1.

(a) S = {BB, BG, GB, GG}

(b) (i) A = {BB, BG, GB}

(ii) B = {BG, GB}

(iii) $\bar{A}$ = {GG}

(c) (i) |A| = 3

(ii) |B| = 2

2.

(a) S = {BBB, BBG, BGB, GBB, GGB,GBG, BGG, GGG}

(b) (i) A = {BBB, BBG, BGB, GBB, GGB,GBG, BGG}

(ii) B = {GGB, GBG, BGG}

(iii) $\bar{A}$ = {GGG}

(c) (i) |A| = 7

(ii) |B| = 3

(**Notice**: if the number of children is more than 3, let's say 4, 5, 6...,
a brand new listing method, which is called **permutation** will be introduced.
The topic of permutation will be covered later in the Advanced Maths course.)

3. LHS = $|A|/|S| + |\bar{A}|/|S|$
$= (|A|+|\bar{A}|)/|S|$
$= |S|/|S|$
= 1 = RHS

4. (a)
P(sum>8) = 10/36
= 5/18

(b)
P(sum = 8) = 5/36

(c)
P(sum>8 or sum = 8) = 5/18+5/36 - 0 (P(sum>8 and sum=8) = 0)
= 5/12

5.

(a)
P(multiple of 3) = 16/50
= 8/25

(b)
P(multiple of 5) = 10/50
= 1/5

(c)
P(multiple of 3 or 5) = 8/25+1/5-3/50
= 23/50

6.

(a) P(G) = 1-1/2-1/3
=1/6

(b) P(R or B) = 1/2+1/3
= 5/6

(c) P(R or G) = 1/2+1/6
= 2/3

7.

(a) P(sum=5 or sum>9) = 4/36+6/36
= 5/18

(b) P(sum=5 or 2 odd numbers) = 4/36+1/4
=13/36

(c) P(sum=9 or 2 odd numbers) = 6/36+1/4-1/36
= 7/18

8. (a) (i)
{2,4,6,8,10,12,14,16,18,20}
P(A)=1/2

(ii)
{16,17,18,19,20}
P(B)=1/4

(iii)
{3,6,9,12,15,18}
P(C)=3/10

(iv)
{1, 2, 3, 4, 5, 6, 7, 8, 9}
P(D)=9/20

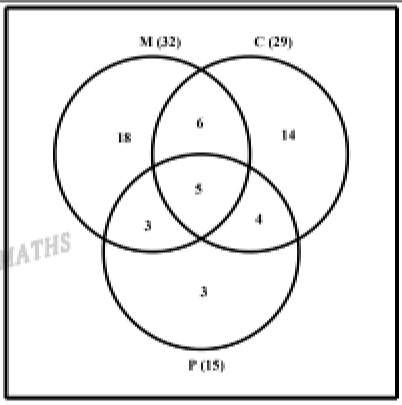
(b) (i) P(A or B) = 1/2+1/4-3/20
= 3/5

(ii) P(A or C) = 1/2+3/10-3/20
=13/20

(iii) P (B or D) = 1/4+9/20-0
= 7/10

9.

(a)

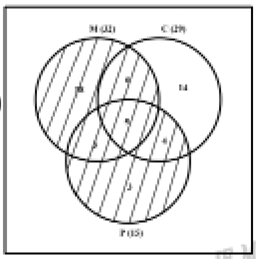


(b) (i) P(M or P) = P(M∪P)

= (18+6+5+3+4+3)/60

= 39/60

= 13/20

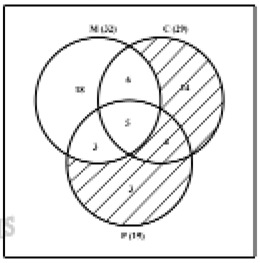


(ii) P(C or P or not M) = P(C∪P∪ $\overline{M}$)

= (14+4+3)/60

= 21/60

= 7/20

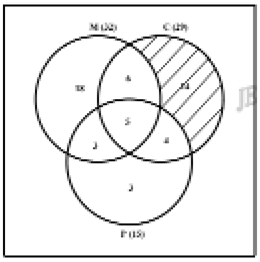


(iii) P(C or not P or not M) = P(C∪ $\overline{P}$ ∪ $\overline{M}$)

= 14/60

= 7/30

Notice: it is the same as finding P(C only)



- Set theory (multiplication law): $P(A\ and\ B) = P(A \cap B) = P(A) \times P(B \mid A)$

10.

