Foundation stage 1:

1. Use the midpoint formula: $x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$ to find the midpoint of each interval AB:

   (a) A(3, 7) and B(7, 12)

   (b) A(-4, 3) and (5, -8)

   (c) A(-3, -1) and B(-5, -6)

   (d) A(2, -13) and (-11, -4)

2. Use the distance formula: $AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ to find the distance of each interval AB:

   (a) A(3, 7) and B(7, 12)

   (b) A(-4, 3) and (5, -8)

   (c) A(-3, -1) and B(-5, -6)

   (d) A(2, -13) and (-11, -4)

3. Given that the midpoint of AB is C, where A(-2, 7) and B(-5, -3), show that AC = BC.

4. Given that A(-2, -1), B(-1, 3) and C(3, 4), show that Δ ABC is an isosceles triangle.

AB = $\sqrt{(1^2+4^2)}$ = $\sqrt{17}$

BC = $\sqrt{(4^2+1^2)}$ = $\sqrt{17}$

AC = $\sqrt{(5^2+5^2)}$ = $5\sqrt{2}$

AB = BC ≠ AC

ans: Δ ABC is an isosceles triangle

5. Given that A(-5, -6), B(4, 6) and C(20, -6).

(a) Show that Δ ABC is a right-angled triangle by using the distance formula only.

AB = $\sqrt{(9^2+12^2)}$ = $\sqrt{225}$ = 15

BC = $\sqrt{(16^2+12^2)}$ = $\sqrt{400}$ = 20

AC = $\sqrt{(25^2+0^2)}$ = $\sqrt{625}$ = 25

$AB^2+BC^2 = 15^2+20^2 = 625 = 25^2 = AC^2$

ans: Δ ABC is a right-angled triangle

(b) Hence, state which angle is 90°.

AC is the hypotenuse

ans: ∠B is 90°

(c) Given that the midpoing of AC is D, find the coordinate of D.

D(x, y), where x = (-5+20)/2 = 7.5, y = -6

D(7.5, -6)

(d) Hence, show that AD = CD = BD.

AD = $\sqrt{[(-5-7.5)^2+0^2]}$ = 12.5

CD = $\sqrt{[(20-7.5)^2+0^2]}$ = 12.5

BD = $\sqrt{[(4-7.5)^2+(6--6)^2]}$ = $\sqrt{(625/4)}$ = 12.5

ans: AD = CD = BD

Foundation stage 2:

1. Use the gradient formula: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to find the gradient of each interval AB:

(a) A(3, 7) and B(7, 12)

(b) A(-4, 3) and (5, -8)

(c) A(-3, -1) and B(-5, -6)

(d) A(2, -13) and (-11, -4)

(e) A(-5, -6), B(20, -6)

(f) A(-5, 20), B(-5, -6)

2. Find the gradient of a line parallel to the given line with the gradient:

(a) 3

(b) $\dfrac{3}{4}$

(c) $-\dfrac{5}{3}$

(d) $-2\dfrac{1}{3}$

3. Find the gradient of a line perpendicular to the given line with the gradient:

(a) 3

(b) $\dfrac{3}{4}$

(c) $-\dfrac{5}{3}$

(d) $-2\dfrac{1}{3}$

4. Show that A(-1, -5), B(1, -4), C(11, -6) and D(9, -7) form a parallelogram by finding the gradient of AB, BC, CD and DA.

5. Given that A(-5, -6), B(4, 6) and C(20, -6). Show that Δ ABC is a right-angled triangle by using the gradient formula only.

6. Givne that A(2, 2), B(-2, 1), C(-1, -3) and D(3, -2).

(a) Show that ABCD is a rectangle by finding the gradient of AB, BC, CD and DA.

(b) Hence, show that ABCD is also a square by finding the distance of AB, BC, CD and DA.

