Foundation stage 1:

1. Evaluate without using a calculator:

   (a) 3^4 (b) $\left(\dfrac{2}{5}\right)^3$

2. Evaluate without using a calculator:

   (a) 1^0 (b) 0.9^1

3. Evaluate involving negative indices without using a calculator:

   (a) 7^{-1} (b) $\left(\dfrac{2}{5}\right)^{-1}$

   (c) 6^{-3} (d) $\left(\dfrac{3}{4}\right)^{-3}$

4. Simplify the following involving multiplication in index form:

   (a) $4^8 \times 4^4$ (b) $x^5 \times x^7$

   (c) $9^3 \times 9^{-8}$ (d) $x^8 \times x^{-4} \times x^2$

5. Simplify the following involving division in index form:

   (a) $4^8 \div 4^4$ (b) $x^5 \div x^7$

   (c) $9^3 \div 9^{-8}$ (d) $x^8 \div x^{-4} \div x^2$

6. Expand the following involving power of power in index form:

(a) $x^2 \times x^2 \times x^2$

(b) $x^{-3} \times x^{-3} \times x^{-3}$

(c) $(x^3)^9$

(d) $(x^{-4})^3$

(e) $(x^{-5})^4$

(f) $(x^{-8})^{-3}$

7. Expand the following expressions in index form:

(a) $(4x)^3$

(b) $(3xy)^5$

(c) $\left(\dfrac{1}{x}\right)^3$

(d) $\left(\dfrac{2y}{3x}\right)^4$

8. Solve the following index equations:

(a) $4^x = 64$

(b) $4^x = \dfrac{1}{64}$

(c) $16^x = 32$

(d) $16^x = \dfrac{1}{32}$

(e) $(\dfrac{1}{3})^x = 81$

(f) $(\dfrac{4}{9})^x = \dfrac{27}{8}$

Foundation stage 2:

1. Simplify the following into a fraction, if necessary:

(a) $9^{\frac{1}{2}}$

(b) $\left(\dfrac{1}{25}\right)^{\frac{1}{2}}$

(c) $1000000^{\frac{1}{3}}$

(d) $\left(\dfrac{16}{81}\right)^{\frac{1}{4}}$

(e) $125^{\frac{2}{3}}$

(f) $\left(\dfrac{125}{8}\right)^{\frac{4}{3}}$

2. Evaluate the following by using the calculator, correct to 3 siginificant figures:

(a) $7^{\frac{1}{3}}$

(b) $\left(\dfrac{2}{5}\right)^{-\frac{5}{2}}$

3. Simplify the following expressions by using index laws:

(a) $x^{\frac{1}{2}} \times x^{\frac{1}{2}}$

(b) $x^{1\frac{1}{2}} \div x^{\frac{1}{2}}$

(c) $x^{1\frac{1}{2}} \div x^{-\frac{1}{2}}$

(d) $x^{\frac{3}{2}} \div x^{-\frac{1}{2}} \times x^{\frac{1}{2}}$

(e) $(x^{\frac{1}{2}})^{\frac{1}{2}}$

(f) $(x^{\frac{1}{2}})^{-\frac{6}{5}}$

(g) $(x^{-\frac{2}{3}})^{6}$

(h) $(x^{-1\frac{1}{2}})^{-\frac{1}{2}}$

4. Change the following index numbers into surds:

(a) $x^{\frac{1}{2}}$

(b) $6x^{\frac{1}{3}}$

(c) $x^{\frac{5}{3}}$

(d) $6x^{-\frac{2}{3}}$

5. Change the following surds into index numbers:

(a) $\sqrt{x}$

(b) $\sqrt[3]{x}$

(c) $3\sqrt{x}$

(d) $\sqrt{3x}$

(e) $\dfrac{3}{\sqrt{x}}$

(f) $\dfrac{1}{\sqrt[3]{3x}}$

6. Solve the following index equations:

(a) $16^x = 2$

(b) $\left(\dfrac{1}{9}\right)^x = \dfrac{1}{243}$

(c) $25^x = \sqrt{5}$

(d) $\left(\dfrac{1}{36}\right)^x = \sqrt[3]{6}$

Foundation stage 3:

1. Change the following into index form and then find the value of x:

(a) $\log_2 8 = x$

(b) $\log_{10} \dfrac{1}{10} = x$

(c) $\log_3 1 = x$

(d) $\log_8 32 = x$

(e) $x = \log_9 9$

(f) $x = \log_{32} \sqrt{64}$

2. Change the following into index form and then find the value of x:

(a) $\log_8 x = 3$

(b) $\log_x 8 = 3$

(c) $\log_6 x = -2$

(d) $\log_x \dfrac{1}{27} = 3$

(e) $\log_4 x = -\dfrac{5}{2}$

(f) $\log_x \dfrac{1}{64} = \dfrac{3}{2}$

3. Use the log law $\log_a x + \log_a y = \log_a xy$, $\log_a x - \log_a y = \log_a \dfrac{x}{y}$ and $\log_a x^n = n\log_a x$

to simplify:

(a) $\log_6 54 + \log_6 4$

(b) $\log_8 128 - \log_8 2$

(c) $\log_4 \dfrac{1}{9} + \log_4 9$

(d) $\log_2 4 + \log_2 6 - \log_2 \dfrac{3}{4}$

4. Write the following in terms of $\log_a 2$:

(a) $\log_a 4$

(b) $\log_a \dfrac{1}{2}$

(c) $\log_a \dfrac{1}{64}$

(d) $\log_a \dfrac{1}{\sqrt{32}}$

5. Express the following in terms of $\log_2 3$ and $\log_2 5$: (hint: $\log_2 2 = 1$)

(a) $\log_2 15$

(b) $\log_2 6$

(c) $\log_2 30$

(d) $\log_2 7.5$

Foundation stage 1:

1. (a)
   81

   (b)
   8/125

2. (a)
   1

   (b)
   0.9

3. (a)
   1/7

   (b)
   5/2

   (c)
   1/216

   (d)
   $(4/3)^3$
   $= 64/27$

4. (a)
   4^{12}

   (b)
   x^{12}

   (c)
   9^{-5}

   (d)
   x^6

5. (a)
   4^{12}

   (b)
   x^{-2}

   (c)
   $9^{3-(-8)}$
   $= 9^{11}$

   (d)
   $x^{8-(-4)-2}$
   $= x^{10}$

6. (a)
   x^6

   (b)
   x^{-9}

   (c)
   x^{27}

   (d)
   x^{-12}

   (e)
   x^{-20}

   (f)
   x^{24}

7. (a)
   64x³

   (b)
   $243x^5y^5$

   (c)
   1/x³

   (d)
   $16y^4/81x^4$

8. (a)
   $4^x = 4^3$
   x = 3

   (b)
   $4^x = 4^{-3}$
   x = -3

(c)
$2^{4x} = 2^5$
4x = 5
x = 5/4

(d)
$2^{4x} = 2^{-5}$
4x = -5
x = -5/4

(e)
$3^{-x} = 3^4$
-x = 4
x = -4

(f)
$(2/3)^{2x} = (2/3)^{-3}$
2x = -3
x = -3/2

Foundation stage 2:

1. (a)
   3

   (b)
   1/5

   (c)
   100

   (d)
   4/3

   (e)
   $5^2 = 25$

   (f)
   $(5/2)^4 = 625/16$

2. (a) $7^{\frac{1}{3}}$
   1.91

   (b)
   9.88

3. (a)
   x

   (b)
   x

   (c)
   x^2

   (d)
   $x^{3/2+1/2+1/2}$
   $= x^{5/2}$

   (e)
   $x^{1/4}$

   (f)
   $x^{-3/5}$

   (g)
   x^{-4}

   (h)
   $x^{3/4}$

4. (a)
   $\sqrt{x}$

   (b)
   $6\sqrt[3]{x}$

   (c)
   $\sqrt[3]{x^5}$

   (d)
   $6/\sqrt[3]{x^2}$

5. (a)
   $x^{1/2}$

   (b)
   $x^{1/3}$

   (c)
   $3x^{1/2}$

   (d)
   $3^{1/2}x^{1/2}$

   (e)
   $3x^{-1/2}$

   (f)
   $3^{-1/3}x^{-1/3}$

6. (a)
   $2^{4x}=2^1$

   (b)
   $3^{-2x}=3^{-5}$

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$4x = 1$

$x = 1/4$

$-2x = -5$

$x = 5/2$

(c)

$5^{2x}=5^{1/2}$

$2x = 1/2$

$x = 1/4$

(d)

$6^{-2x}=6^{1/3}$

$-2x = 1/3$

$x = -1/6$

Foundation stage 3:

1. (a)
    $2^x=8$
    x=3

    (b)
    $10^x=10^{-1}$
    x = -1

    (c)
    $3^x=1$
    x=0

    (d)
    $8^x=32$
    $2^{3x}=2^5$
    3x = 5
    x = 5/3

    (e)
    $9^x=9$
    x = 1

    (f)
    $32^x=64^{1/2}$
    $2^{5x}=2^{6/2}$
    5x = 3
    x = 3/5

2. (a)
    x = 8³ = 512

    (b)
    x³ = 8
    x = 2

    (c)
    x = 6^{-2}
    x = 1/36

    (d)
    x³ = 1/27
    x = 1/3

    (e)
    x = $4^{-5/2}$
    x = 1/32

    (f)
    $x^{3/2}$ = 1/64
    x = $(1/64)^{2/3}$
    x = (1/4)²
    x = 1/16

3. (a)
    $log_6 216$
    $= log_6 6^3$
    $= 3$

    (b)
    $log_8 64$
    $= log_8 8^2$
    $= 2$

    (c)
    $log_4 1$
    $= 0$

    (d)
    $log_2(4×6÷3/4)$
    $= log_2 32$
    $= log_2 2^{25}$
    $= 5$

4. (a)
    $2log_a 2$

    (b)
    $-log_a 2$

    (c)

    (d)

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$-6log_a2$ $-5/2 \times log_a2$

5. (a)
log_23+log_25

(b)
log_23+log_22
$= log_23+1$

(c)
$log_23+log_25+log_22$
$= log_23+log_25+1$

(d)
$log_23+log_25-log_22$
$= log_23+log_25-1$