

6.002x

# CIRCUITS AND ELECTRONICS

Dependent Sources  
Amplifiers



# Review

- Nonlinear circuits — can use the node method
- Small signal trick resulted in linear response

## Today

- Dependent sources
- Amplifiers

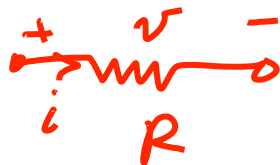
**Reading:** Chapter 7.1, 7.2



# Dependent Sources

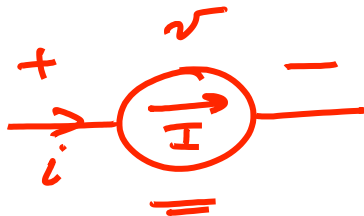
# Elements we have seen previously

Resistor



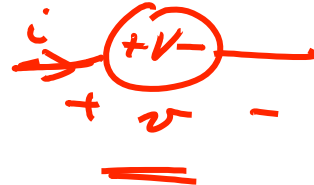
$$i = \frac{v}{R}$$

Independent  
Current source



$$i = I$$

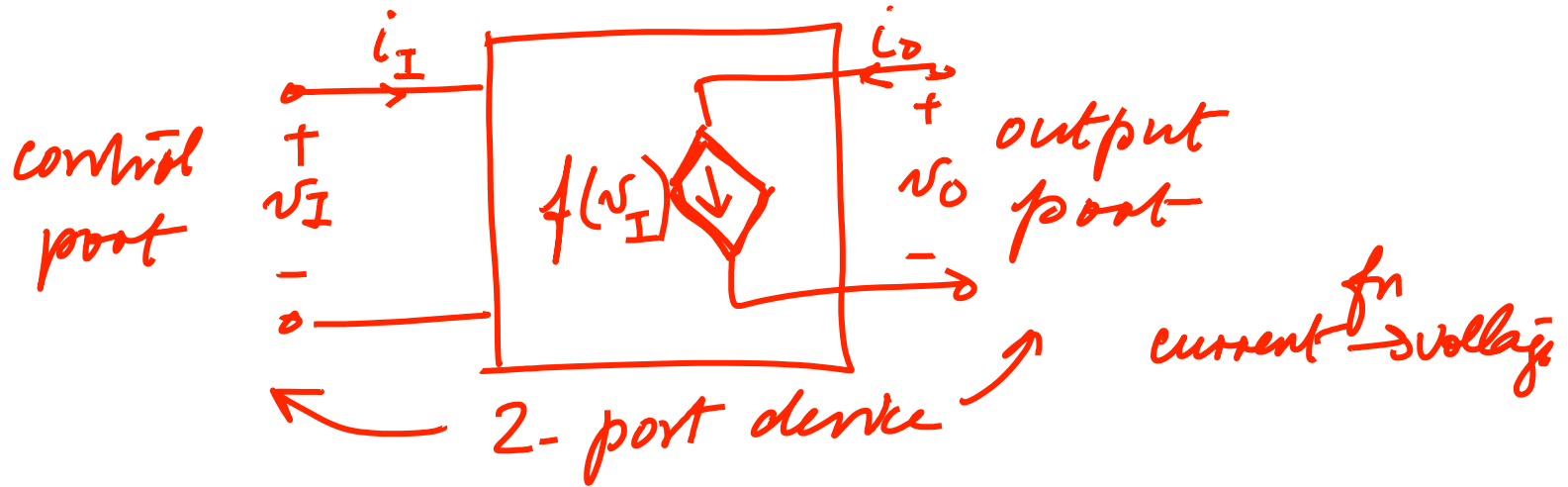
Independent  
voltage source



2-terminal 1-port devices

# Dependent sources

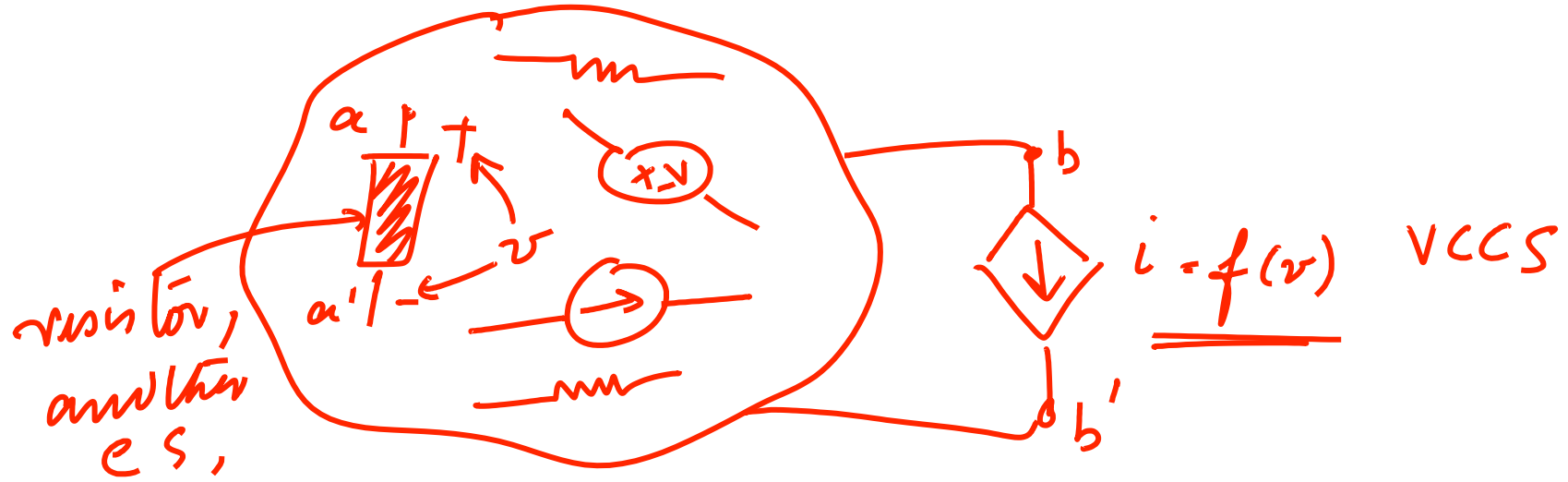
A new element for our toolchest; can be linear or nonlinear



