Extra Enrichment:

1. Given $F(x) = f^2(g(x))$, g(1) = 2, g'(1) = 3, f(2) = 4 and f'(2) = 5, find F'(1).

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2. If F(t) = f(g(t)), $f'(t) = \frac{1}{t^2 + 1}$, $g'(t) = \frac{10}{|t^4| + 1}$ and g(0) = 3, what is F'(0)?

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3. Let h(x) = g(f(x)) and j(x) = f(g(x)), where f(x) and g(x) are differentiable functions for all real x values. Fill in the missing entries in the table below.

x	f(x)	f'(x)	g(x)	g'(x)	h(x)	h'(x)	j(x)	j'(x)
0		-3			1		1	$-\frac{3}{2}$
1	0			$\frac{3}{2}$	0	$\frac{1}{2}$		



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Solutions:

1.
$$F(x) = f^2(g(x)) \Rightarrow F'(x) = 2 \times f(g(x)) \times \frac{d}{dx} f(g(x))$$

$$= 2 \times f(g(x)) \times f'(g(x)) \times g'(x)$$

$$\Rightarrow F'(1) = 2 \times f(g(1)) \times f'(g(1)) \times g'(1)$$

$$= 2 \times f(2) \times f'(2) \times 3$$

$$= 2 \times 4 \times 5 \times 3$$

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2.
$$F(t) = f(g(t)) \Rightarrow F'(t) = f'(g(t)) \times g'(t)$$

$$= \frac{1}{(g(t))^2 + 1} \times \frac{10}{t^4 + 1}$$

$$\Rightarrow F'(0) = \frac{1}{(g(0))^2 + 1} \times \frac{10}{0 + 1}$$

$$= \frac{1}{9 + 1} \times 10$$

$$= 1$$

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3.

x	f(x)	f'(x)	g(x)	g'(x)	h(x)	h'(x)	j(x)	j'(x)
0	1	-3	0	$\frac{1}{2}$	ĩ	$-\frac{9}{2}$	1	$-\frac{3}{2}$
1	0	1	1	$\frac{3}{2}$	0	$\frac{1}{2}$	0	$\frac{3}{2}$

$$h(x) = g(f(x)) \implies h'(x) = g'(f(x)) \cdot f'(x)$$

$$j(x) = f(g(x)) \implies j'(x) = f'(g(x)) \cdot g'(x)$$

Given:
$$f(1)=0$$
, $f'(0)=-3$

$$g'(1)=\frac{3}{2}$$

$$h(0)=1, h(1)=0, h'(1)=\frac{1}{2}$$

$$j(0)=1, j'(0)=-\frac{3}{2}$$

$$j(0)=1, j'(0)=-\frac{3}{2}$$

$$j(0)=1, j'(0)=-\frac{3}{2}$$

$$h(1) = 0 \implies g(f(1)) = 0$$

$$\Rightarrow g(0) = 0 \quad (as f(1)) = 0$$

$$\downarrow g(0) = 0 \quad (as f(1)) = 0$$

$$j(0)=1 \Rightarrow f(g(0))=1$$

 $\Rightarrow f(0)=1 \text{ (as } g(0)=0)$

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$$f'(0) = -\frac{3}{2} \implies f'(g(0)) \cdot g'(0) = -\frac{3}{2}$$

$$\int_{\mathbb{R}} MATHS \implies f'(0) \cdot g'(0) = -\frac{3}{2} \qquad (as_{\mathbb{R}} MAg(0) = 0)$$

$$\implies -3 \times g'(0) = -\frac{3}{2} \qquad (as_{\mathbb{R}} f'(0) = -3)$$

$$\implies g'(0) = \frac{1}{2}$$

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$$H'(1) = \frac{1}{2} \implies g'(f(1)) \cdot f'(1) = \frac{1}{2}$$

$$\implies g'(0) \cdot f'(1) = \frac{1}{2} \quad \text{(as } f(1) = 0)$$

$$\implies \frac{1}{2} \times f'(1) = \frac{1}{2}$$

$$\implies f'(1) = 1$$

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$$\implies f'(1) = 1$$

$$h(0)=1 \Rightarrow g(f(0))=1$$
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 $\Rightarrow g(1)=1$

$$j(1) = f(g(1))$$

$$= f(1)$$

$$= 0$$

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$$= MATHS$$

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$$H(0) = g'(f(0)) \times f'(0)$$

$$= g'(1) \times f'(0)$$

$$= \frac{3}{2} \times (-3)$$

$$= -\frac{9}{2}$$

$$= -\frac{9}{2}$$

$$f'(1) = f'(g(1)) \times g'(1)$$

$$= f'(1) \times g'(1)$$

$$= 1 \times \frac{3}{18}$$

$$= \frac{3}{2}$$

$$= \frac{3}{2}$$

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