Stage 1

1. Use Parametric Differentiation to find $\frac{dy}{dx}$ in terms of the parameter t, then evaluate $\frac{dy}{dx}$ when $t=-2$.

(a) $x=3t,\quad y=8t^2$

(b) $x=5t-4,\quad y=7-2t$

(c) $x=10t^2,\quad y=5t^2-2t$

(d) $x=t^3-t,\quad y=4-t^2$

(e) $x = t^3, \quad y = t^2 - 1$

(f) $x = 3t^2 + 1, \quad y = t^3 - 2t^2$

(g) $x = 2\sqrt{t}, \quad y = \dfrac{1}{t\sqrt{t}}$

2. Given the parametric equation of the curve

$$x = t^3 - 8t, \quad y = t^2$$

where t is a parameter. Given that the point A has parameter $t = -1$,
(a) find the coordinate of A

The line l is a tangent to the curve at point A.
(b) Find the equation of line l.

The line l also intersects the curve at point B.
(c) Find the coordinate of point B.

3. Use parametric differentiation to find the equation of tangent to the curve at the given the point.

(a) $x = 3t^2$, $y = t - 6t^2$ at the point where $t = 1$

(b) $x = (t+2)^2$, $y = (t-1)^3$ at the point where $t = -1$

(c) $x=\dfrac{1}{1+t}$, $y=\dfrac{1}{1-t}$ at the point where $t=\dfrac{1}{2}$

4. Use parametric differentiation to find the equation of the normal to the curve at the given point.

(a) $x=t^2+1$, $y=3(t+1)$ at the point where $t=2$.

(b) $x = t^2$ and $y = t^2 + 6t - 7$ at $t = 12$

Stage 2

1. Use $\frac{dy}{dx} = \frac{1}{dx/dy}$ to differentiate:

(a) $y = \sqrt{x+2}$

(b) $y = \sqrt[3]{x}$

(c) $y = \sqrt[4]{2x}$

(d) $y = \frac{1}{\sqrt{6 - x}}$

(e) $y = \frac{1}{\sqrt[3]{x}}$

(f) $y = \frac{6}{\sqrt[4]{x}}$

Stage 1

1. (a) $x = 3t \Rightarrow \frac{dx}{dt} = 3$

$y = 8t^2 \Rightarrow \frac{dy}{dt} = 16t$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{16t}{3}$$

(b) $x = 5t - 4 \Rightarrow \frac{dx}{dt} = 5$

$y = 7 - 2t \Rightarrow \frac{dy}{dt} = -2$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2}{5}$$

(c) $x = 10t^2 \Rightarrow \frac{dx}{dt} = 20t$

$y = 5t^2 - 2t \Rightarrow \frac{dy}{dt} = 10t - 2$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{10t-2}{20t} = \frac{5t-1}{10t}$$

(d) $x = t^3 - t \Rightarrow \frac{dx}{dt} = 3t^2 - 1$

$y = 4 - t^2 \Rightarrow \frac{dy}{dt} = 0 - 2t = -2t$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2t}{3t^2-1}$$

(e) $x = t^3 \Rightarrow \frac{dx}{dt} = 3t^2$

$y = t^2 - 1 \Rightarrow \frac{dy}{dt} = 2t$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t}{3t^2} = \frac{2}{3t}$$

(f) $x = 3t^2 + 1 \;\Rightarrow\; \dfrac{dx}{dt} = 6t$

$y = t^3 - 2t^2 \;\Rightarrow\; \dfrac{dy}{dt} = 3t^2 - 4t$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2 - 4t}{6t} = \frac{3t-4}{6}$$

(g) $x = 2\sqrt{t} = 2t^{\frac{1}{2}} \;\Rightarrow\; \dfrac{dx}{dt} = t^{-\frac{1}{2}}$

$y = \dfrac{1}{t\sqrt{t}} = \dfrac{1}{t^{\frac{3}{2}}} = t^{-\frac{3}{2}} \;\Rightarrow\; \dfrac{dy}{dt} = -\dfrac{3}{2}t^{-\frac{5}{2}}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-\frac{3}{2}t^{-\frac{5}{2}}}{t^{-\frac{1}{2}}} = -\frac{3}{2}t^{-2} = -\frac{3}{2t^2}$$