E.g., Voltage Controlled Current Source **VCCS**

Current at output port is a function of voltage at the input port

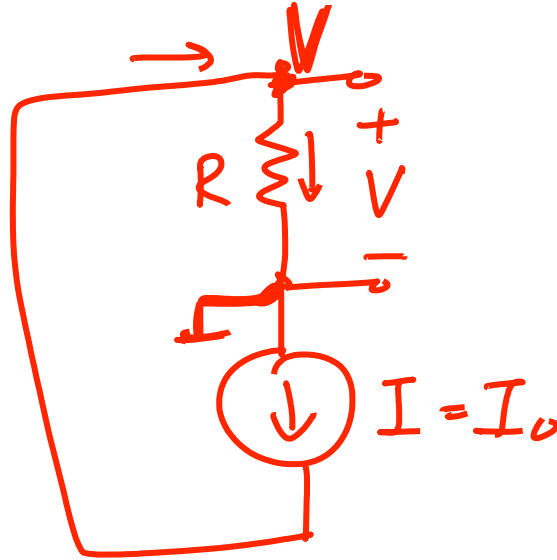
# Dependent source in a circuit



# First, an Example with an independent source

Example 1: Find  $V$

*independent  
current  
source*



*Node method:*

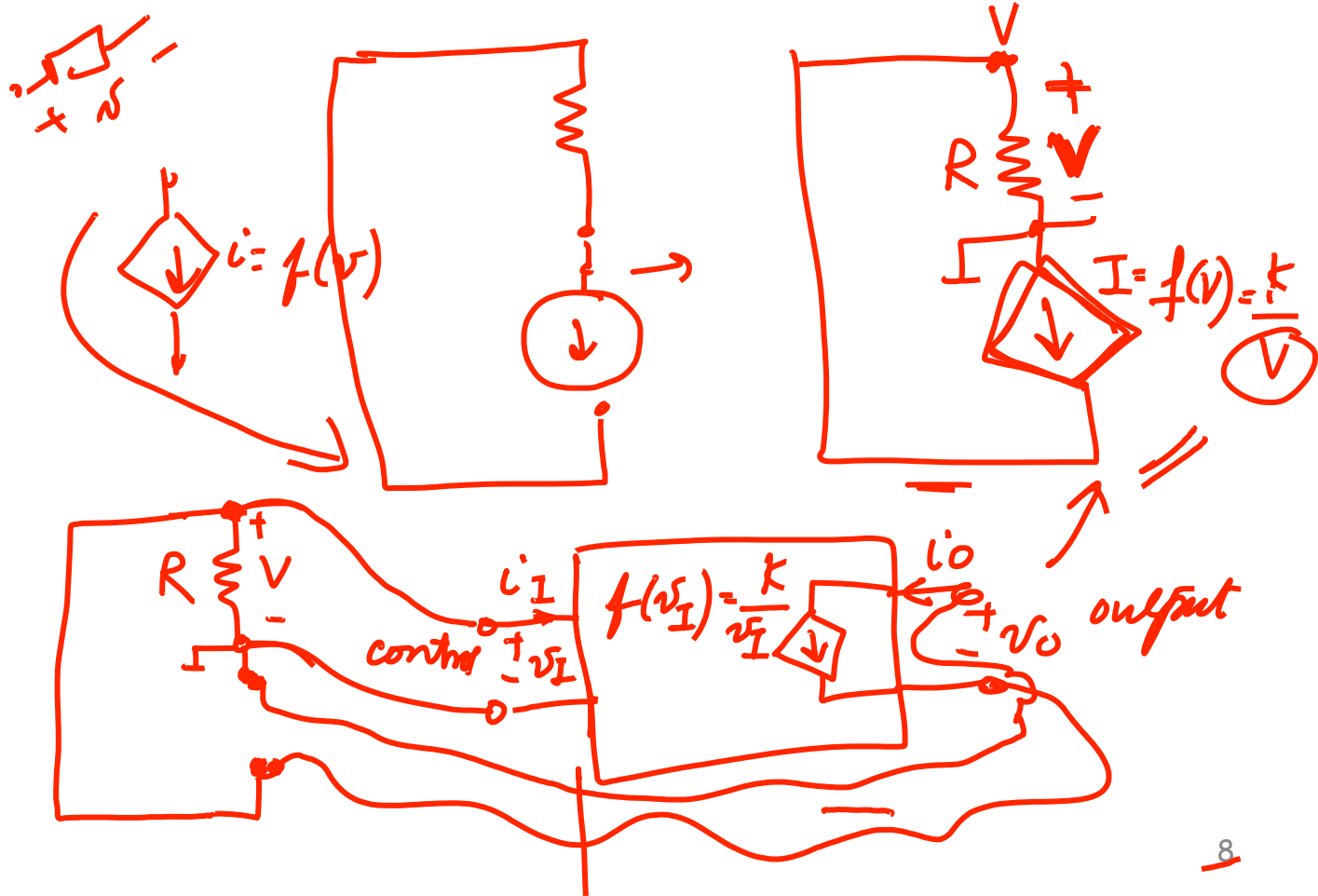
$$\frac{V}{R} - I_0 = 0$$

$V = I_0 R$

# Dependent Sources: Example

Example 2: Find  $V$

voltage  
controlled  
current  
source



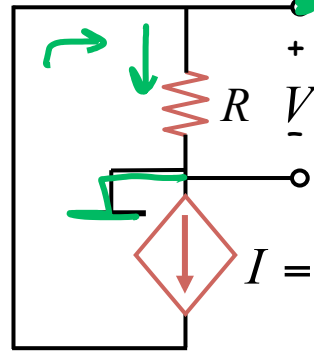
Note that above is  
abstracted view of:



# Dependent Sources: Examples

Example 2: Find  $V$

voltage  
controlled  
current  
source



e.g.

$K = 10^{-3} \text{ Amp} \cdot \text{Volt}$ ,

$R = 1 \text{ k}\Omega$

Node method:  $\frac{V}{R} - I = 0$

$$V = IR$$

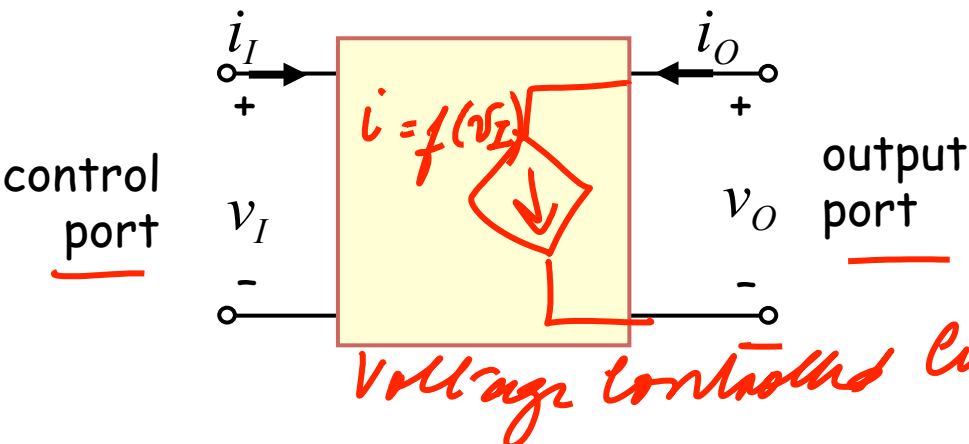
$$V = \left(\frac{K}{V}\right) \cdot R$$

