Foundation stage 1: (Basic probability and sample space)

1. A letter is chosen at random from the 26 letters of the English alphabet.
   Find the probability that is
   (a) a vowel. (b) a consonant.

   (c) a letter of the word 'AARON'. (d) not a letter of the word 'AARON'.

2. A letter is randomly selected from the word 'PIMPLE'.
   (a) List the sample space.

   (b) Find the probability that it is:
   (i) a P. (ii) also from the word 'APPLE'.

3. A box containing a light bulb has chance of $\frac{1}{15}$ of holding a defective bulb.
   (a) If 180 boxes were checked, how many would you expect to hold defective bulbs?

   (b) Hence, find the probability that the box holds that works?

4. (a) When a die is rolled, the theoretical probability of throwing a six is $\frac{1}{6}$. Find the expected number, correct to one decimal place, that you will get if a die is thrown 100 times.

   (b) In an experiment, a certain die was thrown 100 times and a six turned up 16 times.
   Find the relative frequency (experimental probability) of throwing a six?

   (c) Hence, check if the die is biased or fair. Why?

Foundation stage 2: (Using arrays and tree diagram)

1. A fair die is tossed twice.
   (a) Use arrays on a number plane to list all possible outcomes.

   (b) Find the probability of getting:
   (i) a '1' and a '6'.
   (ii) a '1' or a '6'.

2. From a group of four students, Andy, Bessie, Cathy, Danny, two are chosen at random, one after another, to be the captain and vice-captain.
   (a) Use a tree diagram to list all possible outcomes.

   (b) Find the probability of getting:
   (i) Andy is chosen.
   (ii) Bessie is captain.

   (iii) Cathy and Danny are chosen.
   (iv) Cathy or Danny are chosen.

Foundation stage 3: (The addition rule)

1. Given that the universal set E = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10},
   A is a set of odd numbers within 10 = {1,3,5,7, 9} and
   B is a set of prime numbers within 10 = {2, 3, 5, 7}.
   (a) Find:

   (i) $|A|$ (ii) $\overline{B}$

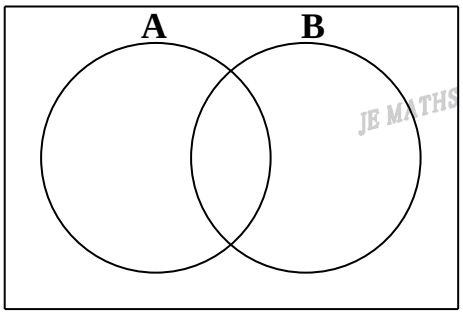
   (iii) $A \cup B$ (iv) $|A \cup B|$

   (v) $A \cap B$ (vi) $|A \cap B|$

   (b) If C is a set of even numbers within 10, check if the following statement is true or not:
   (i) B∩C = ∅ (ii) C⊂E

   (iii) B∪A = E (iv) |A| = |B|

   (b) Hence, make a Venn diagram (labeled) and two-way table below:



	A	$\bar{A}$	T
B			
$\bar{B}$			
T			

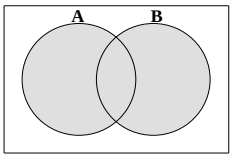
   (c) Find:

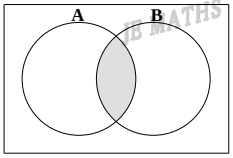
   (i) $|A \cap \overline{B}|$ (ii) $P(A\ only)$

   (iii) $|\overline{A} \cap B|$ (iv) $P(B\ only)$

   (v) $|\overline{A} \cap \overline{B}|$ (vi) $P(neither\ A\ nor\ B)$

2. Match up with lines:

(a) A *and* B (α) A∪B (A) 

(b) A *or* B (β) A∩B (B) 

3. A die is thrown.
Let A be the event that an even number appears, B be the event that an odd number appears.
(a) Check if A and B are mutually exclusive or non-mutually exclusive?
mutually exclusive

(b) Find

(i) $P(A \text{ and } B)$ (ii) $P(A \text{ or } B)$

4. Two cards are selected from **two** packs of 52 cards seperately.
Let A be the event that an Ace card appears and B be the event that a black card appears.
(a) Check if A and B are mutually exclusive or non-mutually exclusive?

(b) Find

(i) $P(\textit{black Ace})$ (ii) $P(\textit{black or Ace})$

5. Use the addition rule $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ to answer the following questions

(a) $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{12}$, find $P(A \cup B)$.