2. (a) When $t = -1$, $x = -1 - (-8) = 7$, $y = 1 \;\Rightarrow\; A(7,1)$

(b) $x = t^3 - 8t \;\Rightarrow\; \dfrac{dx}{dt} = 3t^2 - 8$

$y = t^2 \;\Rightarrow\; \dfrac{dy}{dt} = 2t$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t}{3t^2 - 8} \quad\Rightarrow\quad \left.\frac{dy}{dx}\right|_{t=-1} = \frac{2}{5}$$

$l_T:\; y - 1 = \dfrac{2}{5}(x - 7) \;\Rightarrow\; 2x - 5y - 9 = 0$

3. (a) When $t = 1$, $x = 3$, $y = 1 - 6 = -5 \;\Rightarrow$ at point $(3, -5)$

$x = 3t^2 \;\Rightarrow\; \dfrac{dx}{dt} = 6t$

$y = t - 6t^2 \;\Rightarrow\; \dfrac{dy}{dt} = 1 - 12t$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1-12t}{6t} \quad\Rightarrow\quad \left.\frac{dy}{dx}\right|_{t=1} = -\frac{11}{6}$$

$l_T:\quad y+5=-\frac{11}{6}(x-3) \;\Rightarrow\; 11x+6y-3=0$

(b) When $t=-1,\ x=1,\ y=(-2)^3=-8 \;\Rightarrow$ at point $(1,-8)$

$x=(t+2)^2 \;\Rightarrow\; \frac{dx}{dt}=\frac{dx}{du}\times\frac{du}{dt}$ (by chain rule if let $u=t+2$ and $x=u^2$)

$=2(t+2)\times 1$

$=2(t+2)$

$y=(t-1)^3 \;\Rightarrow\; \frac{dy}{dt}=\frac{dy}{du}\times\frac{du}{dt}$ (by chain rule if let $u=t-1$ and $y=u^3$)

$=3(t-1)^2\times 1$

$=3(t-1)^2$

$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{3(t-1)^2}{2(t+2)} \;\Rightarrow\; \left.\frac{dy}{dx}\right|_{t=-1}=\frac{12}{2}=6$$

$l_T:\quad y+8=6(x-1) \;\Rightarrow\; 6x-y-14=0$

(c) When $t=\frac{1}{2},\ x=\frac{1}{1+\frac{1}{2}}=\frac{2}{3},\ y=\frac{1}{1-\frac{1}{2}}=2 \;\Rightarrow$ at point $\left(\frac{2}{3},2\right)$

$x=\frac{1}{1+t}=(1+t)^{-1} \;\Rightarrow\; \frac{dx}{dt}=\frac{dx}{du}\times\frac{du}{dt}$ (by chain rule if let $u=1+t$ and $x=u^{-1}$)

$=-(1+t)^{-2}\times 1$

$=-\frac{1}{(1+t)^2}$

$y=\frac{1}{1-t}=(1-t)^{-1} \;\Rightarrow\; \frac{dy}{dt}=\frac{dy}{du}\times\frac{du}{dt}$ (by chain rule if let $u=1-t$ and $y=u^{-1}$)

$=-(1-t)^{-2}\times(-1)$

$=\frac{1}{(1-t)^2}$

$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{\dfrac{1}{(1-t)^2}}{-\dfrac{1}{(1+t)^2}}=-\frac{(1+t)^2}{(1-t)^2} \quad\Rightarrow\quad \left.\frac{dy}{dx}\right|_{t=\frac{1}{2}}=-\frac{\dfrac{9}{4}}{\dfrac{1}{4}}=-9$$

$$l_T:\ y-2=9\left(x-\frac{2}{3}\right) \ \Rightarrow\ 9x-y-4=0$$

4. (a) When $t=2$, $x=5$, $y=9$ $\Rightarrow$ at point $(5,9)$

$$x=t^2+1 \ \Rightarrow\ \frac{dx}{dt}=2t$$

$$y=3(t+1) \ \Rightarrow\ \frac{dy}{dt}=3+0=3$$

$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{3}{2t} \Rightarrow \left.\frac{dy}{dx}\right|_{t=2}=\frac{3}{4}$$

$$\Rightarrow\ m_N=-\frac{4}{3} \text{ at point } (5,9) \qquad (\text{as } m_T\times m_N=-1)$$