(a) (i) P(RR) = P(R)×P(R|R)

= 16/30×16/30

= 64/225

(ii) P(RB) = P(R)×P(B|R)

= 16/30×14/30

= 56/225

(b) (i) P(RR) = P(R)×P(R|R)

= 16/30×15/29

= 8/29

(ii) P(RB) = P(R)×P(B|R)

= 16/30×14/29

= 112/435

JE MATHS

- Application of set theory (multiplication law) in probability:

11.

(a) P(all gold) = $6/12 \times 6/12 \times 6/12$
$= 1/8$

(b) P(all the same) = P(all gold)+P(all silver)+P(all bronze)
$= (1/2)^3 + 3/12 \times 3/12 \times 3/12 + 3/12 \times 3/12 \times 3/12$
$= (1/2)^3 + (3/12)^2 + (3/12)^2$
$= 5/32$

(c) P(2 gold 1 silver) = P(GGS)+P(GSG)+P(SGG)
$= 3 \times$ P(GGS)
$= 3 \times (6/12 \times 6/12 \times 3/12)$
$= 3/16$

(d) P(none are gold) = P(no gold) $\times$ P(no gold) $\times$ P(no gold)
$= (1-6/12) \times (1-6/12) \times (1-6/12)$
$= 1/2 \times 1/2 \times 1/2$
$= 1/8$

12.

(a) P(both yellow) = $8/50 \times 7/49$
$= 4/175$

(b) P(one is yellow) = P(yellow) $\times$ P(not yellow)+P(not yellow) $\times$ P(yellow)
$= 8/50 \times 42/49 + 42/50 \times 8/49$
$= 48/175$

(c) P(neither is yellow) = 1-P(both yellow)-P(one is yellow)
$= 1-4/175-48/175$
$= 123/175$

- Question involving multiple selections:

13.

(a) P(HHHH) = $(1/2)^4$
= 1/16

(b) P(TTTT) = $(1/2)^4$
= 1/16

(c) P(at least one head) = 1 – P(no head)
= 1 – 1/16
= 15/16

(d) P(first three coins are heads) = P(HHHH)+P(HHHT)
= 1/16+1/16
= 1/8

(e) P(middle two are tails) = P(HTTH)+P(HTTT)+P(TTTH)+P(TTTT)
= 1/16+1/16+1/16+1/16=1/4

(f)

No of heads	0	1	2	3	4
Probability	1/16	4/16	6/16	4/16	1/16

P(no head) = P(TTTT) = 1/16
P(one head) = P(HTTT)+P(THTT)+P(TTHT)+T(TTTH) = 4/16
P(two heads) = P(HHTT)+P(HTHT)+P(THHT)+P(HTTH)+P(THTH)+P(TTHH) = 6/16
P(three heads) = P(HHHT)+P(HHTH)+P(HTHH)+P(THHH) = 4/16
P(four heads) = P(HHHH) = 1/16
Notice: sum is 1.

JB MATHS

14.

(a) P(WWW) = $3/5 \times 3/5 \times 3/5$
= 27/125

(b) P(WWB)+P(WBW)+P(BWW) = $3/5 \times 3/5 \times 2/5 + 3/5 \times 2/5 \times 2/3 + 2/5 \times 2/3 \times 2/3$
= 542/1125

Notice: P(WWB)≠P(WBW)≠ P(BWW)

(c) P(WBB)+P(BWB)+P(BBW) = $3/5 \times 2/5 \times 1/3 + 2/5 \times 2/3 \times 1/3 + 2/5 \times 1/3 \times 3/4$
= 121/450

