- Shift left/right: $y = f(x)$ to $y = f(x-h)$, $h > 0$

1. (a) $y = (x+1)^3$

   (b) y = 1/(x-3)

   (c) y = 2^(x+1)

   (d) (x-7) - y = 1
   x - y = 8

- Shift up/down: $y = f(x)$ to $y = f(x)+k$ or $(y-k) = f(x)$, $k > 0$

2. (a) $y = x^3 - 2$ or $(y+2)=x^3$

   (b) y = 1/x+4 or (y-4) = 1/x

   (c) y = 2^x-6 or (y+6) = 2^x

   (d) x + (y-8) = 1
   x + y = 9

3. (a) (x-h)

   (b) (x+h)

   (c) (y-k)

   (d) (y+k)

   (e) (y-k)=f(x-h)

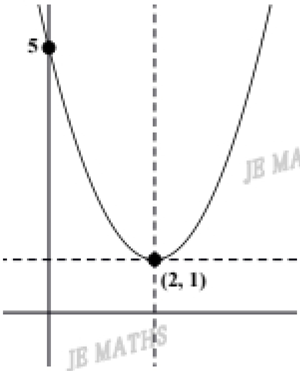
   (f) (y+k)=f(x+h)

- Mixed translation: $y = f(x) \xrightarrow[k\ up]{h\ right} (y-k) = f(x-h)$

4. (a) x→x-2, y→y-1

$y-1=(x-2)^2$

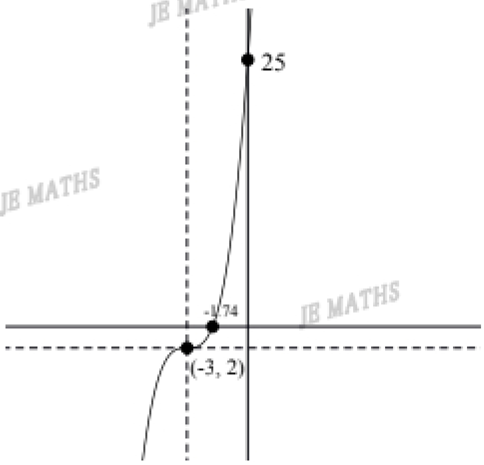
$y = (x-2)^2+1$



(b) x→x+3, y→y+2

$y+2=(x+3)^3$

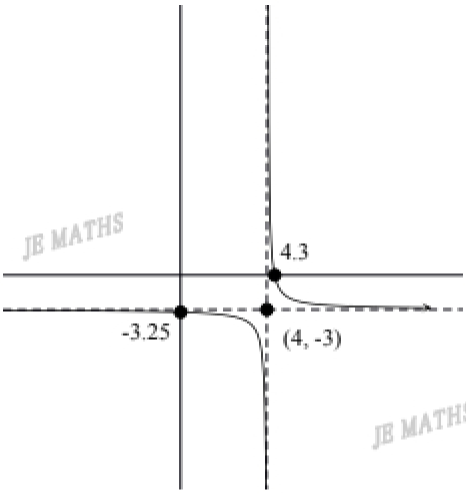
$y = (x+3)^3-2$



(c) x→x-4, y→y+3

$y+3=1/(x-4)$

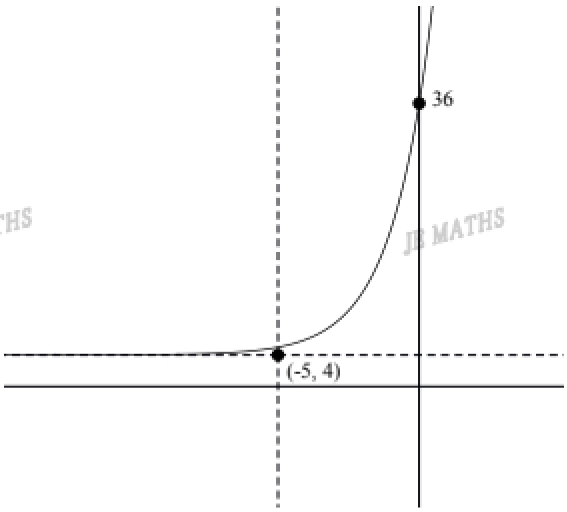
$y=1/(x-4)-3$



(d) x→x+5, y→y-4

$y-4=2^{(x+5)}$

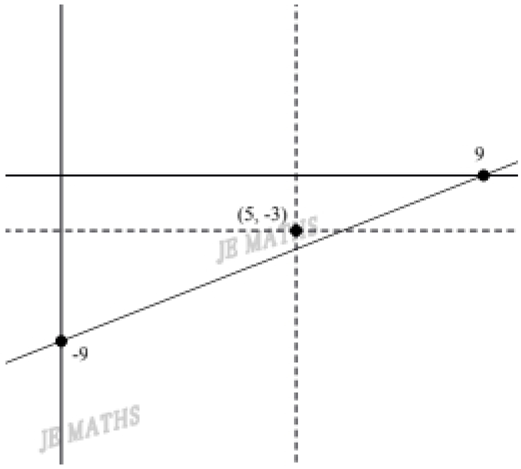
$y=2^{(x+5)}+4$



5. (a) $x \rightarrow x-5$, $y \rightarrow y+3$

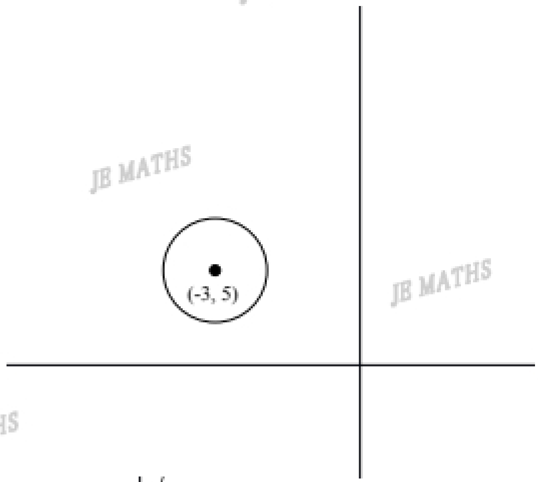
$(x-5)-(y+3)=1$

$x-y=9$



(b) $x \rightarrow x+3$, $y \rightarrow y-5$

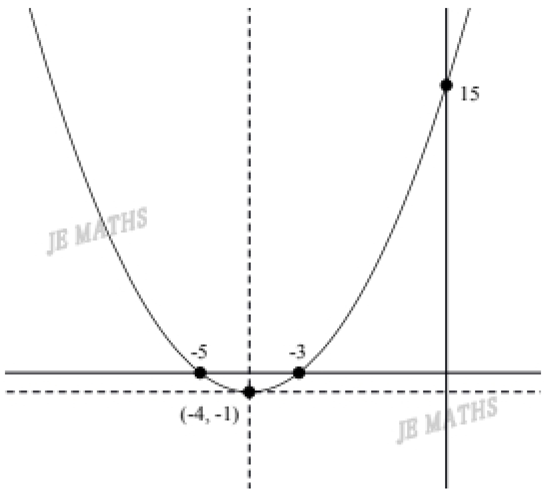
$(x+3)^2+(y-5)^2=1$



(c) $x \rightarrow x+4$, $y \rightarrow y+2$

$y+2=(x+4)^2+1$

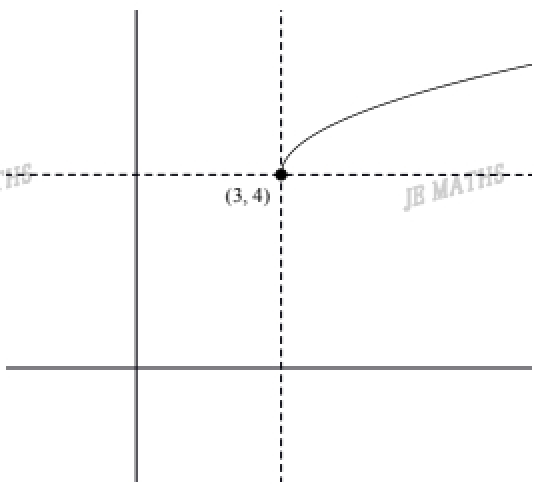
$y=(x+4)^2-1$



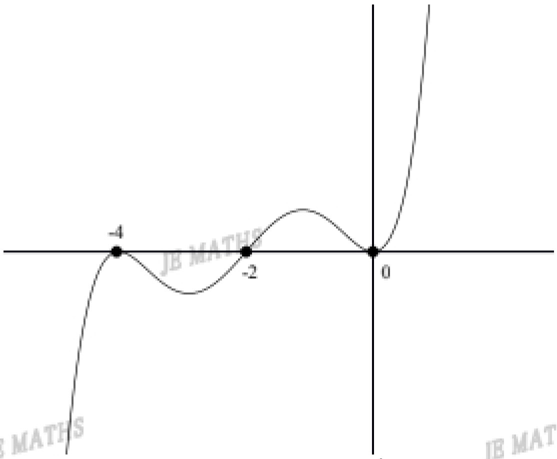
(c) $x \rightarrow x-2$, $y \rightarrow y-4$

$y-4=\sqrt{(x-2-1)}$