(b) $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{1}{4}$, find $P(A \cap B)$.

Foundation stage 4: (The product rule)

1. A fair die is tossed twice.
   Find the probability of getting:
   (a) a double, given that the number '6 is selected.

   (b) a number '1' and '6', given that the number '1' or '6' are selected.

2. From a group of four students, Andy, Bessie, Cathy, Danny, two are chosen at random, one after another, to be the captain and vice-captain.
   Find the probability of getting:
   (a) Bessie is captain and Andy is a vice-captain, given that Andy is selected.

   (b) Cathy and Danny, given that Cathy or Danny are selected.

3. True or false:

   (a) If two events A, B are independent event, then $P(A\ and\ B) = P(A) \times P(B)$.

   (b) If two events A, B are dependent event, then $P(A\ and\ B) = P(A) \times P(B, given\ A)$.

4. A coin is tossed 4 times.
   (a) Check if the events are independent event or dependent event.

   (b) List the outcome in the sample space of getting:
   (i) 1 heads. (ii) 2 heads.

   (c) Hence, use the product rule to find the probability of getting:
   (i)1 heads. (ii) 2 heads.

5. Two cards are selected from **two** packs of 52 cards seperately.
   Let A be the event that an Ace card appears and B be the event that a black card appears.
   (a) Check if A and B are independent event or dependent event.

   (b) Hence, use the product rule to find the probability of getting
   (i) a black Ace. (ii) a red club.

6. Two cards are selected from **one** pack of 52 cards.
   Let A be the event that an Ace card appears and B be the event that a black card appears.
   (a) Check if A and B are independent event or dependent event.

   (b) Hence, use the product rule to find the probability of getting
   (i) a black Ace.

   (ii) a red club.

Foundation stage 5: (Probability tree diagram and condition probability)

1. A bag contains six red marbles and four blue marbles.
   Three marbles are selected in succession.
   (a) Find the probability of getting:
   (i) three red marbles.

   (ii) first red, second blue and third red marble.

   (iii) two red marble and 1 blue marble.

   (iv) at least two red marbles.

   (v) two red marbles given that at least two red marbles is selected. ie, find $P(2R|\geq 2R)$.

   (f) If the marble is red it is replaced and it is blue it is not replaced, find the probability of getting two red marble.

Foundation stage 1: (Basic probability and sample space)

1. (a)
   P(A, E, I, O, U) = 5/26

   (b)
   1- P(vowel) = 1-5/26 = 21/26

   (c)
   P(A, R, O, N) = 4/26 = 2/13

   (d)
   1- P(A, R, O, N) = 1 – 2/13 = 11/13

2. (a) {P, I, M, L, E}

   (b)
   (i)
   P(P) = 2/6 = 1/3

   (ii)
   P(P, L, E) = 3/6 = 1/2

3. (a) 180 ×1/15 = 12

   (b) 180-12 = 168

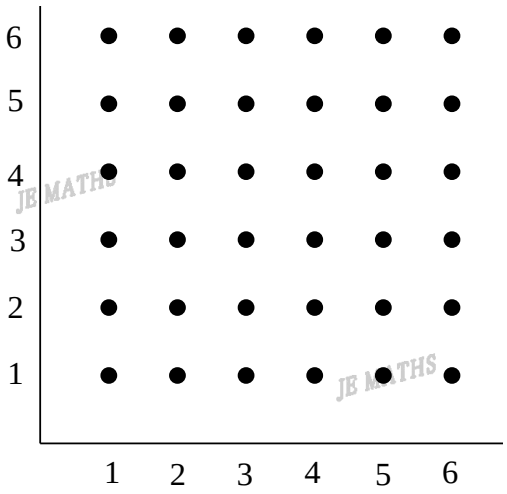
4. (a) 100 ×1/6 = 16.7

   (b) 16/100 = 0.16

   (c) 1/6≠0.16, it is biased.