7. Use the formula $m = \tan\alpha$ to find the gradient of a line, to two decimal places if necessary, given that its angle of inclination is:

(a) 25°

(b) 72.5°

(c) 135°

(d) 102°53'

8. Use the formula $m = \tan\alpha$ to find the angle of linclination of a line, to the nearest degreee if necessary, given that its gradient is:

(a) 1

(b) $\sqrt{3}$

(c) $-\frac{\sqrt{3}}{3}$

(d) -3

9. Find the angle of inclination, to the nearest degree, of the following interval AB:

(a) A(3, 7) and B(7, 12)

(b) A(-4, 3) and (5, -8)

(c) A(-3, -1) and B(-5, -6)

(d) A(2, -13) and (-11, -4)

Foundation stage 3:

1. Find the x-intercept and y-intercept of the following lines:

(a) $y = 3x - 4$

(b) $2x - 5y + 10 = 0$

(c) $3x + 4y = 12$

(d) $-3 = 5y + 2x$

2. Use the gradient intercept form $y = mx + c$ to find the equation of the line with:

(a) gradient 5 and y-intercept (0, 3).

(b) gradient -2/5 and cuts y-axis at -7.

3. (a) Find the gradient of the following lines by first changing into the gradient intercept form:

(i) $y = 3x - 4$

(ii) $2x - 5y + 10 = 0$

(iii) $3x + 4y = 12$

(iv) $-3 = 5y + 2x$

(b) Hence, find the angle of inclination, to the nearest degree, of the following lines by using the formula $m = \tan\alpha$:

(i) $y = 3x - 4$

(ii) $2x - 5y + 10 = 0$

(iii) $3x + 4y = 12$

(iv) $-3 = 5y + 2x$

4. Use the gradient-point form $y-y_1=m(x-x_1)$, to find the equation of a line given gradient and one point: (rearrange into the general form: $ax+by+c=0$)

(a) gradient 3 and passes through (2, 5).

(b) gradient $-\frac{2}{5}$ and passes through (-3, 2).

5. Find the equation of a line, in the gradient-point form,

(a) parallel to $y=3x-2$ and though (2, 3).

(b) parallel to $y=-2x+5$ and though (-2, 3).

(c) perpendicular to $y=\frac{3}{2}x-5$ and though (2, -3).

(d) perpendicular to $y=-\frac{3}{2}x+5$ and though (-2, -3).

6. Find the equation of a line given two points: (rearrange into the general form)

(a) (3, 5) and (5, 9)

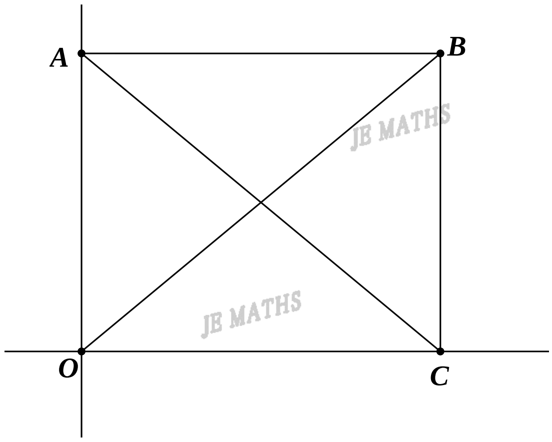
(b) (-4, 4) and (5, -8)

7. Given that a lines l passes two points A(-2, 7) and B(-8, 3), find the equation of a line that is perpendicular to l and passes C(-3, 3).

Foundation stage 4:

1. Given that O(0, 0), A(0, a), B(b, a) and C(b, 0) form a rectangle.

(a) Find OA, AB, BC and CO in terms of letters.



$OA = \sqrt{(0+a^2)} = a$

$AB = \sqrt{(b^2+0)} = b$

$BC = \sqrt{(0+a^2)} = a$

$CO = \sqrt{(b^2+0)} = b$

(b) Find the length of OB and AC in terms of letters.