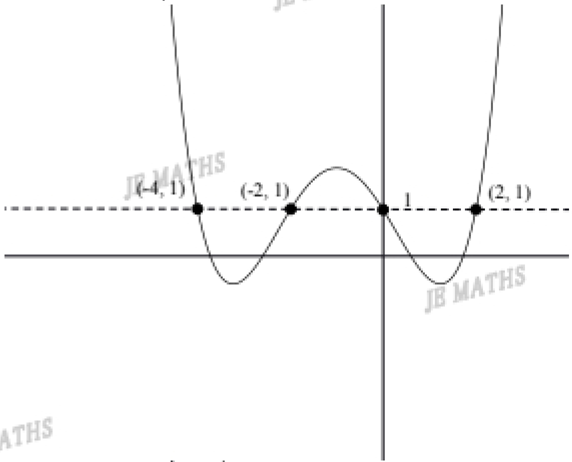
$y=\sqrt{(x-3)}+4$



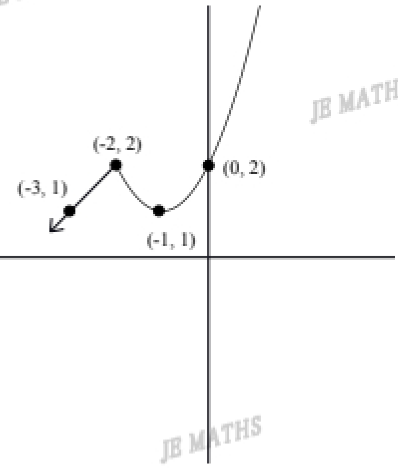
6. (a) translate by 2 units left
(-4, 0)
(-2, 0)
(0, 0)



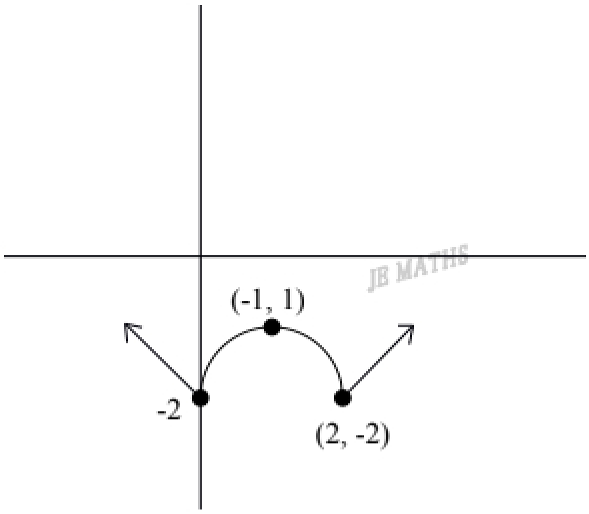
(b) translate by 1 unit up
(-4, 1)
(-2, 1)
(0, 1)
(0, 1)



(c) translate by 2 units left and 1 unit up
(-3, 1)
(-2, 2)
(-1, 1)
(0, 2)



(d) translate by 1 unit right and 2 units down
(0, -2)
(-1, 1)
(-2, -2)



1.

(a) S = {BB, BG, GB, GG}

(b) (i) A = {BB, BG, GB}

(ii) B = {BG, GB}

(iii) $\bar{A}$ = {GG}

(c) (i) |A| = 3

(ii) |B| = 2

2.

(a) S = {BBB, BBG, BGB, GBB, GGB,GBG, BGG, GGG}

(b) (i) A = {BBB, BBG, BGB, GBB, GGB,GBG, BGG}

(ii) B = {GGB, GBG, BGG}

(iii) $\bar{A}$ = {GGG}

(c) (i) |A| = 7

(ii) |B| = 3

(**Notice**: if the number of children is more than 3, let's say 4, 5, 6...,
a brand new listing method, which is called **permutation** will be introduced.
The topic of permutation will be covered later in the Advanced Maths course.)

3. LHS = $|A|/|S| + |\bar{A}|/|S|$
$= (|A|+|\bar{A}|)/|S|$
$= |S|/|S|$
= 1 = RHS

4. (a)
P(sum>8) = 10/36
= 5/18

(b)
P(sum = 8) = 5/36

(c)
P(sum>8 or sum = 8) = 5/18+5/36 - 0 (P(sum>8 and sum=8) = 0)
= 5/12

5.

(a)
P(multiple of 3) = 16/50
= 8/25

(b)
P(multiple of 5) = 10/50
= 1/5

(c)
P(multiple of 3 or 5) = 8/25+1/5-3/50
= 23/50

6.

(a) P(G) = 1-1/2-1/3
=1/6

(b) P(R or B) = 1/2+1/3
= 5/6

(c) P(R or G) = 1/2+1/6
= 2/3

7.

