Enrichment stage 1:

1. The formulae $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$ allows you to find the acute angle θ between two straight lines where m_1, m_2 are the gradients of each line. Use this formula to find the acute angle of:

   (a) y = 3x – 9 and 2x - 3y – 5 = 0, to the nearest degree.

   (b) 2x – y – 5 = 0 and x – 2y – 8 = 0 , to the nearest minute.

2. Given that a line can be written in the form: $(1+k)x + (k\sqrt{2} - \sqrt{2})y + (3-k) = 0$.

   (a) Find the value of k when this line is inclined at 45º to the positive direction of the x-axis.

   (b) Hence, show that the general equation of this line is $(2-\sqrt{2})x + (\sqrt{2}-2)y + \sqrt{2} = 0$.

3. The points A(1, 5), B(-1, 2) and C(2, -3) represent the triangle ABC.

(a) Show that the equation of the perpendicular bisector BC is $3x - 5y - 4 = 0$.

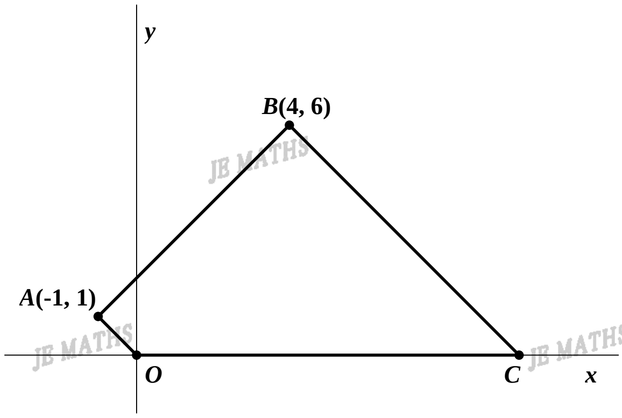
(b) If the equation of the perpendicular bisector AB is $4x + 6y - 21 = 0$, find the co-ordinates of the centre of the circle which passes through all the vertices of triangle ABC.

4. The lines $2x + y + 4 = 0$ and $x - 2y - 3 = 0$ intersect at point P.

(a) Write an equation which describes the set of lines which pass through point P.

(b) Find, in general form, the equations of the lines which pass through P such that their shortest distance to point A(2, -3) is $\sqrt{2}$ units.

5. In the diagram below, OABC is a trapezium with OA∥CB. The coordinates of O, A and B are (0, 0), (-1, 1) and (4, 6) respectively.



(a) Write down the gradient of the line OA.

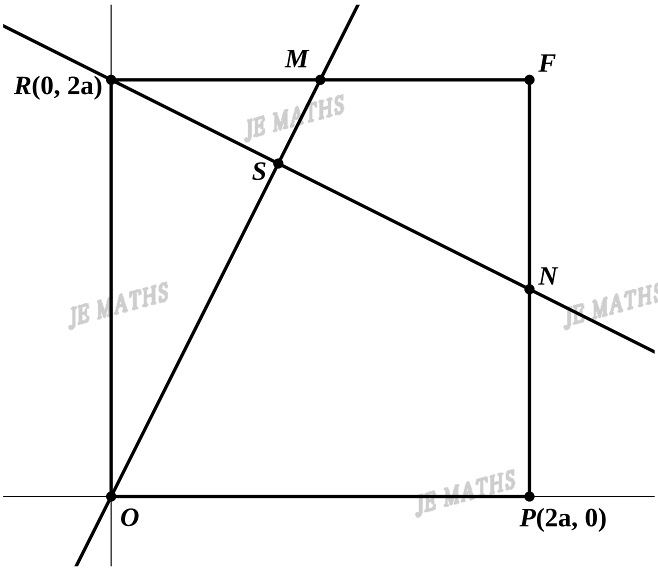
(b) Find the equation of the line BC.

(c) Show that the perpendicular distance from O to the line BC is $5\sqrt{2}$ by using the distance formula $d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

(d) Hence, or otherwise, calculate the area of the trapezium OABC.

Enrichment stage 2:

1. Given OPQR is a square where P(2a, 0) and R(0, 2a) as shown in the diagram below. The points M and N are the midpoints of sides QR and QP respectively. OM and RN intersect at S.