$OB = \sqrt{(b^2+a^2)}$

$AC = \sqrt{(b^2+a^2)}$

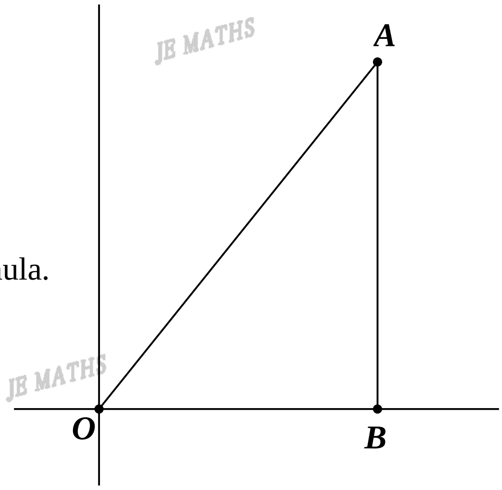
(c) Hence, show that the diagonals of a rectangle are equal.

Since $OB = AC$

so the diagonals of a rectangle are equal

2. Given that O(0, 0), A(a, b) and B(a, 0) form a right-angled triangle.

(a) Find C as the midpoint of AO.



$C(x, y)$, where $x = (a+0)/2 = a/2$, $y = (b+0)/2 = b/2$

$C(a/2, b/2)$

(b) Show that CO = CA = CB by using the distance formula.

(c) Hence, show that the midpoint of the hypotenuse of a right-angled triangle is the centre of a circle through all three vertice.

Foundation stage 1:

1. (a)
$x = (3+7)/2 = 5$
$y = (7+12)/2 = 9.5$
$(5, 9.5)$

(b)
$x = (-4+5)/2 = 0.5$
$y = (3-8)/2 = -2.5$
$(0.5, -2.5)$

(c)
$x = (-3-5)/2 = -4$
$y = (-1-6)/2 = -3.5$
$(-4, -3.5)$

(d)
$x = (2-11)/2 = -4.5$
$y = (-13-4)/2 = -8.5$
$(-4.5, -8.5)$

2. (a)
$AB = \sqrt{[(3-7)^2+(7-12)^2]}$
$= \sqrt{(4^2+5^2)}$
$= \sqrt{41}$

(b)
$AB = \sqrt{[(-4-5)^2+(3--8)^2]}$
$= \sqrt{(9^2+11^2)}$
$= \sqrt{202}$

(c)
$AB = \sqrt{[(-3--5)^2+(7-12)^2]}$
$= \sqrt{(2^2+5^2)}$
$= \sqrt{29}$

(d)
$AB = \sqrt{[(2--11)^2+(-13--4)^2]}$
$= \sqrt{(13^2+9^2)}$
$= \sqrt{250}$
$= 5\sqrt{10}$

3. $C(x, y)$, where $x = (-2-5)/2 = -3.5$, $y = (7-3)/2 = 2$
$C(-3.5, 2)$
$AC = \sqrt{[(-2--3.5)^2+(7-2)^2]} = \sqrt{(9/4+25)} = \sqrt{109/4}$
$BC = \sqrt{[(-5--3.5)^2+(-3-2)^2]} = \sqrt{(9/4+25)} = \sqrt{109/4}$
ans: $AC = BC$.

4. $AB = \sqrt{(1^2+4^2)} = \sqrt{17}$
$BC = \sqrt{(4^2+1^2)} = \sqrt{17}$
$AC = \sqrt{(5^2+5^2)} = 5\sqrt{2}$
$AB = BC \neq AC$
ans: ΔABC is an isosceles triangle

5. (a)

$AB = \sqrt{(9^2+12^2)} = \sqrt{225} = 15$

$BC = \sqrt{(16^2+12^2)} = \sqrt{400} = 20$

$AC = \sqrt{(25^2+0^2)} = \sqrt{625} = 25$

$AB^2+BC^2= 15^2+20^2= 625 = 25^2= AC^2$

ans: Δ ABC is a right-angled triangle

(b)