(a) P(sum=5 or sum>9) = 4/36+6/36
= 5/18

(b) P(sum=5 or 2 odd numbers) = 4/36+1/4
=13/36

(c) P(sum=9 or 2 odd numbers) = 6/36+1/4-1/36
= 7/18

8. (a) (i)
{2,4,6,8,10,12,14,16,18,20}
P(A)=1/2

(ii)
{16,17,18,19,20}
P(B)=1/4

(iii)
{3,6,9,12,15,18}
P(C)=3/10

(iv)
{1, 2, 3, 4, 5, 6, 7, 8, 9}
P(D)=9/20

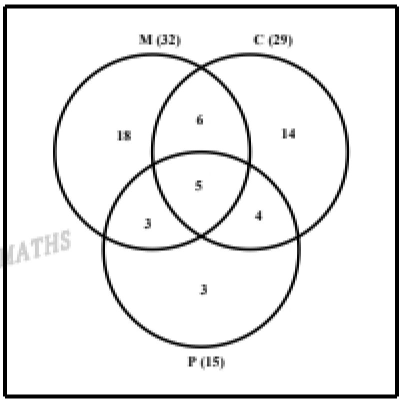
(b) (i) P(A or B) = 1/2+1/4-3/20
= 3/5

(ii) P(A or C) = 1/2+3/10-3/20
=13/20

(iii) P (B or D) = 1/4+9/20-0
= 7/10

9.

(a)

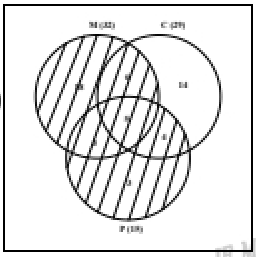


(b) (i) $P(M \text{ or } P) = P(M \cup P)$

$= (18+6+5+3+4+3)/60$

$= 39/60$

$= 13/20$

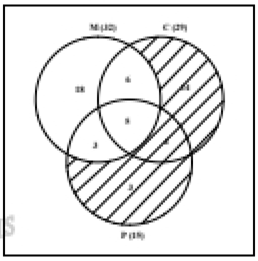


(ii) $P(C \text{ or } P \text{ or not } M) = P(C \cup P \cup \bar{M})$

$= (14+4+3)/60$

$= 21/60$

$= 7/20$

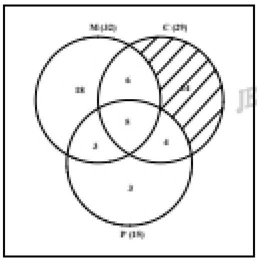


(iii) $P(C \text{ or not } P \text{ or not } M) = P(C \cup \bar{P} \cup \bar{M})$

$= 14/60$

$= 7/30$

Notice: it is the same as finding P(C only)



- Set theory (multiplication law): $P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B \mid A)$

10.

(a) (i) $P(RR) = P(R) \times P(R|R)$

$= 16/30 \times 16/30$

$= 64/225$

(ii) $P(RB) = P(R) \times P(B|R)$

$= 16/30 \times 14/30$

$= 56/225$

(b) (i) $P(RR) = P(R) \times P(R|R)$

$= 16/30 \times 15/29$

$= 8/29$

(ii) $P(RB) = P(R) \times P(B|R)$

$= 16/30 \times 14/29$

$= 112/435$

- Application of set theory (multiplication law) in probability:

11.

(a) P(all gold) $= 6/12 \times 6/12 \times 6/12$
$= 1/8$

(b) P(all the same) = P(all gold)+P(all silver)+P(all bronze)
$= (1/2)^3 + 3/12 \times 3/12 \times 3/12 + 3/12 \times 3/12 \times 3/12$
$= (1/2)^3 + (3/12)^2 + (3/12)^2$
$= 5/32$

(c) P(2 gold 1 silver) = P(GGS)+P(GSG)+P(SGG)
$= 3 \times$ P(GGS)
$= 3 \times (6/12 \times 6/12 \times 3/12)$
$= 3/16$

(d) P(none are gold) = P(no gold) $\times$ P(no gold) $\times$ P(no gold)
$= (1-6/12) \times (1-6/12) \times (1-6/12)$
$= 1/2 \times 1/2 \times 1/2$
$= 1/8$

12.

(a) P(both yellow) $= 8/50 \times 7/49$
$= 4/175$

(b) P(one is yellow) = P(yellow) $\times$ P(not yellow)+P(not yellow) $\times$ P(yellow)
$= 8/50 \times 42/49 + 42/50 \times 8/49$
$= 48/175$

(c) P(neither is yellow) = 1-P(both yellow)-P(one is yellow)
$= 1-4/175-48/175$
$= 123/175$

1. (a) $f(-x)=2(-x)^2=2x^2$

(b) $f(-x)=(-x)^4-(-x)=x^4+x$

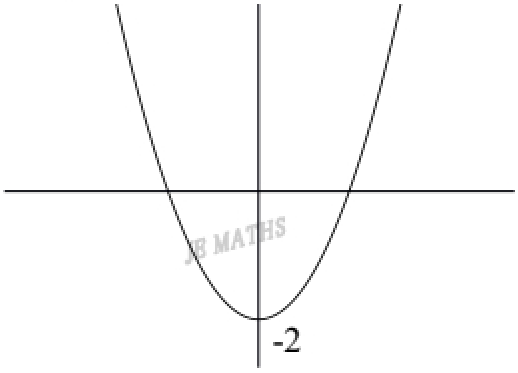
(c) $f(-x)=(-x)^3+3(-x)=-x^3-3x$

(d) $f(-x)=(-x)^5-(-x)^3+1=-x^5+x^3+1$

2. (a)

$f(-x)=(-x)^2-2=x^2-2=f(x)$

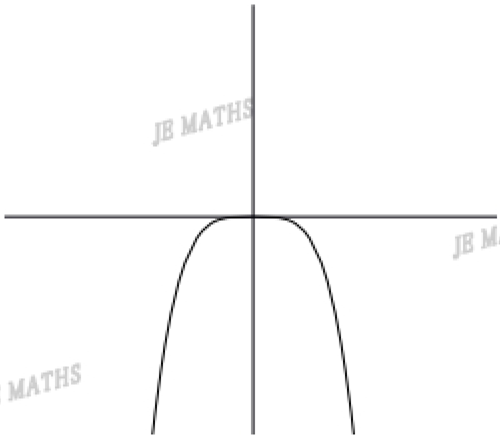
Sketch $f(x) = x^2 - 2$



(b)

$f(-x)=-(-x)^4= -x^4 = f(x)$

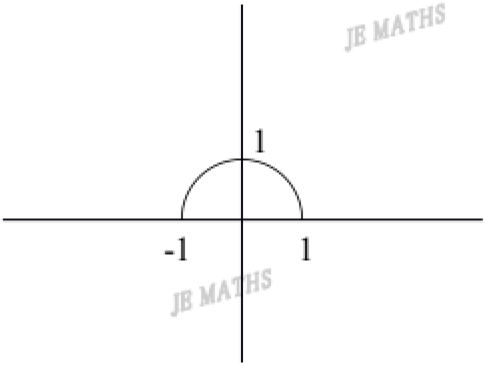
Sketch $f(x) = -x^4$



(c)

