- Find the exact missing angles and sides.

1. (a) $\sin25^\circ=x/3$
   $x=3\sin25^\circ$

   (b) $\cos20^\circ=4/x$
   $x=4/\cos20^\circ$

   (c) $\tan66^\circ=7/x$
   $x=7/\tan66^\circ$

2. (a) $\tan35^\circ=x/CB$
   $CB = x/\tan35^\circ$

   (b) $\tan35^\circ=BD/x$
   $BD = x\tan35^\circ$

   (c) $CD = CB - BD$
   $= x/\tan35^\circ - x\tan35^\circ$

3. (a) $\sin\theta=2/5$
   $\theta=sin^{-1}(2/5)$

   (b) $\cos\theta=13/21$
   $\theta=cos^{-1}(13/21)$

   (c) $\tan\theta=1/1.2$
   $\theta=tan^{-1}(5/6)$

4. (a) $\tan\angle BCD= 5/7$
   $\angle BCD = tan^{-1}(5/7)$

   (b) $\tan\angle BCA=10/7$
   $\angle BCA = tan^{-1}(10/7)$

   (c) $\angle ACD = \angle BCA - \angle BCD$
   $= tan^{-1}(10/7) - tan^{-1}(5/7)$
   $= 19.4703\ldots$
   $= 19^\circ28'13''$
   $= 19^\circ28'$

5. (a) $AC = \sqrt{[1^2+(2\sqrt{2})^2]} = \sqrt{9} = 3$

(b) (i) $\cos\alpha = 2\sqrt{2}/3$
(ii) $\sin\alpha = 1/3$

(c) LHS $= (1/3)^2 + (2\sqrt{2}/3)^2$
$= 1/9 + 8/9 = 1$
= RHS

6. (a) $\sec\alpha = 2\sqrt{3}/3$, $\cos\alpha = 3/2\sqrt{3}$
$BC = \sqrt{[(2\sqrt{3})^2-(3)^2]} = \sqrt{3}$

(b)(i) $= 1/\sin\theta = 2\sqrt{3}/\sqrt{3} = 2$

(ii) $= 1/\tan\theta = 3/\sqrt{3} = 3\sqrt{3}/3 = \sqrt{3}$

(c) LHS $= 1+(\sqrt{3})^2 = 1+3 = 4$
RHS $= 2^2 = 4$
LHS = RHS

7. (a) (17.30)

(b) $13/\tan 13.5 = 54.15$

- Special angles for 6 trigratios: 30°, 45°, 60°.

8. (a)
$= (\sqrt{2}/2)^2+(\sqrt{2}/2)^2$
$= 1/2+1/2$
$= 1$

(b)
$= 1/2 \times 1/2 - \sqrt{3}/2 \times \sqrt{3}/2$
$= 1/4-3/4$
$= -1/2$

(c)
$= 2\sqrt{3}/[1-(\sqrt{3})^2]$
$= 2\sqrt{3}/(-2)$
$= -\sqrt{3}$

(b)
$= 1/\sin 60^\circ - 1/\tan 60^\circ$
$= (2/\sqrt{3})^2-(1/\sqrt{3})^2$
$= 4/3-1/3 = 1$

9. (a)
LHS $= 1 - (\sqrt{3}/2)^2 = 1-3/4 = 1/4$
RHS $= (1/2)^2 = 1/4$
LHS = RHS

(b)
LHS $= 1 + 1^2 = 2$
RHS $= 1/\cos 45^\circ = (\sqrt{2})^2 = 2$
LHS = RHS

(c)
LHS $= 1/2$
RHS $= 1 - 2\times(1/2)^2 = 1/2$
LHS = RHS

(d)
LHS $= 1 + 1/\tan 30^\circ = 1 + (\sqrt{3})^2 = 1 + 3 = 4$
RHS $= 1/\sin 30^\circ = (2)^2 = 4$
LHS = RHS

- Six trig ratios in the unit circle:

10. (a) $x = -1$, $y = 1$, $r = \sqrt{[(-1)^2+1^2]}=\sqrt{2}$
$\sin\theta = 1/\sqrt{2}=\sqrt{2}/2$, $\operatorname{cosec}\theta = 2$
$\cos\theta = -1/\sqrt{2}=-\sqrt{2}/2$, $\sec\theta = -\sqrt{2}$
$\tan\theta = 1/-1=-1$, $\cot\theta = -1$

(b) $x = \sqrt{3}$, $y = -3$, $r = \sqrt{[(\sqrt{3})^2+(-3)^2]}=\sqrt{12}=2\sqrt{3}$
$\sin\theta = -3/2\sqrt{3}=-\sqrt{3}/2$, $\operatorname{cosec}\theta = -2\sqrt{3}/3$
$\cos\theta = \sqrt{3}/2\sqrt{3}=1/2$, $\sec\theta = 2$
$\tan\theta = -3/\sqrt{3}=-\sqrt{3}$, $\cot\theta = -\sqrt{3}/3$

- Boundary angles.