AC is the hypotenuse

ans: $\angle B$ is 90 °

(c)

D(x, y), where x = (-5+20)/2 = 7.5, y = -6

D(7.5, -6)

(d)

$AD = \sqrt{[(-5-7.5)^2+0^2]} = 12.5$

$CD = \sqrt{[(20-7.5)^2+0^2]} = 12.5$

$BD = \sqrt{[(4-7.5)^2+(6--6)^2]} = \sqrt{(625/4)} = 12.5$

ans: AD = CD = BD

Foundation stage 2:

1. (a)
m = (12-7)/(7-3)
= 5/4

(b)
m = (-8-3)/(5--4)
= -11/9
= -11/9

(c)
m = (-6--1)/(-5--3)
= -5/-2
= 5/2

(d)
m = (-4--13)/(-11-2)
= -9/13

(e)
m = 0/25
= 0

(f)
m = (-20-6)/0
= undefined

2. (a)
3

(b)
3/4

(c)
-5/3

(d)
-2 1/3

3. (a)
-1/3

(b)
-4/3

(c)
3/5

(d)
3/7

4. m_{AB} = (-4--5)/(1--1) = 1/2
m_{BC} = (-6--4)/(11-1) = -2/10 = -1/5
m_{CD} = (-7--6)/(9-11) = -1/-2 = 1/2
m_{DA} = (-7--5)/(9--1) = -2/10 = -1/5
$m_{AB} = m_{CD}$ and $m_{BC} = m_{DA}$
AB||CD and BC||DA
ans: ABCD is a parallelogram

5. m_{AB} = (6--6)/(4--5) = 12/9 = 4/3
m_{BC} = (-6-6)/(20-4) = -12/16 = -3/4
m_{CA} = (-6--6)/(20--5) = 0
$m_{AB} \times m_{BC}$ = 4/3×-3/4 = -1
AB⊥BC
∠B is a right angle
ans: Δ ABC is a right-angled triangle

6. (a)

m_{AB} = (1-2)/(-2-2) = -1/-4 = 1/4

m_{BC} = (-3-1)/(-1--2) = -4/1 = -4

m_{CD} = (-2--3)/(3--1) = -1/4

m_{DA} = (-2-2)/(3-2) = -4/1 = -4

$m_{AB} = m_{CD}$ and $m_{BC} = m_{DA}$

AB||CD and BC||DA

ABCD is a parallelogram

$m_{AB} \times m_{BC}$ = 1/4×-4 = -1

AB⊥BC

∠B is a right angle

ans: ABCD is a rectangle

(b)

AB = √(4²+1²) = √17

BC = √(1²+4²) = √17

CD = √(4²+1²) = √17

DA = √(1²+4²) = √17

AB = BC = CD = DA

ans: ABCD is a square

7. (a)

m = tan 25 °= 0.47

(b)

m = tan 72.5 °= 3.17

(c)

m = tan 135 °= -1

(d)

m = tan 102°53'= -4.37

8. (a)

tanα = 1

α = 45 °

(b)

tanα =√3

α = 60 °

(c)

tanα = -√3/3

α = -30 °

= -30 °+ 180 °= 150 °

(d)

tanα = -3

α = -72 °

= -72 °+ 180 °= 108 °

9. (a)

$m = (12-7)/(7-3)$

$= 5/4$

$5/4 = \tan\alpha$

$\alpha = 51°$

(b)

$m = (-8-3)/(5--4)$

$= -11/9$

$= -11/9$

$-11/9 = \tan\alpha$

$\alpha = -51°$

$= -51° + 180° = 129°$

(c)

$m = (-6--1)/(-5--3)$

$= -5/-2$

$= 5/2$

$5/2 = \tan\alpha$

$\alpha = 68°$

(d)