$f(-x)=\sqrt{1-(-x)^2}=\sqrt{1-x^2}=f(x)$

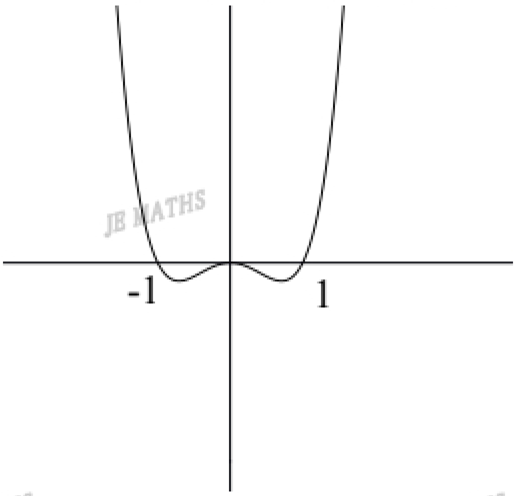
Sketch $\sqrt{1-x^2}$



(d)

$f(-x)=(-x)^4-(-x)^2= x^4-x^2= f(x)$

Sketch $f(x) = x^2(x^2-1)=x^2(x+1)(x-1)$



3. (a) $-f(x)=-2^x$

(b) $-f(x)=-1/(x+1)$

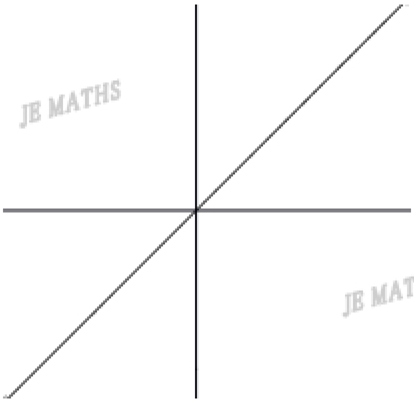
(c) $-f(x)=-(x^3-x+1)=-x^3+x-1$

(d) $-f(x)=-x/(x^2+1)$

4. (a)

$f(-x) = -x = -f(x)$

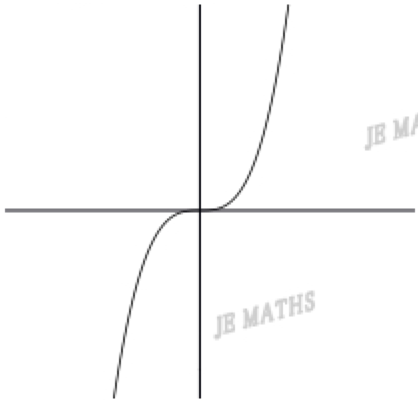
Sketch $f(x) = x$



(b)

$f(-x) = 3(-x)^3 = -3x^3 = -f(x)$

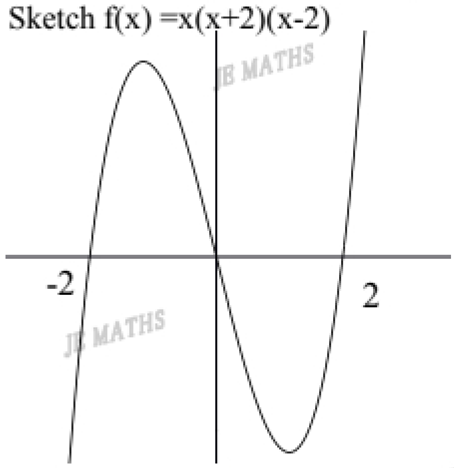
Sketch $f(x) = 2x^3$



(c) $f(x) = x^3 - 4x$

$f(-x)=(-x)^3-4(-x)=-(x^3-4x)=-f(x)$

Sketch $f(x) = x(x+2)(x-2)$

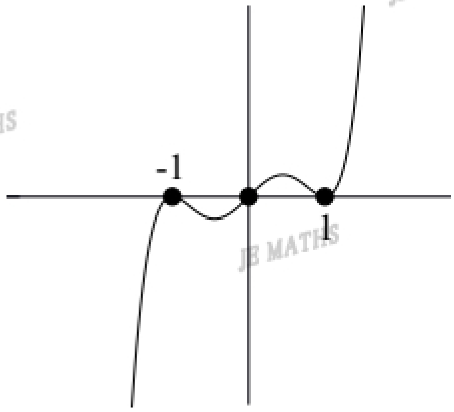


(d)

$f(-x)=(-x)^5-2(-x)^3+(-x)=-x^5+2x^3-x$

$= -(x^5-2x^3+x)=-f(x)$

Sketch $f(x) = x(x^2-1)^2=x(x-1)^2(x+1)^2$



5. (a)

$f(-x) = (-x)^2+2^{(-x)}$

$= x^2+2^{-x} \neq f(x)$

$\neq -f(x)$

Neither even nor odd.

(b)

$f(-x)=(-x)^4-2(-x)^2+1$

$=x^4-2x^2+1$

$=f(x)$

Even.

(c)

$f(-x) = 3^{(-x)}-3^x$

$= -[3^x-3^{(-x)}]$

$= -f(x)$

Odd.

(d)

sub $-x$ in, $(-x)^2y^2=x^2y^2=1$

meets → even

sub $-x$ and $-y$ in, $(-x)^2(-y)^2=x^2y^2=1$

meets → odd

Both even and odd.

6. (a) Let: f(x) is even, then f(-x)=f(x); g(x) is even, then g(-x)=g(x)
h(-x) =f(-x)+g(-x) (since f(-x)=f(x) and g(-x)=g(x))
= f(x)+g(x) = h(x)
h(x) is even.

(b) Let: f(x) is odd, then f(-x)=-f(x); g(x) is odd, then g(-x)=-g(x)
h(-x) =f(-x)+g(-x) (since f(-x)=f(x) and g(-x)=g(x))
= -f(x)-g(x) = -[f(x)+g(x)] = - h(x)
h(x) is odd.

(c) Let: f(x) is even, then f(-x)=f(x); g(x) is odd, then g(-x)=-g(x)
h(-x) =f(-x)+g(-x) (since f(-x)=f(x) and g(-x)=g(x))
= f(x)-g(x) $\neq$ h(x)
$\neq$ -h(x)
h(x) is neither even nor odd.

7. (a) 7

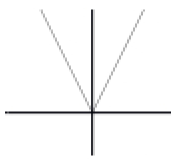
(b) 7

(c) |4|=4

(d) |-4|=4

8. (a) Graph $y = |2x|$ by using a table of values.

