

Problem Set 9

Written by Andy Tran, James Bang for the MaPS Correspondence Program

Instructions

- Some of these problems are based off the notes “*Methods of Counting*”. Some other are revision problems for the previous notes.
- They are in roughly difficulty order and get quite difficult, so you are **not** expected to be able to solve every problem.
- However, please attempt as many questions as you can and submit your solutions to your mentor for marking and feedback.
- You may (and encouraged to) submit incomplete solutions if you can not solve a problem completely.
- You may type your solutions or submit a pdf of a **clear** scan/photo of **legible** written solutions.
- Feel free to discuss these problems with your peers and on the forum but the solutions you submit must be written by yourself.

Problems

1. Noting that a 4-digit number cannot start with a 0:
 - (a) How many 4-digit numbers are there?
 - (b) How many 4-digit numbers are there that are odd?
 - (c) How many 4-digit numbers are there such that all the digits are odd?
 - (d) How many 4-digit numbers are there with distinct digits?
 - (e) How many 4-digit numbers are there with digits in strictly descending order?
 - (f) How many 4-digit numbers are there with digits in strictly ascending order?
2. For all real x , show that
$$\frac{x^2 + 2}{\sqrt{x^2 + 1}} \geq 2.$$
3.
 - (a) How many squares of any size can be found in an 8×8 grid of unit squares?
 - (b) How many squares of any size can be found in a 2022×2022 grid of unit squares?
 - (c) How many rectangles of any size can be found in an 8×8 grid of unit squares?
 - (d) How many rectangles of any size can be found in a 2022×2022 grid of unit squares?
4. Haowen is tiling his rectangular roof with 1×4 and 2×2 rectangular tiles (which may be rotated). He had a set of tiles which would allow him to complete this task with no remaining tiles. Unfortunately, Andy stole one of the tiles, and replaced it with a tile of the opposite kind. Is it possible for Haowen to still successfully tile the roof, or is he doomed by the next thunderstorm?
5. Oliver lives at the bottom left corner of an $n \times n$ grid and his school is at the top-right corner of the grid. He is late for school, so he needs to run from his home to school and will only travel in “up” and “right” directions.
 - (a) How many paths can Oliver take?
 - (b) If Oliver is superstitious and will only travel inside the triangle formed by his home, his school and the bottom right corner, how many paths can Oliver take?