$m = (-4--13)/(-11-2)$

$= -9/13$

$-9/13 = \tan\alpha$

$\alpha = -35°$

$= -35° + 180° = 145°$

Foundation stage 3:

1. (a)
x-int: (4/3, 0)
y-int: (0, -4)

(b)
x-int: (-5, 0)
y-int: (0, 2)

(c)
x-int: (4, 0)
y-int: (0, 3)

(d)
x-int: (-3/2, 0)
y-int: (0, -3/5)

2. (a)
y = 5x + 3

(b)
y = -2x/5 – 7

3. (a)
(i)
m = 3

(ii)
5y = 2x + 10
y = 2x/5 + 2
m = 2/5

(iii)
4y = -3x + 12
y = -3x/4 + 3
m = -3/4

(iv)
5y = -2x - 3
y = -2x/5 - 3/5
m = -2/5

(b)
(i)
$\tan\alpha = 3$
$\alpha = 72°$

(ii)
$\tan\alpha = 2/5$
$\alpha = 22°$

(iii)
$\tan\alpha = -3/4$
$\alpha = -37°$
$= -37° + 180° = 143°$

(iv)
$\tan\alpha = -2/5$
$\alpha = -22°$
$= -22° + 180° = 158°$

4. (a)
y-5=3(x-2)
y-5=3x-6
3x-y-1=0

(b)
y-2 = -2/5×(x+3)
5y-10 = -2x – 6
2x+5y = 4
2x+5y-4=0

5. (a)
m = 3, (2, 3)
y-3 = 3(x-2)

(b)
m = -2, (-2, 3)
y-3 = -2(x+2)

(c)
m = -2/3, (2, -3)
y+3 = -2/3×(x-2)

(d)
m = 2/3, (-2, -3)
y+3 = 2/3×(x+2)

6. (a)
m = 4/2 = 2
y-5 = 2(x-3)
y-5 = 2x-6
2x-y-1 = 0

(b)
m = -12/9 = -4/3
y-4 = -4/3×(x+4)
3y-12 = -4x-16
4x+3y+4 = 0

7. m_{AB} = (3-7)/(-8--2) = -4/-6 = 2/3
m = -3/2, (-3, 3)
y-3 = -3/2×(x+3)
2y-6 = -3x-9
3x+2y+3 = 0

Foundation stage 4:

1. (a)

$OA = \sqrt{(0+a^2)} = a$
$AB = \sqrt{(b^2+0)} = b$
$BC = \sqrt{(0+a^2)} = a$
$CO = \sqrt{(b^2+0)} = b$

(b)

$OB = \sqrt{(b^2+a^2)}$
$AC = \sqrt{(b^2+a^2)}$

(c)

Since OB = AC
ans: the diagonals of a rectangle are equal

2. (a)

$C(x, y)$, where $x = (a+0)/2 = a/2$, $y = (b+0)/2 = b/2$
$C(a/2, b/2)$

(b)

$CO = \sqrt{[(a/2-0)^2+(b/2-0)^2]} = \sqrt{(a^2/4+b^2/4)} = \sqrt{[(a^2+b^2)/4]} = \frac{1}{2}\times\sqrt{(a^2+b^2)}$
$CA = \sqrt{[(a-a/2)^2+(b-b/2)^2]} = \sqrt{(a^2/4+b^2/4)} = \sqrt{[(a^2+b^2)/4]} = \frac{1}{2}\times\sqrt{(a^2+b^2)}$
$CB = \sqrt{[(a-a/2)^2+(0-b/2)^2]} = \sqrt{(a^2/4+b^2/4)} = \sqrt{[(a^2+b^2)/4]} = \frac{1}{2}\times\sqrt{(a^2+b^2)}$
$CO = CA = CB$

(c)

since $CO = CA = CB$
then C is the center of a circle which go through A, B and C, as three vertices of this RtΔ