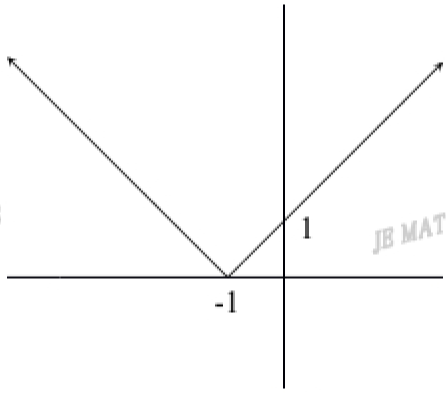
x	-2	-1	0	1	2
|2x|	4	2	0	2	4



(b) Write down the equations of the two branches.
y = 2x, 2x≥0, x≥0
y = -2x, 2x<0, x<0

9. (a) Sketch.

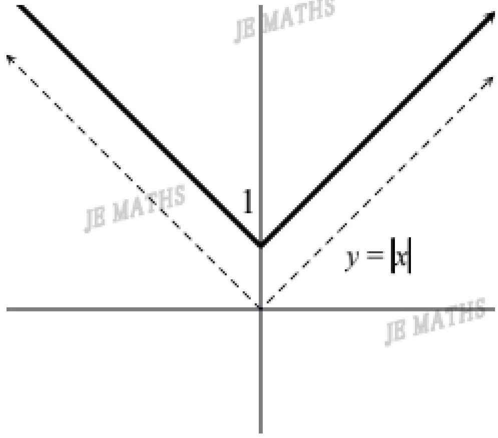
x	-2	-1	0	1	2
$\|x+1\|$	1	0	1	2	3



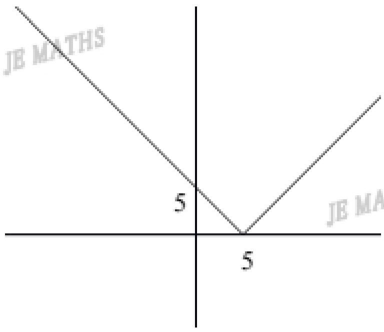
(b) $y = x+1,\ x+1 \geq 0,\ x \geq -1$
$y = -(x+1) = -x-1,\ x+1<0,\ x<-1$

(c) $x \rightarrow x+1$, translate 1 unit left.

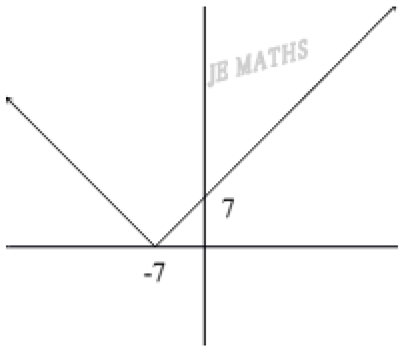
(d) Sketch.



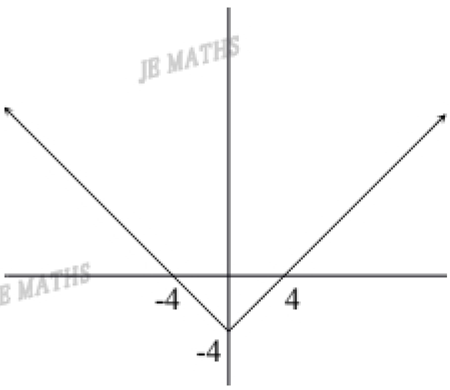
10. (a)



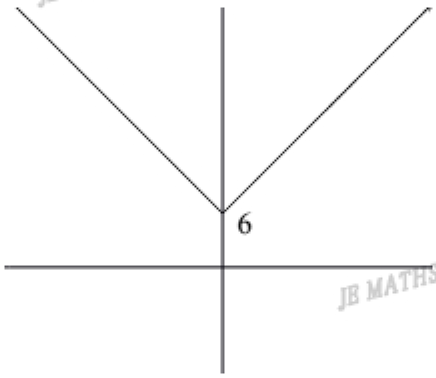
(b)



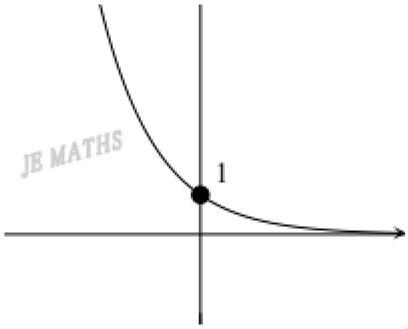
(c)



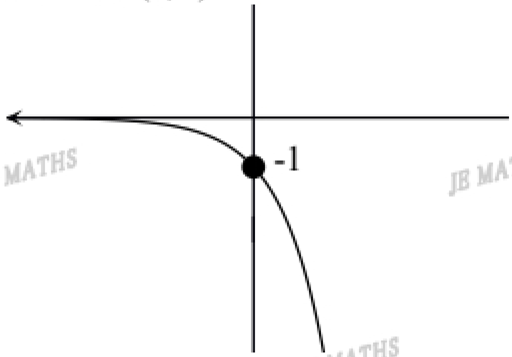
(d)



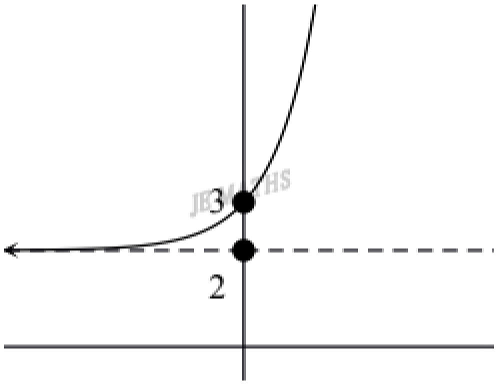
1. (a)
New centre (0, 0)



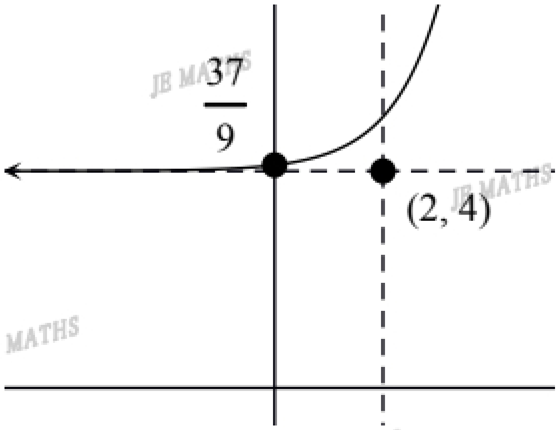
(b)
New centre (0, 0)



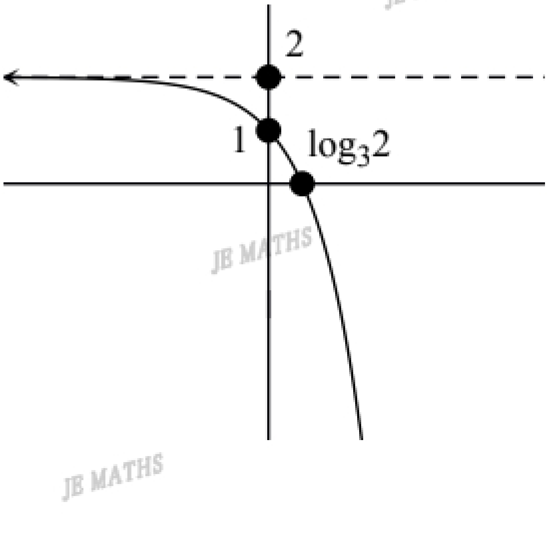
(c)
New centre (0, 2)



