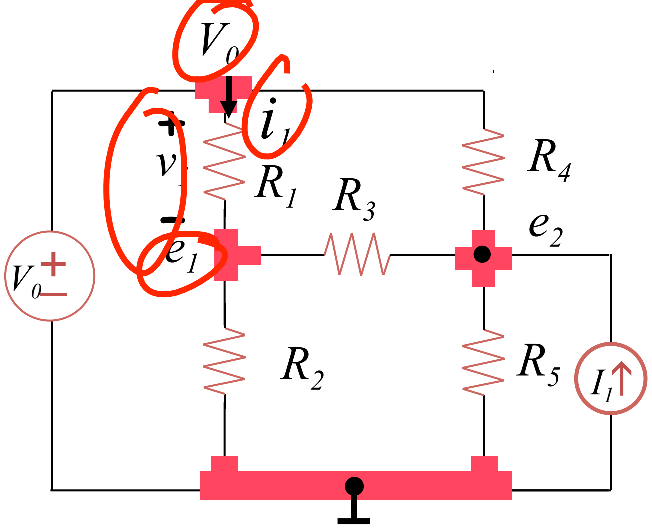
Once you have solved for e_1 and e_2, easy to find branch v's and i's

For example:

$$v_1 = V_0 - e_1$$

$$i_1 = \frac{v_1}{R_1} = \frac{(V_0 - e_1)}{R_1}$$

⋮



5. Back solve for branch voltages and currents

e's

Revisit Step 4 of Node Method for Cultural Interest

$$e_1(G_1 + G_2 + G_3) + e_2(-G_3) = V_0(G_1) \quad \text{— } 1$$

$$e_1(-G_3) + e_2(G_3 + G_4 + G_5) = V_0(G_4) + I_1 \quad \text{— } 2$$

4. Solve for node voltages

In matrix form:

$$\underbrace{\begin{bmatrix} G_1 + G_2 + G_3 & -G_3 \\ -G_3 & G_3 + G_4 + G_5 \end{bmatrix}}_{\text{conductivity matrix}} \underbrace{\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}}_{\text{unknown node voltages}} = \underbrace{\begin{bmatrix} G_1 V_0 \\ G_4 V_0 + I_1 \end{bmatrix}}_{\text{sources}}$$

Step 4 of Node Method

4. Solve for node voltages

$Ax = b$

$$\begin{bmatrix} G_1+G_2+G_3 & -G_3 \\ -G_3 & G_3+G_4+G_5 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} G_1V_0 \\ G_4V_0+I_1 \end{bmatrix}$$

Solve

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \frac{\begin{bmatrix} G_3+G_4+G_5 & G_3 \\ G_3 & G_1+G_2+G_3 \end{bmatrix} \begin{bmatrix} G_1V_0 \\ G_4V_0+I_1 \end{bmatrix}}{(G_1+G_2+G_3)(G_3+G_4+G_5)-G_3^{\,2}}$$

$$e_1 = \frac{(G_3+G_4+G_5)(G_1V_0)+(G_3)(G_4V_0+I_1)}{G_1G_3+G_1G_4+G_1G_5+G_2G_3+G_2G_4+G_2G_5+G_3^{\,2}+G_3G_4+G_3G_5}$$

$$e_2 = \frac{(G_3)(G_1V_0)+(G_1+G_2+G_3)(G_4V_0+I_1)}{G_1G_3+G_1G_4+G_1G_5+G_2G_3+G_2G_4+G_2G_5+G_3^{\,2}+G_3G_4+G_3G_5}$$

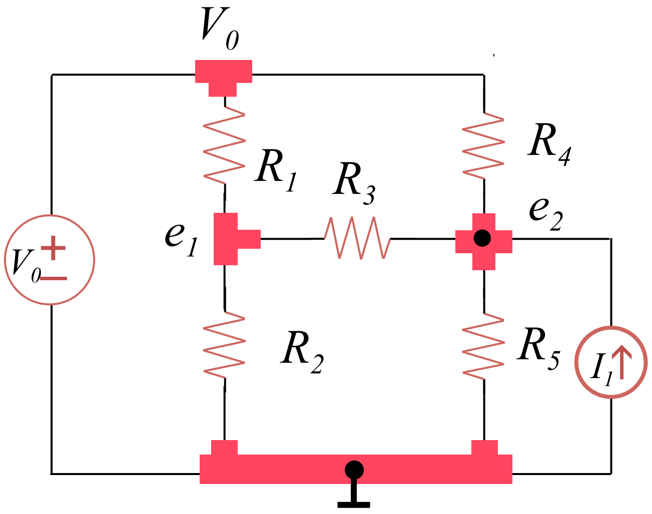
(same denominator)

Notice: linear in V_0, I_1, no negatives in denominator – we will use this later

Step 4 of Node Method

E.g., solve for e_2, given

$\left.\begin{matrix} G_1 \\ G_5 \end{matrix}\right\} = \frac{1}{8.2K}$ $\left.\begin{matrix} G_2 \\ G_4 \end{matrix}\right\} = \frac{1}{3.9K}$ $G_3 = \frac{1}{1.5K}$ $I_1 = 0$



$$e_2 = 0.6V_0$$

If $V_0 = 3V$, then $e_2 = 1.8V$

1.8 V