Foundation stage 2: (Using arrays and tree diagram)

1. (a)

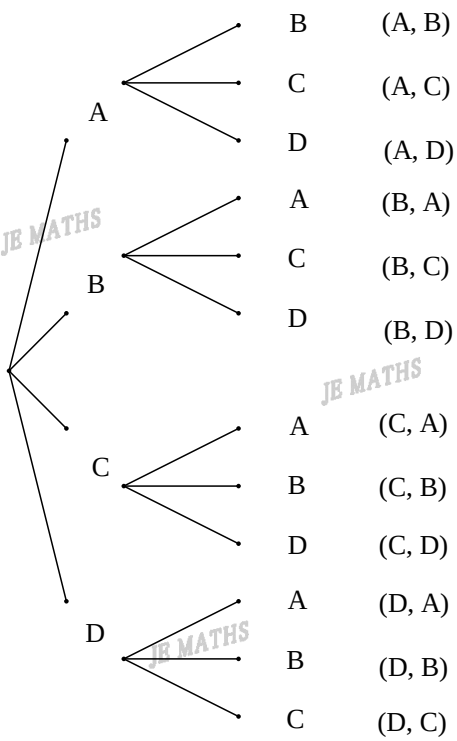


(b)

(i)
P(1 and 6) = 2/36 = 1/18

(ii)
P(1 or 6) = 11/36

2. (a)



(b)

(i)
P(A) = 6/12 = 1/2

|

(ii)
P(B first) = 3/12 = 1/4

(iii)
P(C and D) = 2/12 = 1/6

(iv)
P(C or D) = 10/12 = 5/6

Foundation stage 3: (The addition rule)

1. (a)

(i)
|A| = 5

(ii)
$\bar{B}$ = {1, 4, 6, 8, 9, 10}

(iii)
A∪B = {1, 2, 3, 5, 7, 9}

(iv)
|A∪B| = 6

(v)
A∩B = {3, 5, 7}

(vi)
|A∩B| = 3

(b)

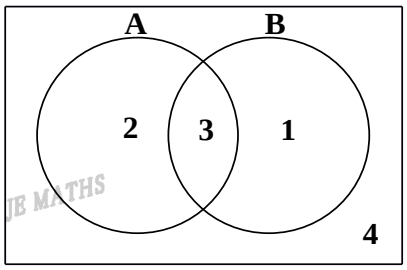
(i)
F

(ii)
T

(iii)
T

(iv)
T

(b)



	A	$\bar{A}$	T
B	(3)	(1)	(4)
$\bar{B}$	(2)	(4)	(6)
T	(5)	(5)	(10)

(c)

(i)
2

(ii)
P(A only) = 2/10 =1/5

(iii)
1

(iv)
P(B only) = 1/10

(v)
4

(vi)
P(neither A nor B) = 4/10 = 2/5

2. (a)-(β)-(B), (b)-(α)-(A)

3. (a) mutually exclusive

(b)

(i)
P(A and B) = P(A∩B)
= ∅

(ii)
P(A or B) = P(A∪B)
= 1/2 + 1/2
= 1

4. (a) non-mutually exclusive

   (b)

   (i)

   P(A and B) = P(A∩B)

   = 2/52

   = 1/26

   (ii)

   P(A or B) = P(A∪B)

   = 26/52 + 4/52 – 2/52

   = 28/52

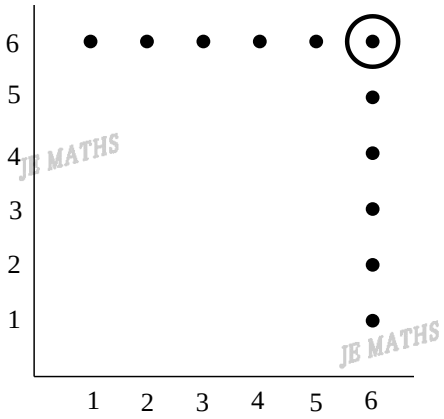
   = 7/13

5. (a) P(A∪B) = 1/2 + 1/3 – 1/12 = 3/4

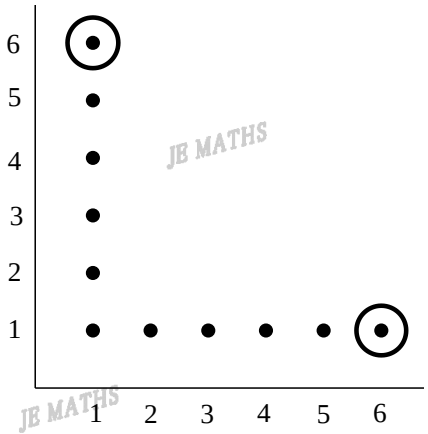
   (b) P(A∩B) = 1/2 + 1/3 – 1/4 = 7/12

Foundation stage 4: (The product rule)