(a) Find the equation of line OM.

M(a, 2a)

mOM = 2a/a = 2

Equation of OM is y = 2x

(b) Show that RN is perpendicular to OM at S.

N(2a, a)

mRN = (2a - a) / (0 - 2a)

= a / -2a

= -1/2

mOM × mRN = 2 × (-1/2) = -1

∴OM ⊥RN at S.

(c) Show that △OPS is isosceles from P.

Equation of RN is y = 2a - x/2

For point S: 2a - x/2 = 2x

4a - x = 4x

x = 4a/5, y = 8a/5

S(4a/5, 8a/5)

SP² = (2a - 4a/5)² + (0 - 8a/5)²

= 36a²/25 + 64a²/25

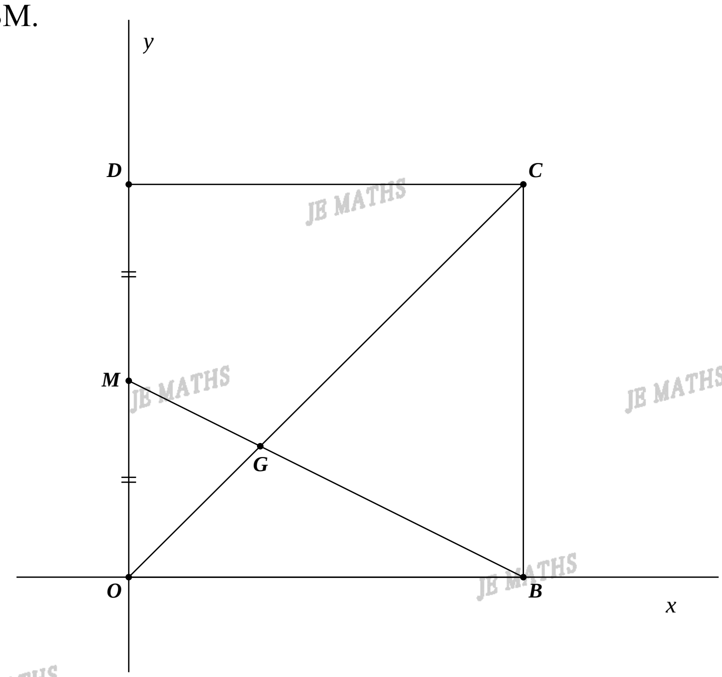
= 4a²

∴SP = 2a

As SP = OP = 2a

∴△OPS is isosceles from P.

2. Given OBCD is a square, where M is midpoint of OD. Point G is the point of intersection of diagonal OC with BM.



Find the ratio of |OGM| : |OBCD|, where |OGM| represents the area of △OGM, etc.

Let B = (a, 0), C = (a, a), D = (0, a).

∴M = (0, a/2)

∴Equation of OC: $y = x$

∴Equation of BM: $y = -x/2 + a/2$

$x = -x/2 + a/2$

$3x/2 = a/2$

$x = a/3$

∴G = (a/3, a/3)

∴|OGM| = $1/2 \times a/3 \times a/2 = a^2/12$ units²

|OBCD| = a^2 units²

|OGM| : |OBCD| = 1 : 12

Enrichment stage 3:

1. Tangents from the origin O touch the circle $\left(x-4\sqrt{3}\right)^2+\left(y-4\right)^2=16$ at two points.

(a) Prove that x axis is a tangent to the circle and write down the coordinates of A, the point of contact of the circle with the x axis.