11.

	0°	90°	180°	270°	360°
x	1	0	-1	0	1
y	0	1	0	-1	0
r	1	1	1	1	1
$\sin\theta$	0/1=0	1/1=1	0/1=0	-1/1=-1	0/1=0
$\cos\theta$	1/1=1	0/1=0	-1/1=-1	0/1=0	1/1=1
$\tan\theta$	0/1=0	1/0=∞	0/1=0	1/0=∞	0/1=0
$\cos ec\theta$	1/0=∞	1/1=1	1/0=∞	1/-1=-1	1/0=∞
$\sec\theta$	1/1=1	1/0=∞	1/-1=-1	1/0=∞	1/1=1
$\cot\theta$	1/0=∞	0/1=0	1/0=∞	0/1=0	1/0=∞

- Sign of trig (ASTC):

12.

Quadrant	1st	2nd	3rd	4th
x	+	-	-	+
y	+	+	-	-
r	+	+	+	+
$\sin\theta$	+/+=+	+/+=+	-/+=-	-/+=-
$\cos\theta$	+/+=+	-/+=-	-/+=-	+/+=+
$\tan\theta$	+/+=+	+/-=-	-/-=+	-/+=-
$\cos ec\theta$	+/+=+	+/+=+	+/-=-	+/-=-
$\sec\theta$	+/+=+	+/-=-	+/-=-	+/+=+
$\cot\theta$	+/+=+	-/+=-	-/-=+	+/-=-

- Find the exact value of a special trig by using the related angle:

13. (a)
= -sin45˚
=-√2/2

(b)
=cos30˚
=√3/2

(c)
=tan(-150˚+360˚)
=tan210˚
=tan30˚
=√3/3

(d)
= cos(-315˚+360˚)
= cos45˚
= √2/2

(e)
=sin(690˚-360˚)
=sin330˚
=-sin30˚
=-1/2

(f)
= tan(600-360)
= tan240˚
= tan60˚
= √3

14. (a)
=1/cos210˚
=1/-cos30˚
=1/(-√3/2)
=-2/√3
=-2√3/3

(b)
=1/sin135˚
= 1/sin45˚
= 1/(√2/2)
=√2

(c)
=cosec(-60˚+360˚)
=cosec300˚
=1/sin300˚
=1/-sin60˚
=-2/√3
=-2√3/3

(d)
= cot(-240˚+360˚)
= cot120˚
= 1/tan120˚
= 1/-tan60˚
= -1/(√3)
= -√3/3

(e)
= cos270˚/sin270˚
=0/1
=0

(f)
= 1/cos90˚
= 1/0
= ∞

- General trig formulae:

15.

	180 º-A	180 º+A	360 º-A
sin	sinA	-sinA	-sinA
cos	-cosA	-cosA	cosA
tan	-tanA	tanA	-tanA
cos*ec*	cosecA	-cosecA	-cosecA
sec	-secA	-secA	secA
cot	-cotA	cotA	-cotA

16. (a)
= sinA/-sinA
= -1

(b)
=cos(360˚-A)/cos(180˚-A)
=cosA/-cosA=-1

(c)
=-sinA ×cosA/sinA ×-cosA
=1

(d)
= -sinA ×sinA/-sinA ×-cosA
= -tanA

17. (a)
= -secA/-secA
= 1

(b)
= secA/-cosecA
= -(sinA/cosA)
= -tanA

18. (a)
=sin165˚=sin15˚=0.25

(b)
=cos195˚=-cos15˚=-0.97

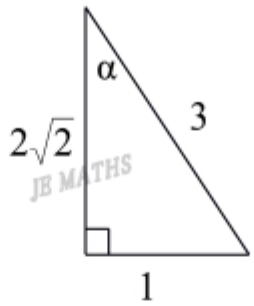
(c)
=2 ×0.25 ×-0.97=-0.485

(d)
=1/-0.97+1/-0.25= -5.03 = -5.0

19. a=-60+360=300˚, b =-120+360=240˚
LHS =sin(300˚+240˚)
=sin540˚
=sin(540˚-360˚)
=sin180˚
=0
RHS = sin300˚cos240+sin240˚cos300˚
=-sin60˚×-cos60˚+(-sin60˚)×cos60˚
= 0
LHS = RHS