$$V^2 = KR$$

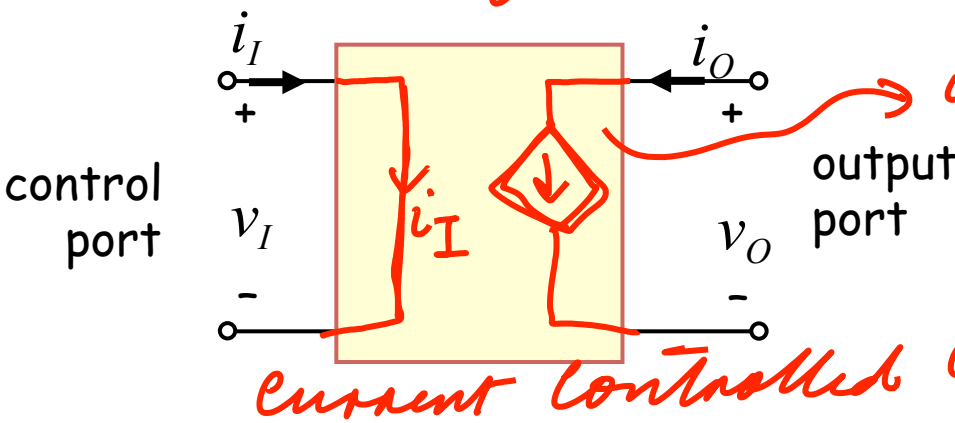
$$V = \sqrt{KR}$$

$$= \sqrt{10^{-3} \cdot 10^3} = 1 \text{ Volt}$$

# Other Types of Dependent Sources



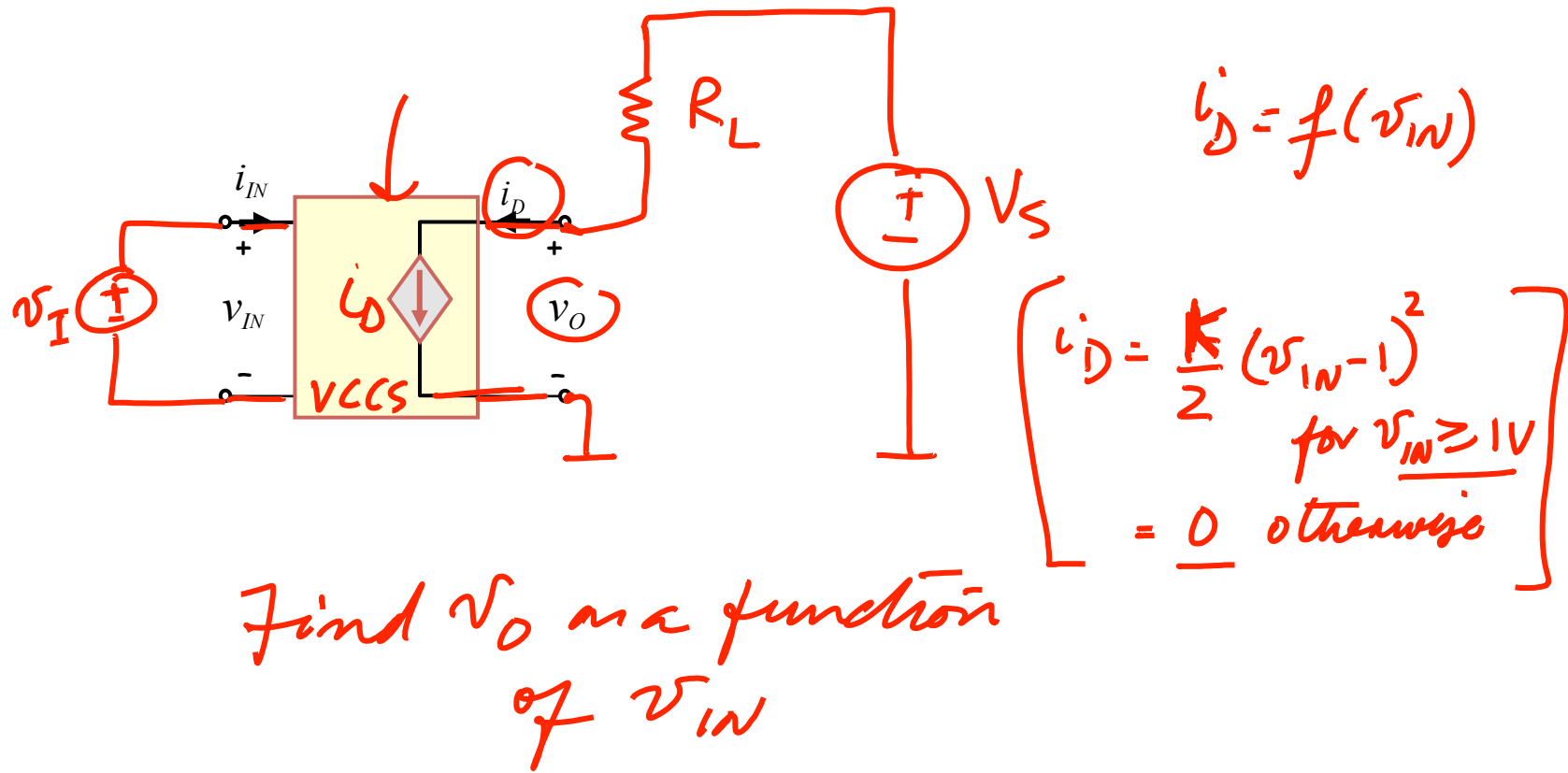
dependent  
voltage  
source  
VCVS, CCVS  
VCCS



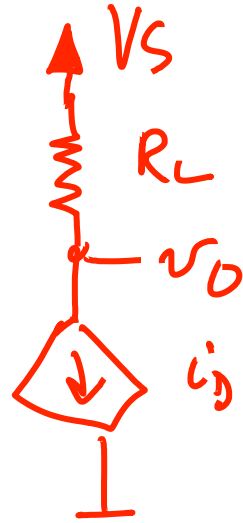
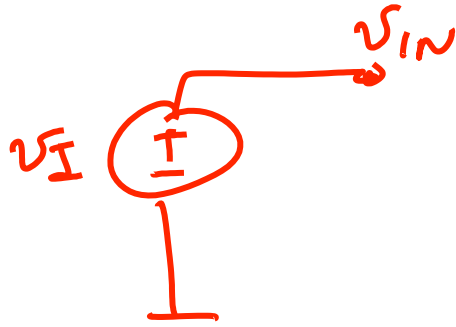
Similarly,  
CCVS, VCVS