(d) P(BBB) = $2/5 \times 1/3 \times 1/4$
= 1/30

15. (a) (i) P(no six) = $5/6 \times 5/6$
= 25/36

(ii) 1 - P(no six) = 1 - 25/36
= 11/36
= 0.306

The purpose of this question is to show that when a die is thrown **2** times, the probability of obtaining **at least one six** is **less than 1/2**. ie 0.306<1/2

(b) (i) P(no six) = $5/6 \times 5/6 \times 5/6 \times 5/6$
= $(5/6)^4$

(ii) 1 - P(no six) = 1 - $(5/6)^4$
= 0.518

The purpose of this question is to show that when a die is thrown **4** times, the probability of obtaining **at least one six** is a **little more than 1/2**. ie 0.518>1/2

(c) (i) P(no double-six) =1 - $(1/6)^2$ = 35/36

(ii) 1 - P(no double-six) = 1 - $(35/36)^{24}$
= 0.491

The purpose of this question is to show that when two dice are thrown **24** times, the probability of obtaining **at least one six** is a **little less than 1/2**. ie, 0.491<1/2

It's worth noting that this well-known problem is also referred to as the Chevalier de Méré Problem, which led to the development of probability theory by Pascal and Fermat.

- Set theory (condition probability formula): $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{|A \cap B|}{|A|}$

16. $P(<3|\text{even}) = |\text{even} \cap <3| / |\text{even}|$

$= 1/3$

- Application of set theory (condition probability formula) in probability:

17.

(a) $P(B|A) = P(B \cap A)/P(A)$

$= (1/6)/(5/12)$

$= 2/5$

(b) $P(A|B) = P(A \cap B)/P(B)$

$= (1/6)/(1/2)$

$= 1/3$

Notice: $P(B \cap A) = P(A \cap B)$, but $P(B|A) \neq P(A|B)$

18.

(a) P(black hair | blue eyes) = P(blue eyes∩black hair)/P(blue eyes)

= 10%/30%

= 33%

(b) P(blue eyes | black hair) = P(blue eyes∩black hair)/P(black hair)

= 10%/20%

= 50%

Notice: $P(B|A) = P(A \cap B)/P(A)$ needs to be used, when $P(B|A) = |A \cap B|/|A|$ is not working.

19. $P(B|A) = P(B \cap A)/P(A)$

$= P(B \cap A)/P(A) \times P(B)/P(B)$

$= P(B \cap A)/P(B) \times P(B)/P(A)$ (Since $P(B \cap A)/P(B) = P(A|B)$)

$= P(A|B) \times P(B)/P(A)$ (Given that $P(A|B) = P(A)$)

$= P(A) \times P(B)/P(A)$

$= P(B)$

Notice: this proof is to show that if A is independent of B then B is also independent of A.

- Application of condition probability formula in tree diagram cases:

20.

(a) P(A wins on his second draw)
= P(A red, B red, A red)+P(A red, B white, A red)
= $3/8 \times 2/7 \times 1/6 + 3/8 \times 5/7 \times 2/6$
= $3/28$

(b) P(B wins on his second draw)
= P(A red, B red, A white, B red) + P(A white, B red, A red, B red)
+ P(A white, B red, A white, B red)
= $3/8 \times 2/7 \times 5/6 \times 1/5 + 5/8 \times 3/7 \times 2/6 \times 1/5 + 5/8 \times 3/7 \times 4/6 \times 2/5$
= $3/28$

(c) P(A|both win on his second draw) = $(3/28)/(3/28+3/28)$
= $1/2$

(d) P(A draws a red ball) = $3/8$
P(A draws a red ball∩neither player has won after two draws)
= P(A red, B red, A white, B white) + P(A red, B white, A white, B white)
+ P(A red, B white, A white, B red)
= $3/8 \times 2/7 \times 5/6 \times 4/5 + 3/8 \times 5/7 \times 4/6 \times 3/5 + 3/8 \times 5/7 \times 4/6 \times 2/5$
= $1/4$
P(neither player has won after two draws | A draws a red ball)
= $(1/4)/(3/8)$
= $2/3$

JE MATHS