20. (a) $\sin A = \sin(180^\circ - \alpha)$
$= \sin\alpha = 1/3$

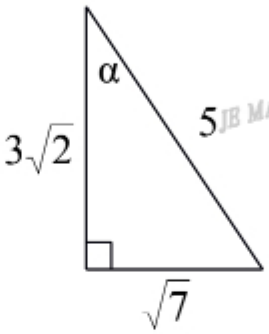
$\cos A = \cos(180^\circ - \alpha)$
$= -\cos\alpha$
$= -2\sqrt{2}/3$



(b) $\tan A = \tan(180^\circ - \alpha)$
$= -\tan\alpha$
$= -1/2\sqrt{2} = -\sqrt{2}/4$

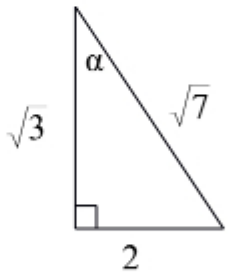
21. (a) $\tan B = \tan(360^\circ - \alpha)$
$= -\tan\alpha = -\sqrt{7}/3\sqrt{2}$
$\tan\alpha = \sqrt{7}/3\sqrt{2}$

$\sin B = \sin(360^\circ - \alpha)$
$= -\sin\alpha$
$= -\sqrt{7}/5$



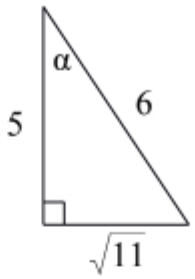
(b) $\cos B = \cos(360^\circ - \alpha)$
$= \cos\alpha$
$= 3\sqrt{2}/5$

22. (a) θ in 2nd and 4th quadrant.



(b) $\sin\theta = \pm\sin\alpha$
$= \pm 2/\sqrt{7}$
$= \pm 2\sqrt{7}/7$

23. (a)θ in 2nd quadrant.



(b) $\tan\theta<0$

$\tan\theta = -\tan\alpha = -\sqrt{11}/5$

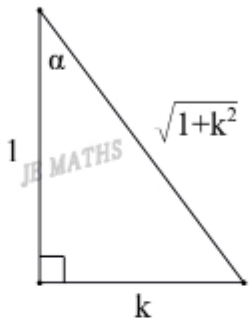
24. $\tan\theta = k = k/1$, $k>0$, θ in 1st and 3rd quadrant.

$\sin\theta = \pm\sin\alpha$

$= \pm k/\sqrt{(1+k^2)}$

$\operatorname{cosec}\theta = 1/\sin\theta$

$= \pm\sqrt{(1+k^2)}/k$



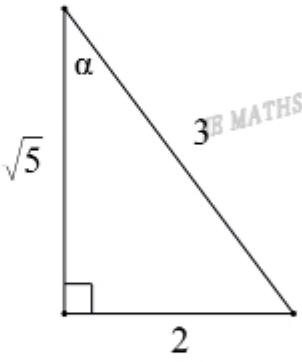
25. (a) $\operatorname{cosec}\theta = -3/2, \sin\theta = -2/3$

$\sin\alpha = 2/3$

$\sec\theta = 1/\cos\theta<0$ (θ in 3rd quadrant)

$= -1/\cos\alpha$

$= -3/\sqrt{5} = -3\sqrt{5}/5$



(b) $\cot\theta = 1/\tan\theta>0$

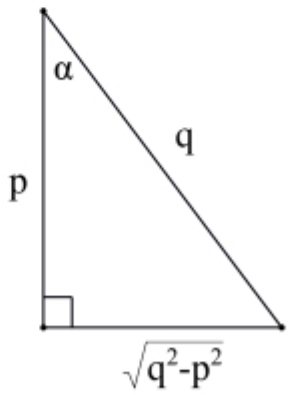
$= 1/\tan\alpha$

$= \sqrt{5}/2$

26. (a) $\sec\theta = q/p$, $\cos\theta = p/q$

$\operatorname{cosec}\theta = 1/\sin\theta<0$ (θ in 4th quadrant)

$= -1/\sin\alpha = -q/\sqrt{(q^2-p^2)}$



(b) $\cot\theta = 1/\tan\theta<0$

$= -1/\tan\alpha = -p/\sqrt{(q^2-p^2)}$