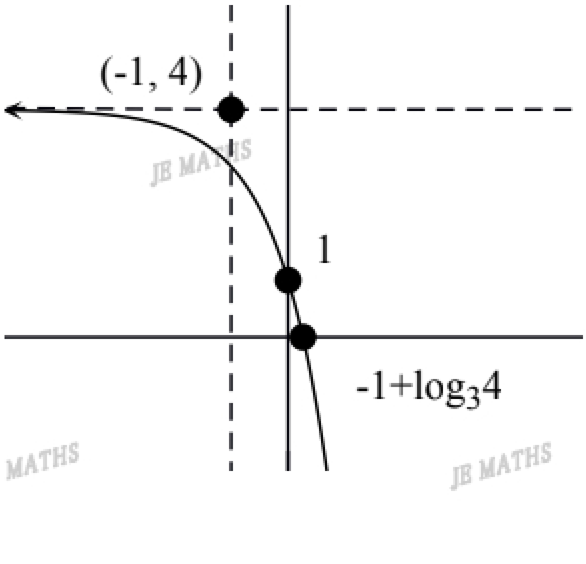
(d)
New centre (2, 4)



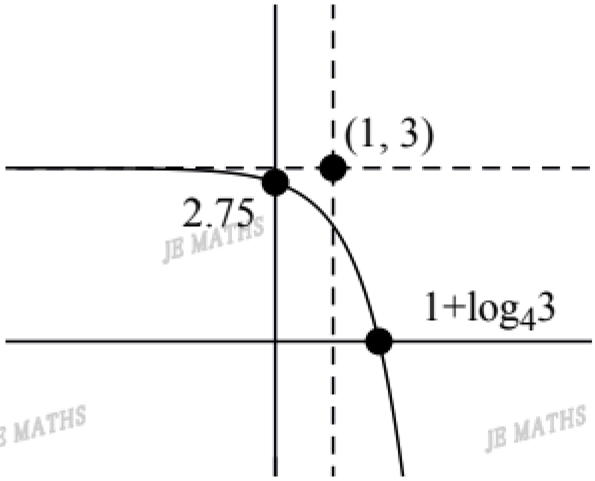
2. (a)
New centre (0, 2)



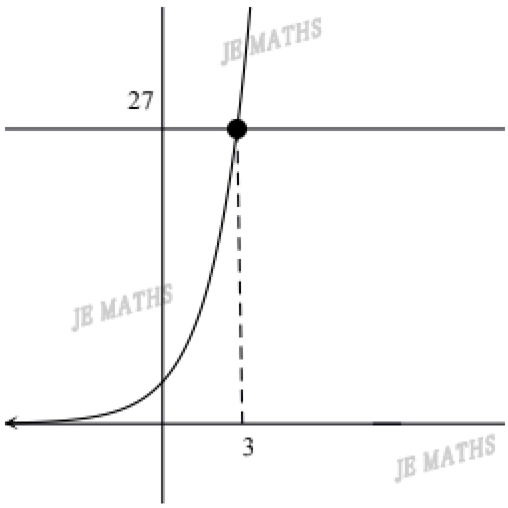
(b)
New centre (-1, 4)



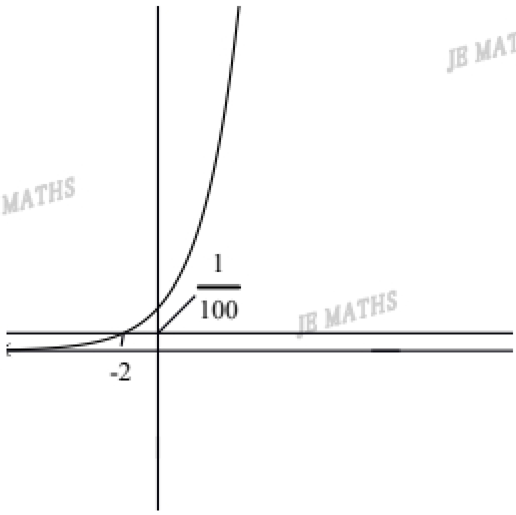
3. New centre: (1, 3)
   y-int: (0, 2.75)
   x-int: $(1 + log_4 3, 0)$



4. (a)
   $3^x \leq 3^3$
   $x \leq 3$



(b)
$10^x > 10^{-2}$
$x > -2$



5. (a)
   $log_{10} 3^x < log_{10} 5$
   $x log_{10} 3 < log_{10} 5$
   $x < log_{10} 5 / log_{10} 3$
   $x < log_3 5$

(b)
$log_{10} 1.01^x \geq log_{10} 0.1$
$x log_{10} 1.01 \geq log_{10} 0.1$
$x \geq log_{10} 0.1 / log_{10} 1.01$
$x \geq log_{1.01} 0.1$

6. (a)

$3^{2x+1} > 3^2$ (take log_3)

$log_3 3^{2x+1} > log_3 3^0$

2x +1 > 2

2x > 1

x > 1/2

(b)

$\log_{10}3^{x+3} < \log_{10}1000$ (take log_{10})

$\log_{10}3^{x+3} < 3\log_{10}10$

$(x+3)\log_{10}3 < 3$ ($\log_{10}3 > 0$)

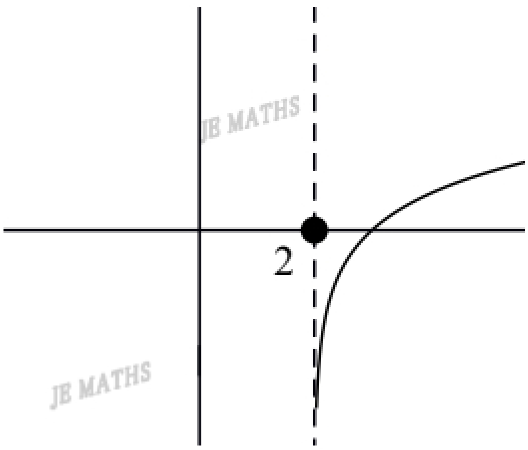
$x + 3 < 3/\log_{10}3$

$x < 3/\log_{10}3 - 3$

If you take log_3, x<3log(3)10−3

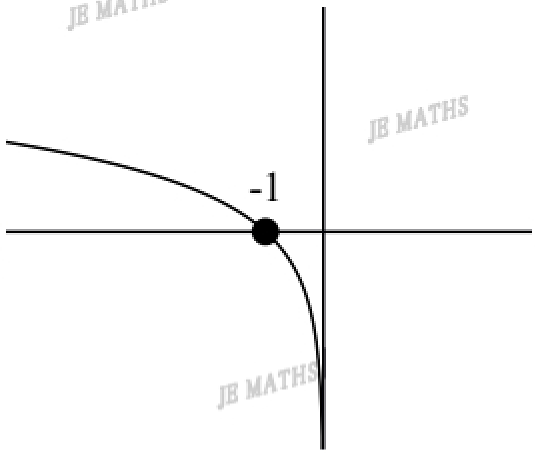
7. (a)