CCCS  
Current Controlled Current Source

## Another dependent source example



# Simplify our Drawing



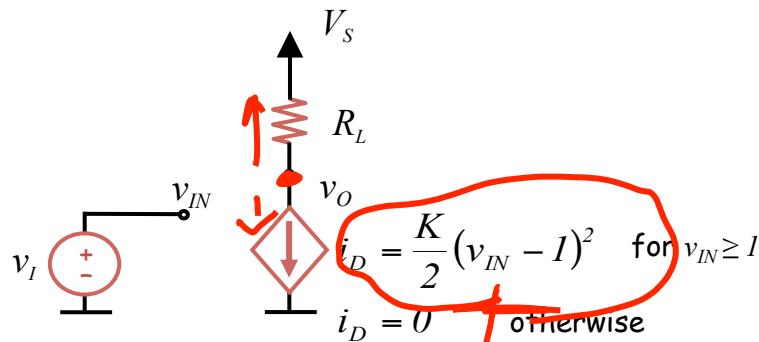
$$i_D = \frac{k}{2} (v_{IN} - 1)^2 \text{ for } v_{IN} \geq 1$$
$$= 0 \text{ otherwise}$$

Solve

$v_O$  vs  $v_I$   
Node method

$$\frac{v_O - V_S}{R_L} + (C_D) = 0$$

$$v_O = V_S - \underline{i_D R_L}$$



$$\underline{v_O} = V_S - \frac{K}{2} (\underline{v_{in}} - 1)^2 R_L \text{ for } v_{in} \geq 1V$$

$$v_O = V_S \text{ for } v_{in} < 1V$$



Hold that thought

Plot  $v_O$  versus  $v_I$  curve for our dependent source example

e.g.  $V_S = 10V$ ,  $K = 2 \frac{mA}{V^2}$ ,  $R_L = 5K\Omega$

For  $v_I > 1V$

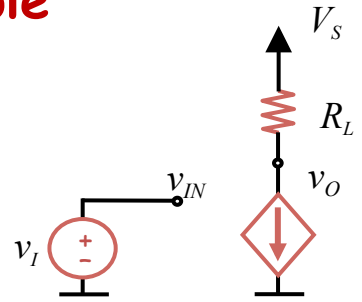
$$v_O = V_S - \frac{K}{2} (v_I - 1)^2 R_L$$

$$= 10 - \frac{2}{2} (v_I - 1)^2 \cdot 5$$

$$v_O = 10 - 5(v_I - 1)^2$$

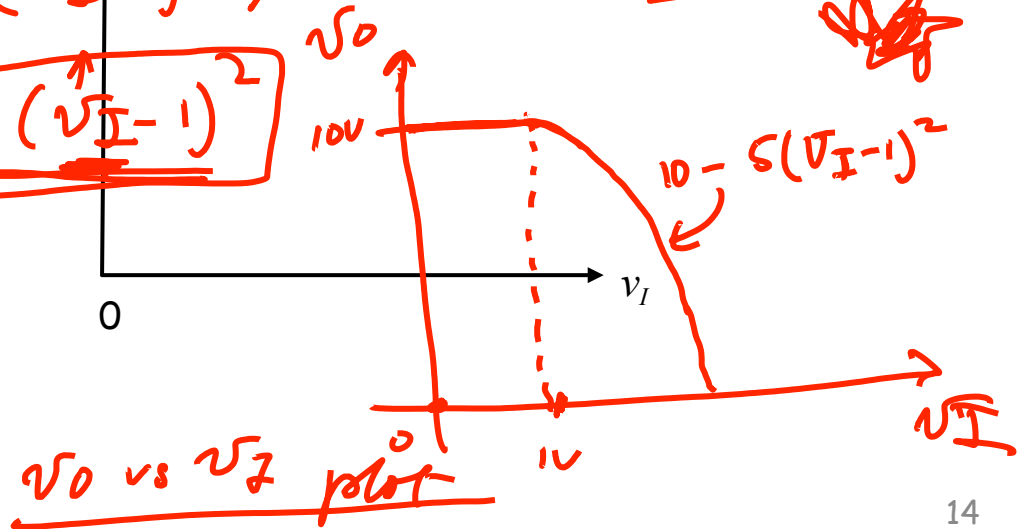
For  $v_I < 1V$

$$v_O = V_S = 10V$$



$$v_O = V_S - \frac{K}{2} (v_I - 1)^2 R_L \quad \text{for } v_I \geq 1V$$

$$v_O = V_S \quad \text{for } v_I < 1V$$



# Superposition with Dependent Sources

linear

One way

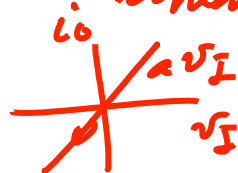
- leave all dependent sources in (note, dependent sources must be linear!)
- solve for one independent source at a time
- [section 3.5.1 of the text]

VCCS

$$i_o = f(v_I)$$

$$i_o = a \cdot v_I$$

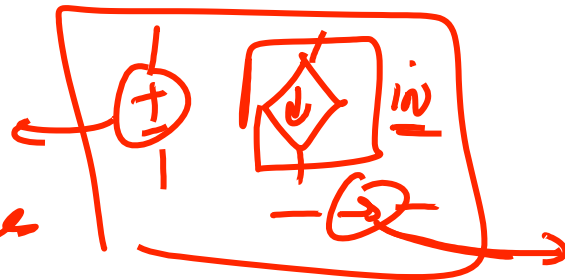
linear



$$i_o = b \cdot v_I^2$$

nonlinear

Linear circuit  
with time nets  
+ indep. sources  
Tutorial end of W2 -



Next, Amplifiers

*big abstraction*

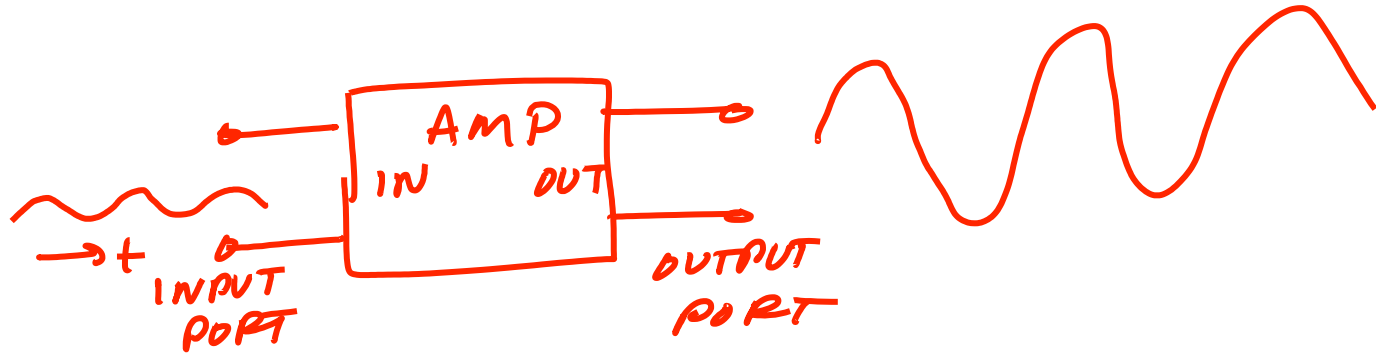
*- gate*  
*-*



# Why amplify?

Signal amplification key to both analog and digital processing.