Circle has radius 4, centre $(4\sqrt{3}, 4)$

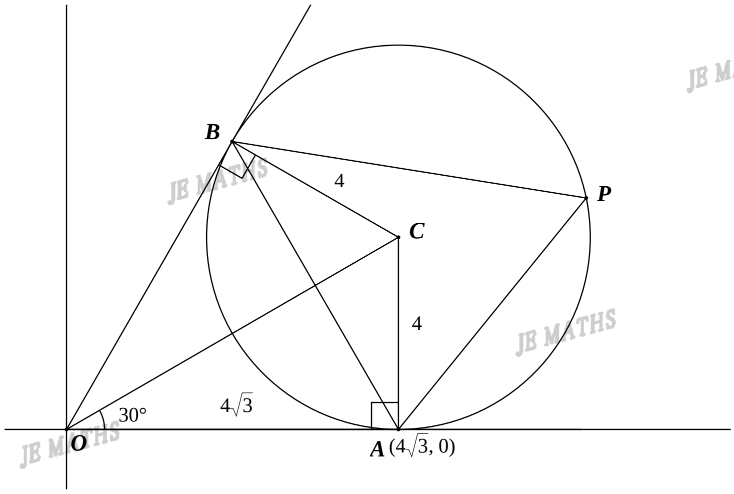
Distance from centre to $y = 0$ is $4/\sqrt{1} = 4$ which is the radius.

Thus, $y = 0$ is a tangent to the circle.

Point of contact has y coordinate of 0.

Therefore $A(4\sqrt{3}, 0)$

(b) The other tangent from O touches the circle at B. Show that the angle AOB is 60° and hence that triangle OAB is equilateral.



$\angle CAO = 90°$ (Radius forms 90° with tangent)

$\tan\angle COA = 4/4\sqrt{3} = 1/\sqrt{3}$

$\therefore \angle COA = 30°$

OC is common.

BC = AC (Same radii)

OB = OA (Tangents from any exterior point are equal)

$\triangle OBC \equiv \triangle OAC$ (SSS)

$\angle BOC = \angle AOC = 30°$ (Corresponding angles in congruent triangles are equal)

$\therefore \angle BOA = 30° + 30° = 60°$

But OB = OA so $\triangle OAB$ is isosceles.

Thus $\angle OBA = \angle OAB$ (Angles opposite equal sides are equal)

$\angle OAB + \angle OBA + \angle BOA = 180°$ (Angle sum of triangle AOB is 180°)

$\angle OAB + \angle OBA = 120°$

$\therefore \angle OAB = \angle OBA = \angle BOA = 60°$

Thus, $\triangle OAB$ is equilateral.

(c) P is a point on the major arc AB of the circle. Find the size of the angle APB.

$\angle BCA = 120°$ (Sum of angles of quadrilateral OACB is 360°)

$\angle APB = 60°$ (Angle at circumference is half angle at centre when standing on the same arc)

Enrichment stage 1:

1. (a)

$m_{BC} = -5/3$

$m \times m_{BC} = -1$

$m = 3/5$

$M_{BC} = (1/2, -1/2)$

Perpendicular bisector of BC:

$y + 1/2 = 3/5 \times (x - 1/2)$

$10y + 5 = 6x - 3$

$6x - 10y - 8 = 0$

$3x - 5y - 4 = 0$

(b)

$3x - 5y - 4 = 0$ (1)

$4x + 6y - 21 = 0$ (2)

$3 \times (2) - 4 \times (1)$:

$12x + 18y - 63 - (12x - 20y - 16) = 0$

$38y = 47$

$y = 47/38$

Substitute $y = 47/38$ into (1):

$3x - 5(47/38) - 4 = 0$

$x = 129/38$

Centre $(129/38, 47/38)$

2. (a) Lines through P are $2x + y + 4 + k(x - 2y - 3) = 0$ where k is real.

(b)

Equation in general form is $(2 + k)x + (1 - 2k)y + (4 - 3k) = 0$

$|(2 + k)(2) + (1 - 2k)(-3) + 4 - 3k| / \sqrt{[(2 + k)^2 + (1 - 2k)^2]} = \sqrt{2}$

$(4 + 2k - 3 + 6k + 4 - 3k)^2 = 2(4 + 4k + k^2 + 1 - 4k + 4k^2)$

$(5k + 5)^2 = 10k^2 + 10$

$25k^2 + 50k + 25 = 10k^2 + 10$

$5k^2 + 10k + 5 = 2k^2 + 2$

$3k^2 + 10k + 3 = 0$

$(3k + 1)(k + 3) = 0$

$k = -1/3, -3$

$5x/3 + 5y/3 + 5 = 0$

$-x + 7y + 13 = 0$

Therefore lines are $x + y + 3 = 0$ and $x - 7y - 13 = 0$.