New centre: (2, 0)



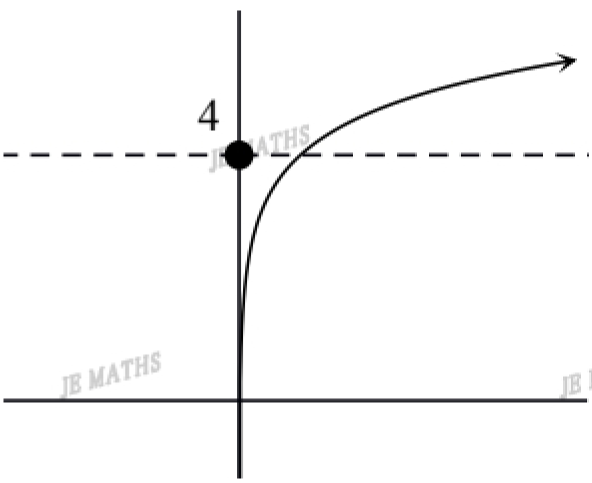
(b)

New centre: (0, 0)



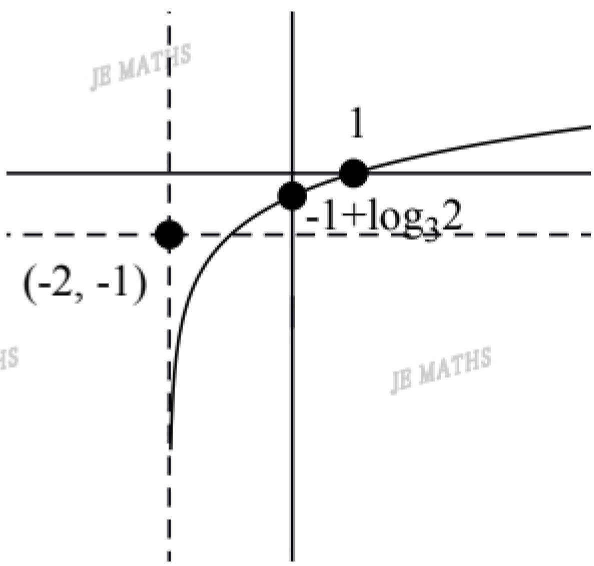
(c)

New Centre: (0, 4)



(d)

New Centre: (-2, -1)

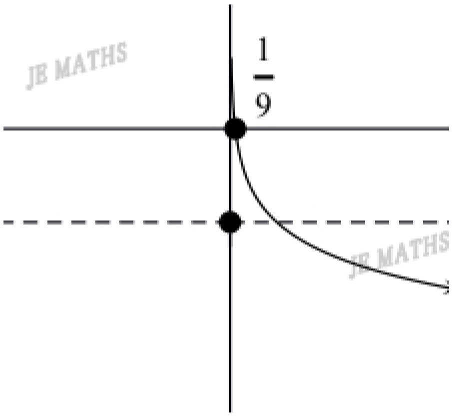


8.

(a)

New Centre: (0, -2)

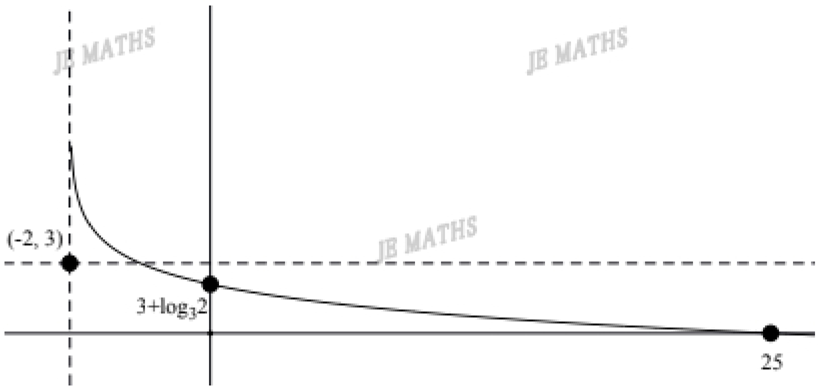
$y = log_3 x + 2 \rightarrow y = -(log_3 x + 2)$



(b)

New Centre: (-2, 3)

$y = log_3(x+2) - 3 \rightarrow y = -(log_3(x+2) - 3)$

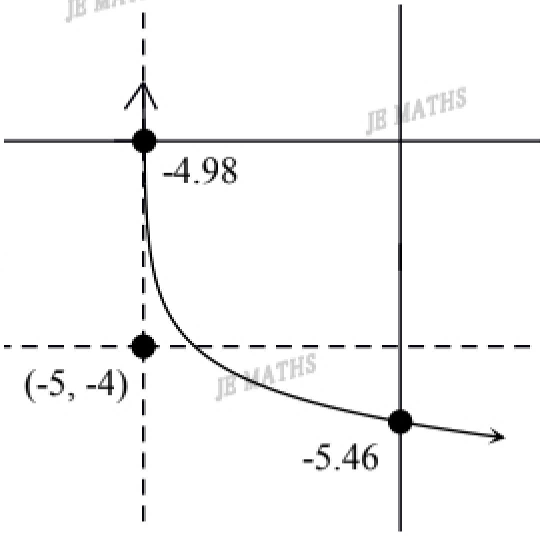


9. $y = -\log_3(x+5) - 4$

New centre: (-5, -4)

x-int: $(3^{-4} - 5, 0)$=(-4.98,0)

y-int: $(0, -4 - log_3 5)$=(0, -5.46)



- Exponential inequalities:

10.

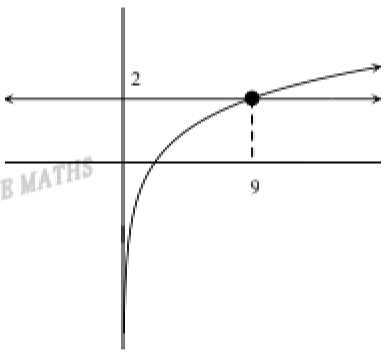
(a)

$x \leq 3^2$

$x \leq 9$

since $x > 0$

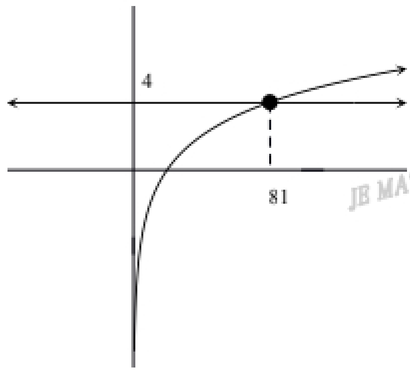
$0 < x \leq 9$



(b)

$x > 3^4$

$x > 81$



- Three reciprocal identities: $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$, $\sec\theta = \frac{1}{\cos\theta}$, $\cot\theta = \frac{1}{\tan\theta}$