Analog:

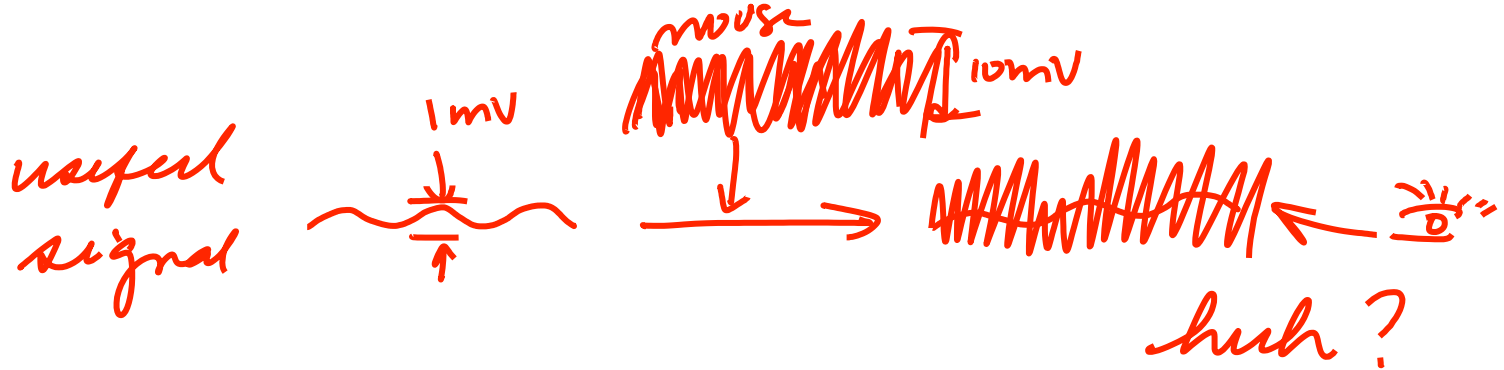


Besides the obvious advantages of being heard farther away, amplification is key to noise tolerance during communication

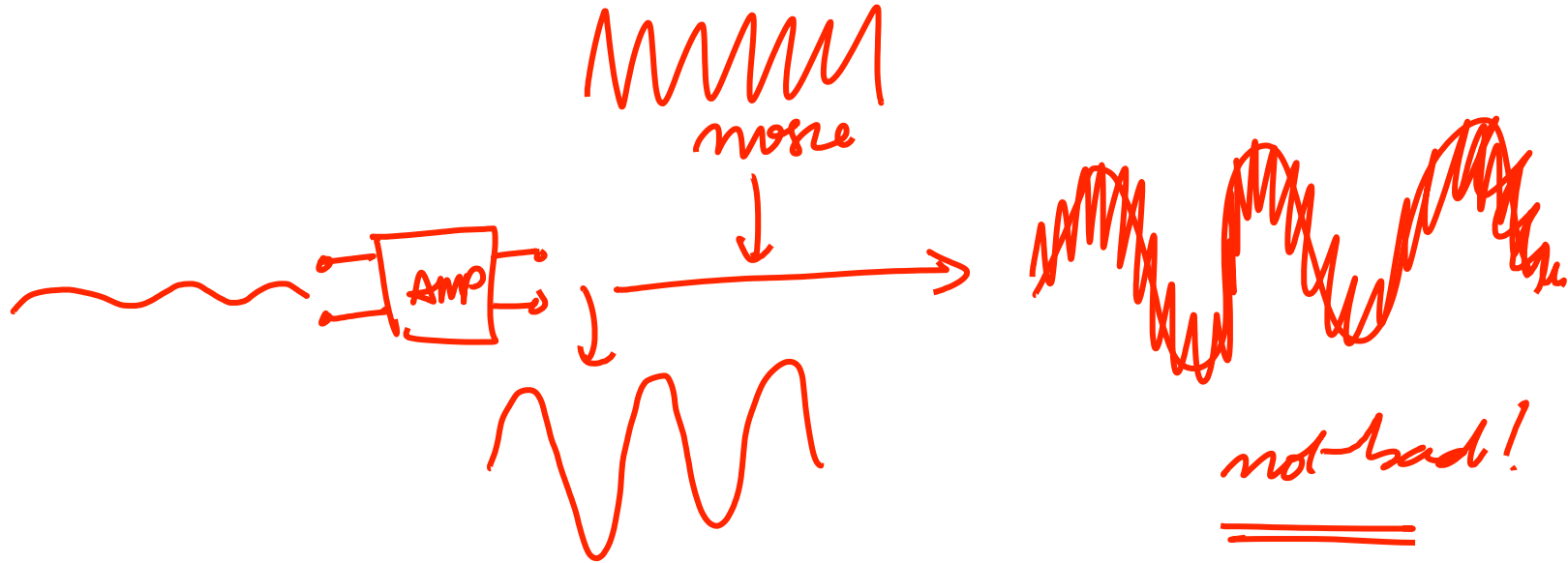
# Why amplify?