$$l_N:\ y-9=-\frac{4}{3}(x-5) \ \Rightarrow\ 4x+3y-47=0$$

(b) When $t=12$, $x=144$, $y=144+72-7=209$ $\Rightarrow$ at point $(144,209)$

$$x=t^2 \ \Rightarrow\ \frac{dx}{dt}=2t$$

$$y=t^2+6t-7 \ \Rightarrow\ \frac{dy}{dt}=2t+6$$

$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{2t+6}{2t} \quad\Rightarrow\quad \left.\frac{dy}{dx}\right|_{t=12}=\frac{24+6}{24}=\frac{5}{4}$$

$$\Rightarrow\ m_N=-\frac{4}{5} \text{ at point } (144,209) \qquad (\text{as } m_T\times m_N=-1)$$

$$l_N:\ y-209=-\frac{4}{5}(x-144) \ \Rightarrow\ 4x+5y-1621=0$$

Stage 2

1. (a) $y=\sqrt{x+2} \Rightarrow y^2 = x+2$

$$\Rightarrow x = y^2 - 2$$

$$\Rightarrow \frac{dx}{dy} = 2y$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

$$= \frac{1}{2y}$$

$$= \frac{1}{2\sqrt{x+2}}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{x+2}}$$

(b) $y = \sqrt[3]{x} \Rightarrow x = y^3$

$$\Rightarrow \frac{dx}{dy} = 3y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

$$= \frac{1}{3y^2}$$

$$= \frac{1}{3\sqrt[3]{x^2}}$$

$$\therefore \frac{dy}{dx} = \frac{1}{3\sqrt[3]{x^2}}$$

(c) $y = \sqrt[4]{2x} \quad \Rightarrow \quad 2x = y^4$

$$\Rightarrow \quad x = \frac{y^4}{2}$$

$$\Rightarrow \quad \frac{dx}{dy} = 2y^3$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

$$= \frac{1}{2y^3}$$

$$= \frac{1}{2\sqrt[4]{(2x)^3}}$$

$$= \frac{1}{2\sqrt[4]{8x^3}}$$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{2\sqrt[4]{8x^3}}$$

(d) $y = \dfrac{1}{\sqrt{6-x}} \quad \Rightarrow \quad y^2 = \dfrac{1}{6-x}$

$$\Rightarrow \quad 6 - x = y^{-2}$$

$$\Rightarrow \quad x = 6 - y^{-2}$$

$$\Rightarrow \quad \frac{dx}{dy} = 0 - (-2)y^{-3}$$

$$= 2y^{-3}$$

$$= \frac{2}{y^3}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

$$= \frac{y^3}{2}$$

$$= \frac{1}{2\sqrt{(6-x)^3}}$$

$$\therefore \ \frac{dy}{dx} = \frac{1}{2\sqrt{(6-x)^3}}$$

(c) $y = \frac{1}{\sqrt[3]{x}} \ \Rightarrow \ y^3 = \frac{1}{x}$

$$\Rightarrow \ x = y^{-3}$$

$$\Rightarrow \frac{dx}{dy} = -3y^{-4}$$

$$= -\frac{3}{y^4}$$

$$\Rightarrow \ \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

$$= -\frac{y^4}{3}$$

$$= -\frac{1}{3\sqrt[3]{x^4}}$$

$$\therefore \ \frac{dy}{dx} = -\frac{1}{3\sqrt[3]{x^4}}$$

(f) $y=\dfrac{6}{\sqrt[4]{x}} \;\Rightarrow\; y^4=\dfrac{1296}{x}$

$$\Rightarrow \; x=1296y^{-4}$$

$$\Rightarrow \; \frac{dx}{dy}=-5184y^{-5}$$

$$=-\frac{5184}{y^5}$$

$$\Rightarrow \; \frac{dy}{dx}=\frac{1}{\frac{dx}{dy}}$$

$$=-\frac{y^5}{5184}$$

$$=-\frac{1}{5184}\times\frac{6^5}{\sqrt[4]{x^5}}$$

$$=-\frac{3}{2x\sqrt[4]{x}}$$

$$\therefore \; \frac{dy}{dx}=-\frac{3}{2x\sqrt[4]{x}}$$