1. (a) (secx)

   (b) (tanx)

   (c) 1/sinx ×sinx=1

   (d) sinx/cosx ×cos/sinx=1

2. (a) sinx ×sinx=$\sin^2 x$

   (b) cosx/sinx ×sinx=cosx

   (c) $(\text{cosx/sinx})^2 = \cot^2 x$

   (d) $(\text{sinx} \times 1/\text{sinx})^2 = 1$

3. (a)
   LHS = secxcosecx
   = RHS

   (b)
   LHS = 1/cosx ×cosx/sinx
   = 1/sinx
   = cosecx
   = RHS

   (c)
   LHS = 1/cosecx+1/secx
   = (secx+cosecx)/cosecxsecx
   = RHS

   (d)
   LHS = tanx-1/tanx
   =$(\tan^2 x - 1)$/tanx
   = RHS

- Ratio identities: $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\sec\theta}{\cos ec\theta}$, $\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{\cos ec\theta}{\sec\theta}$

4. (a)
LHS = cos θ sin θ /cos θ
= sin θ
= RHS

(b)
LHS = sinθcosθ/sinθ
= cosθ
= RHS

(c)
LHS = 1/sin θ ×cos θ
= cos θ /sin θ
= cot θ
= RHS

(d)
LHS = tan θ ×1/sin θ ×cos θ
= sin θ /cos θ ×cos θ /sin θ
= 1
= RHS

- Complementary angle identities:

$\cos(90° - \theta) = \sin\theta$, $\cot(90° - \theta) = \tan\theta$, $\cos ec(90° - \theta) = \sec\theta$

5. (a)
sin70˚=sin(90˚-70˚)
=cos20˚

(b)
cos36˚=cos(90˚-36˚)
=cos54˚

(c)
tan20˚=tan(90˚-20˚)
=cot70˚

(d)
cosec13˚=cosec(90˚-13˚)
=sec77˚

6. (a) cosx

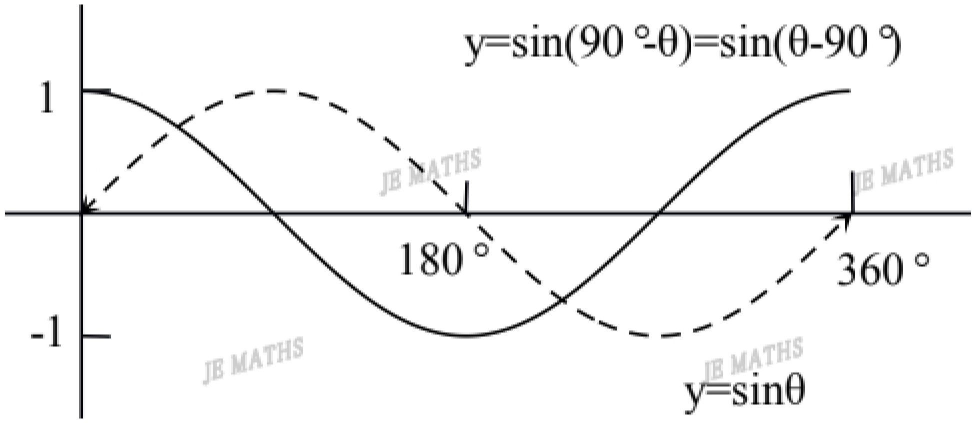
(b) cosecx

(c) 1/tanx = cotx

(d) sinx/cosx=tanx

7. (a) y = sin (90˚-Θ)=-sin(Θ-90˚)

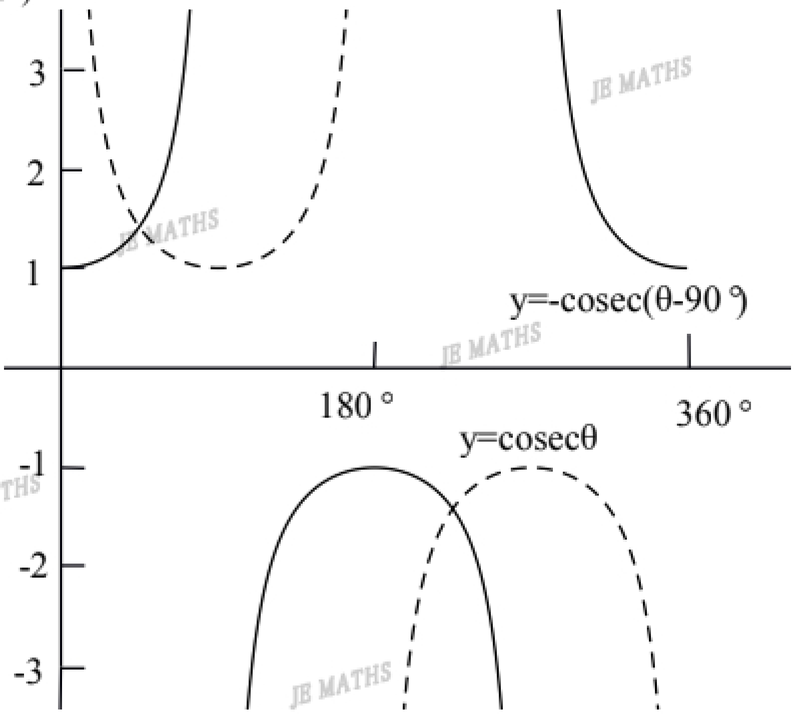
(b) Sketch.



(c) (yes)

8. (a) y = cosec (90˚-Θ)=-cosec(Θ-90˚)

(b) Sketch.



(c) (yes)

9. (a)

LHS = -sinΘsinΘ

= $-\sin^2\Theta$

= RHS

(b)

LHS = tanΘtanΘcosΘ

= tanΘsinΘ/cosΘ×cosΘ

= tanΘsecΘ

= RHS

- Pythagorean identities: $\sin^2\theta + \cos^2\theta = 1$, $1+\tan^2\theta = \sec^2\theta$, $1+\cot^2\theta = \operatorname{cosec}^2\theta$

10. (a)
($\cos^2 A$)

(b)
($\tan^2 A$)

(c)
(1)

(d)
(1+1=2)

11. (a)
$1+\tan^2 A = \sec^2 A$

(b)
$\sin^2\theta \times \operatorname{cosec}^2\theta = \sin^2\theta \times 1/\sin^2\theta = 1$

(c)
1+1 = 2

(d)
$\tan^2 A/\tan^2 A = 1$

12. $1 + \tan^2 A = \sec^2 A$
$\tan A = -\sqrt{(\sec^2 A-1)}$ (A in 4th Q, $\tan A<0$)
$= -\sqrt{(1/9-1)}$
$= -\sqrt{(8/9)}$
$= -2\sqrt{2}/3$

13. (a)
LHS $= \sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta$
$= 1 + 2\sin\theta\cos\theta$
= RHS

(b)
LHS $= 1-\sin^2\theta - \sin^2\theta$
$= 1-2\sin^2\theta$
= RHS