Amplification is key to noise tolerance during communication

*No amplification*

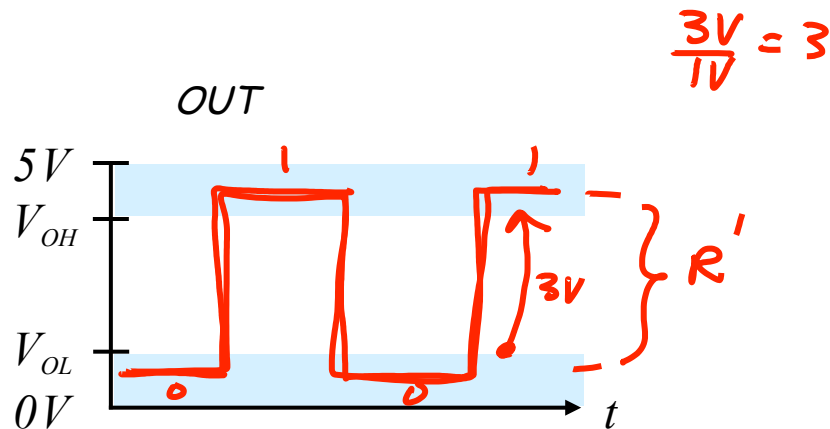
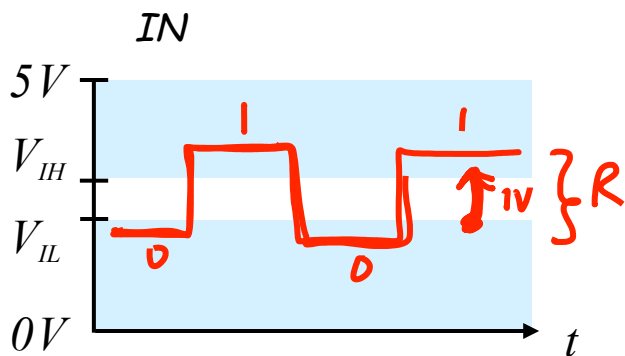
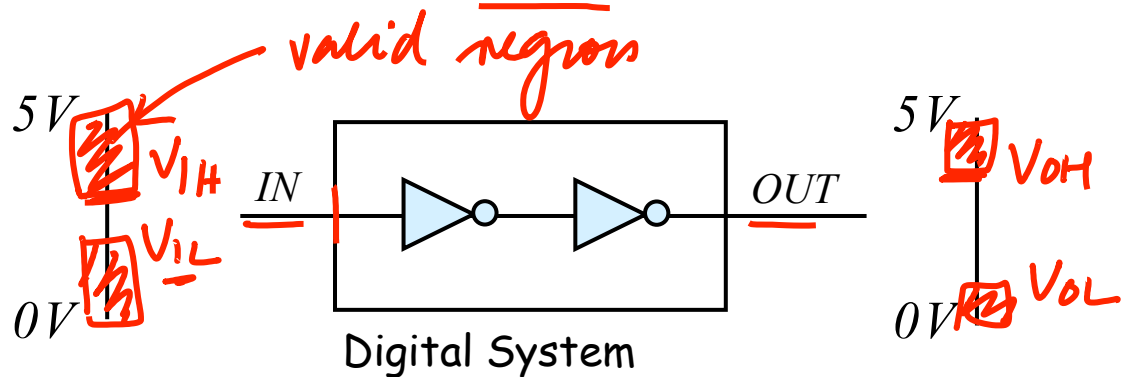


# Try amplification



# Why amplify?

Amplification is fundamental to the digital domain as well

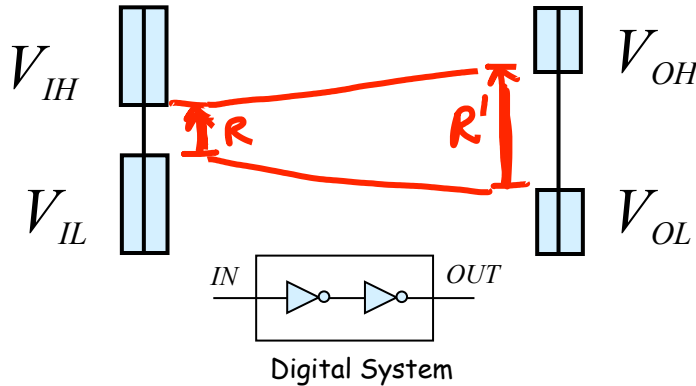


# Why amplify?

Digital domain:

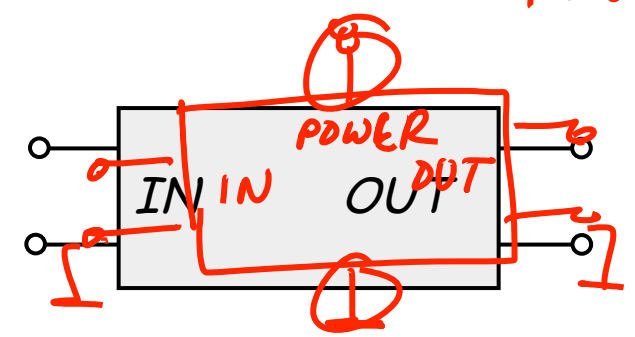
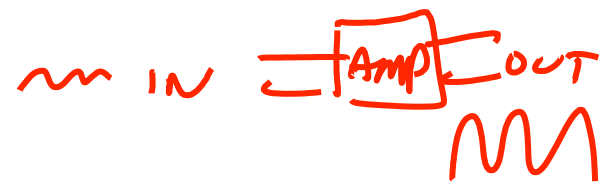
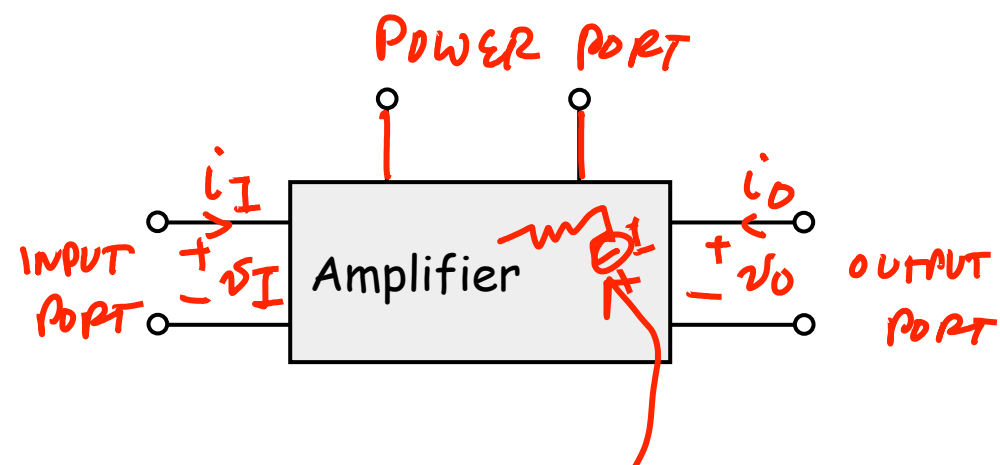
Static discipline requires amplification!

Minimum amplification needed:



$$\frac{R'}{R} = \frac{V_{OH} - V_{OL}}{V_{IH} - V_{IL}}$$

An amplifier is a 3-ported device, actually



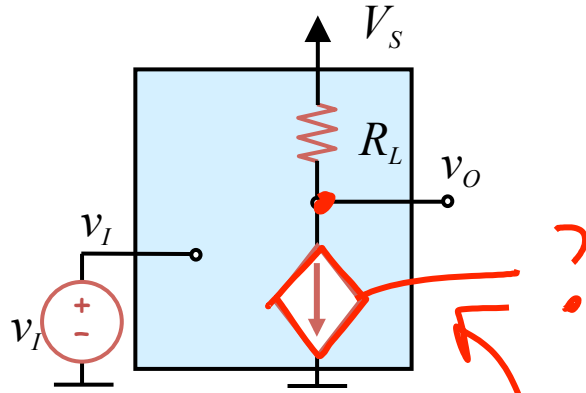
We often don't show the power port.