3. (a)

m_1=3

m_2=-a/b=-(-2)/3=2/3

tanϴ=|(3-2/3)/(1+3×2/3)|=7/9

ϴ=tan^{-1}(7/9)=38˚

(b)

m_1=-a/b=-2/-1=2

m_2=-a/b=-1/-2=1/2

tanϴ=|(2-1/2)/(1+2×1/2)|=3/4

ϴ=tan^{-1}(3/4)= 36°52'

4. (a)

m = -a/b = -(k√2-√2)/(1+k) = (-k√2+√2)/(1+k)

tan45˚ = 1

(-k√2+√2)/(1+k) = 1

-k√2+√2=1+k

√2-1 = k+k√2

√2-1 = k(1+√2)

k = (√2-1)/(√2+1)

= (√2-1)²

= 3-2√2

(b) sub k = 3-2√2 in

a = 1+3-2√2 = 4-2√2=2(2-√2),

b = (3-2√2)√2-2 = 3√2-4-√2 = 2√2-4 = 2(√2-2),

c = 3-(3-2√2) = 2√2

2(2-√2)x + 2(√2-2)y + 2√2=0

÷2: (2-√2)x + (√2-2)y + √2=0

(a)
$m_{OA} = (1 - 0) / (-1 - 0) = -1$

(b)
Since CB∥OA, line BC has gradient -1.
$y - 6 = -1(x - 4)$
$y - 6 = -x + 4$
$x + y - 10 = 0$

(c)
$d = |1(0) + 1(0) - 10| / \sqrt{(1^2 + 1^2)}$
$= |-10| / \sqrt{2}$
$= 10\sqrt{2} / 2$
$= 5\sqrt{2}$ units

(d)
$d_{OA} = \sqrt{[(-1 - 0)^2 + (1 - 0)^2]}$
$= \sqrt{2}$
When $y = 0$, BC has intercepts (0, 10).
$d_{BC} = \sqrt{[(10 - 4)^2 + (0 - 6)^2]}$
$= \sqrt{72}$
$= 6\sqrt{2}$
$A = 1/2 \times 5\sqrt{2} \times (6\sqrt{2} + \sqrt{2})$
$= 35$ units²

Enrichment stage 2:

1. (a)

   M(a, 2a)

   $m_{OM} = 2a/a = 2$

   Equation of OM is $y = 2x$

   (b)

   N(2a, a)

   $m_{RN} = (2a - a) / (0 - 2a)$

   $= a / -2a$

   $= -1/2$

   $m_{OM} \times m_{RN} = 2 \times (-1/2) = -1$

   $\therefore$OM $\perp$RN at S.

   (c)

   Equation of RN is $y = 2a - x/2$

   For point S: $2a - x/2 = 2x$

   $4a - x = 4x$

   $x = 4a/5$, $y = 8a/5$

   S(4a/5, 8a/5)

   $SP^2 = (2a - 4a/5)^2 + (0 - 8a/5)^2$

   $= 36a^2/25 + 64a^2/25$

   $= 4a^2$

   $\therefore SP = 2a$

   As SP = OP = 2a

   $\therefore\triangle$OPS is isosceles from P.

2. Let B = (a, 0), C = (a, a), D = (0, a).

   $\therefore$M = (0, a/2)

   $\therefore$Equation of OC: $y = x$

   $\therefore$Equation of BM: $y = -x/2 + a/2$

   $x = -x/2 + a/2$

   $3x/2 = a/2$

   $x = a/3$

   $\therefore$G = (a/3, a/3)

   $\therefore|OGM| = 1/2 \times a/3 \times a/2 = a^2/12$ units²

   $|OBCD| = a^2$ units²

   $|OGM| : |OBCD| = 1 : 12$

Enrichment stage 3:

1. (a)

Circle has radius 4, centre $(4\sqrt{3}, 4)$

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$\angle OAB + \angle OBA = 120^\circ$

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Thus, $\triangle OAB$ is equilateral.

(c)

$\angle BCA = 120^\circ$ (Sum of angles of quadrilateral OACB is 360º)

$\angle APB = 60^\circ$ (Angle at circumference is half angle at centre when standing on the same arc)