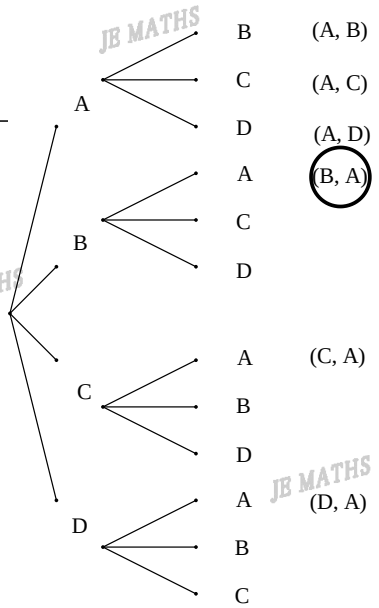
1. (a) P(double, given 6) = 1/11



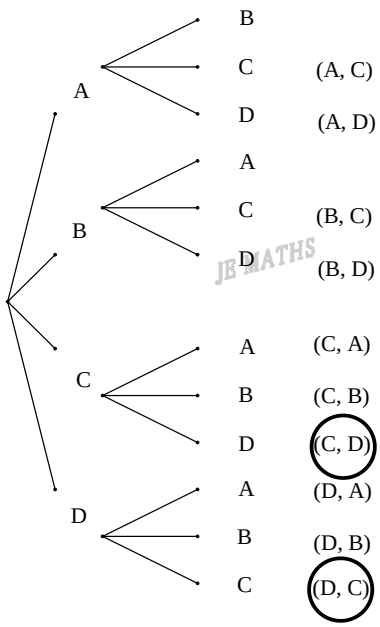
(b) P(1 and 6, given 1 or 6) = 2/11



2. (a) P(BA, given A) = 1/6



(b) P(C and D, given that C or D) = 2/10=1/5



3. True or false:
   (a) (T)

   (b) (T)

4. (a) independent event

   (b)
   (i)
   {HTTT, THTT, TTHT, TTTH}

   (ii)
   {HHTT, HTHT, HTTH, TTHH, THHT, HTTH}

   (c)
   (i)
   $P(1H) = 4 \times (1/2 \times 1/2 \times 1/2 \times 1/2)$
   $= 4/16 = 1/4$

   (ii)
   $P(2H) = 6 \times (1/2 \times 1/2 \times 1/2 \times 1/2)$
   $= 6/16 = 3/8$

5. (a) independent event

   (b)
   (i)
   $P(\text{black Ace}) = P(\text{black}) \times P(\text{Ace})$
   $= 26/52 \times 4/52$
   $= 1/26$

   (ii)
   $P(\text{red club}) = P(\text{red}) \times P(\text{club})$
   $= 26/52 \times 12/52$
   $= 3/26$

6. (a) dependent event

   (b)
   (i)
   $P(\text{alack Ace}) = P(\text{black}) \times P(\text{Ace, given one black card is selected})$
   $= 26/52 \times 4/51$ (1 black card is selected, then 4 Ace cards in the rest of 51 cards)
   $= 2/51$

   (ii)
   $P(\text{red club}) = P(\text{red}) \times P(\text{club, given one red card is selected})$
   $= 26/52 \times 12/51$ (1 red card is selected, then 12 club cards in the rest of 51 cards)
   $= 2/17$

Foundation stage 5: (Probability tree diagram and condition probability)

1. (a)

(i)

$P(RRR) = 6/10 \times 5/9 \times 4/8$

$= 1/6$

(ii)

$P(RBR) = 6/10 \times 4/9 \times 5/8$

$= 1/6$

(iii)

List two red and 1 blue marble: {RRB, RBR, BRR}

$P(\text{two R, 1B}) = 3 \times P(RBR)$

$= 3 \times 1/6$

$= 1/2$

(iv)

At least two red marbles means two red marbles and three red marbles

notice two red marbles means two red and 1 blue marble

$P(\text{at least 2R}) = P(\geq 2R)$

$= P(2R) + P(3R)$

$= 1/2 + 1/6$

$= 2/3$

(v)

$P(2R|\geq 2R) = P(2R \cap \geq 2R)/P(\geq 2R)$

$= (1/2) \div (2/3)$

$= 3/4$

(f)

List two red and 1 blue marble: {RRB, RBR, BRR}

$P(RRB) = 6/10 \times 6/10 \times 4/10 = 18/225$

$P(RBR) = 6/10 \times 4/10 \times 6/9 = 4/25$

$P(BRR) = 4/10 \times 6/9 \times 6/9 = 8/45$

$P(2R) = P(RRB) + P(RBR) + P(BRR)$

$= 18/225 + 4/25 + 8/45$

$= 94/225$