Also, for convenience we commonly observe "the common ground discipline."

In other words, all ports often share a common reference point called "ground."

How do we build one?

# You already have! Remember...



Node method:

$$\frac{v_O - V_S}{R_L} + i_D = 0 \quad \leftarrow$$

$$v_O = V_S - i_D R_L \quad \leftarrow$$



$$v_O = V_S - \frac{K}{2} (v_I - 1)^2 R_L \quad \text{for } v_I \geq 1 \quad \leftarrow$$

$$v_O = V_S \quad \text{for } v_I < 1$$

$$i_D = \frac{K}{2} (v_{IN} - 1)^2 \quad \text{for } v_{IN} \geq 1$$

$$i_D = 0 \quad \text{otherwise}$$

Claim: This is an amplifier

# So, where's the amplification?

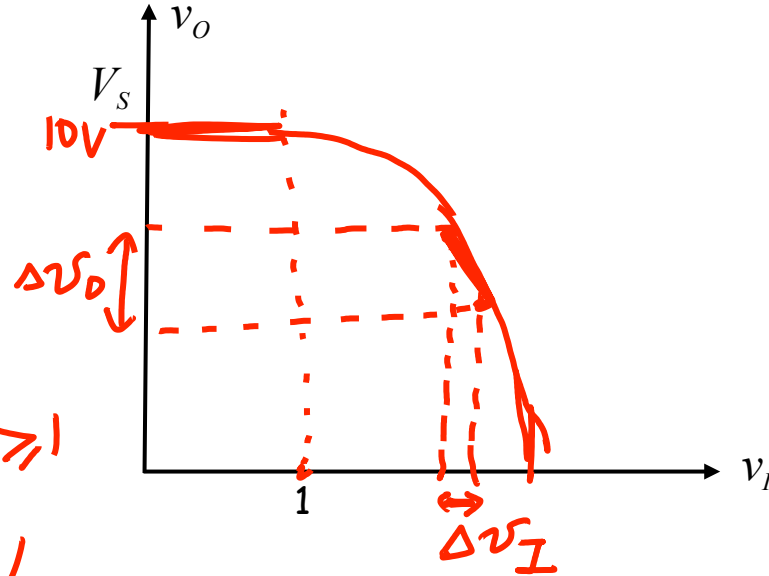
$v_O$  versus  $v_I$  curve

e.g.  $V_S = 10V$ ,  $K = 2 \frac{mA}{V^2}$ ,  $R_L = 5k\Omega$

$$v_O = V_S - \frac{K}{2} R_L (v_I - 1)^2$$

$$v_O = 10 - 5 (v_I - 1)^2 \quad \leftarrow \text{for } v_I \geq 1$$

$$= V_S \quad \text{for } v_I < 1V$$



$$\frac{\Delta v_O}{\Delta v_I} > 1 \Rightarrow \text{amplification}$$



Plot  $v_O$  versus  $v_I$

$$v_O = 10 - 5(v_I - 1)^2 \rightarrow v_I \geq 1V$$

$v_I$	$v_O$
0	10V
1	10V
1.5	8.75V
2V	5V
2.1	4V
2.2	2.8V
2.3	1.5V
2.4	$\sim 0V$

0.1V change  
in  $v_I$

1V change  
in  $v_O$

Gain!

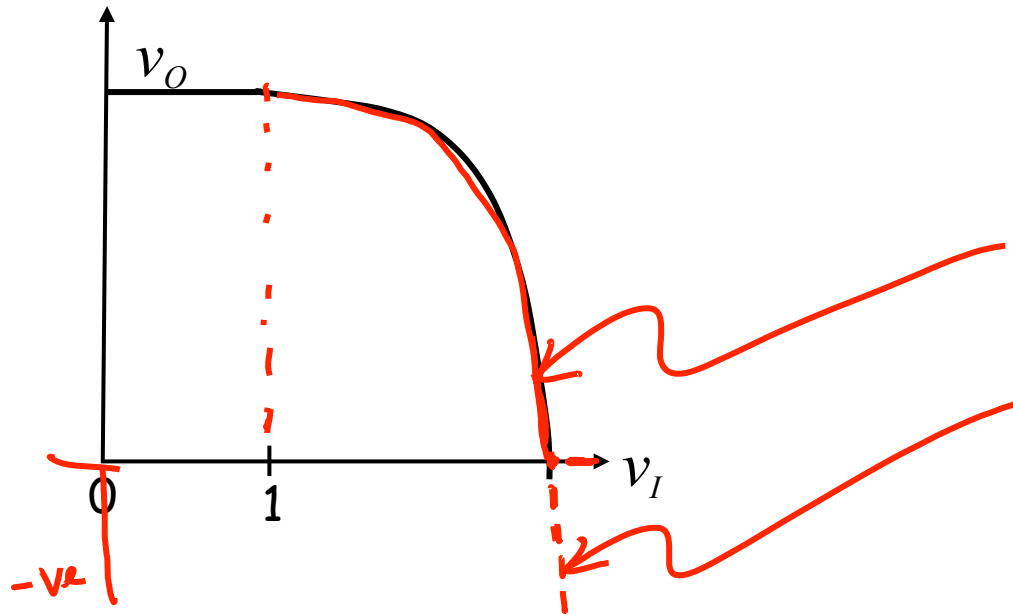
$v_O = V_S$   $v_I < 1V$   
**Demo**

Measure  $v_O$ .

# One nit ...

Mathematically,

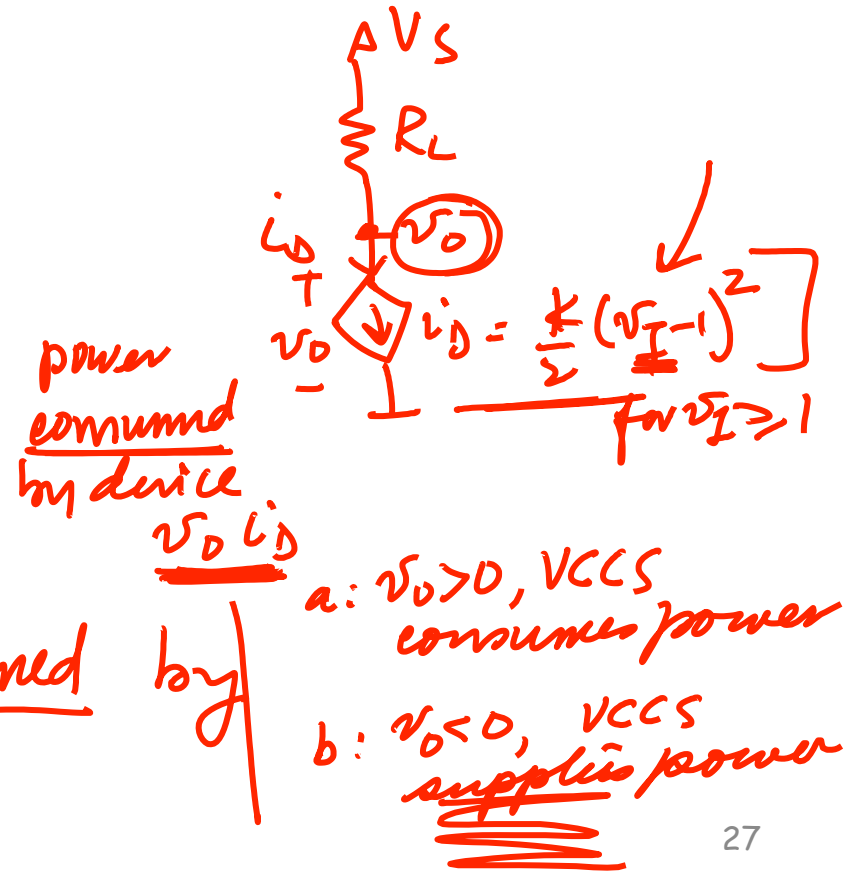
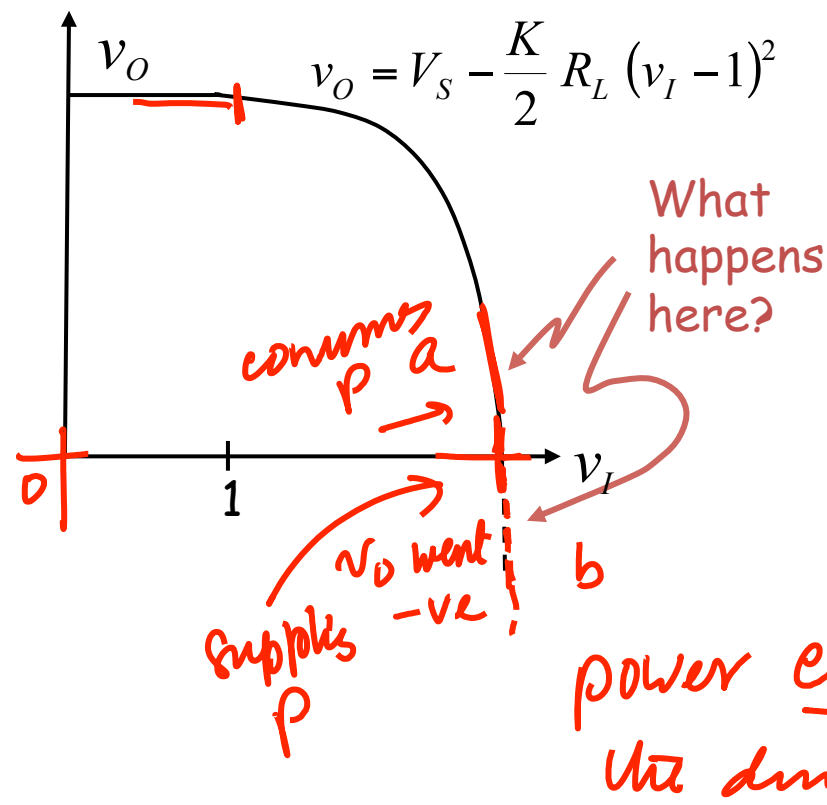
$$v_O = V_S - \frac{K}{2} R_L (v_I - 1)^2$$



So this is mathematically predicted behavior in our amplifier built with an abstract dependent source

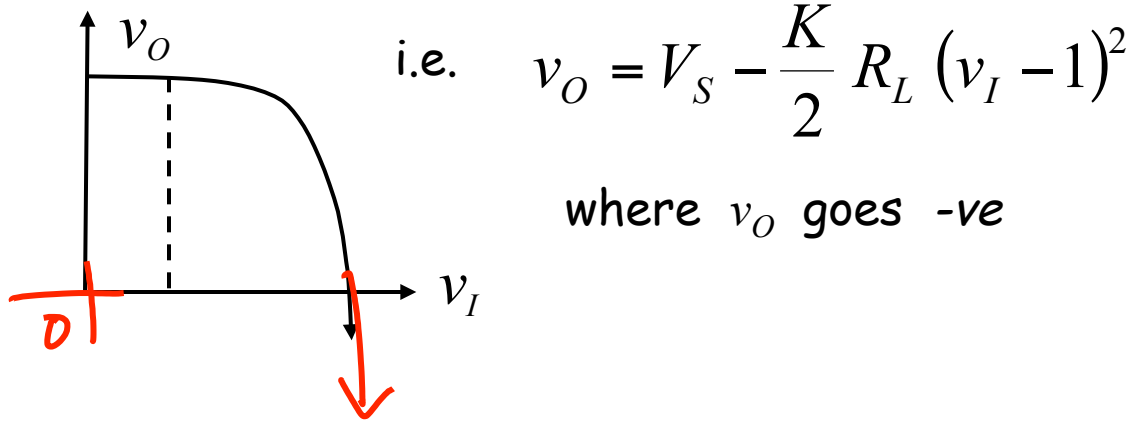
# One nit ...

However, looking at the circuit



If our VCCS is a device that can source (or supply) power

then the mathematically predicted behavior will be observed



However, if our VCCS is a passive device

i.e., it does not provide power gain

Then it cannot source power,  
so  $v_O$  cannot go -ve.

e.g.  $i_D$  saturates (stops increasing)  
and we observe.

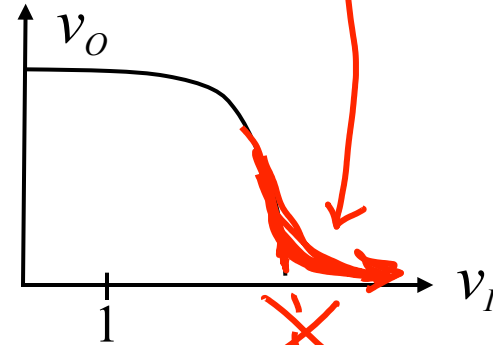
So, something must give!

Turns out, our model breaks down.

Commonly

$$i_D = \frac{K}{2} (v_I - 1)^2$$

will no longer be valid when  $v_O \leq 0$ .



**Demo**

We will look at a practical device shortly...