- Involving circle properties.

1. (a)

$\angle B = 90^\circ$ (angles in a semicircle)

$AC = 2 \times 7 = 14$cm

$\cos\Theta = 8/14 = 4/7$

$\Theta = \cos$^-1(4/7)

$= 55^\circ 9' 0.34''$

$= 55^\circ 9'$

(b)

$\angle OTP = 90^\circ$ (radius is perpendicular to the tangent)

$\tan\Theta = \sqrt{3}/3$

$\Theta = 30^\circ$

2. (a)

$\angle B = 90^\circ$ (angles in a semicircle)

$\sin 60^\circ = \sqrt{2}/AC$

$AC = \sqrt{2}/\sin 60^\circ$

$= \sqrt{2}/(\sqrt{3}/2)$

$= 2\sqrt{2}/\sqrt{3}$

$AO = 2\sqrt{2}/\sqrt{3}$

$= \sqrt{2}/\sqrt{3}$

$= \sqrt{6}/3$ cm

(b) Area of the circle $= \pi \times (\sqrt{2}/\sqrt{3})^2 = 2\pi/3$ cm ^{2}

(c)

ΔABO is an equilateral triangle, ie $AB = AO = \sqrt{2}/\sqrt{3}$ cm

Area of $\Delta ABC = \frac{1}{2} \times \sqrt{2}/\sqrt{3} \times \sqrt{2} = 1/\sqrt{3}$ cm²

Area ratio = ΔABC: Circle O

$= 1/\sqrt{3} : 2\pi/3$

$= \sqrt{3}/3 : 2\pi/3$

$= \sqrt{3} : 2\pi$

3.

(a) (i)

sinΘ= TA/1

TA = sinΘ

(ii)

TB = OA

cosΘ=OA/1

OA = cosΘ = TB

(iii)

∠OTP = 90˚ (radius is perpendicular to the tangent)

tanΘ=TP/1

TP = tanΘ

(iv)

cosΘ = 1/OP

OP = 1/cosΘ

= secΘ

(v)

cos(90-Θ) = 1/QO (complementary identities)

sinΘ = 1/QO

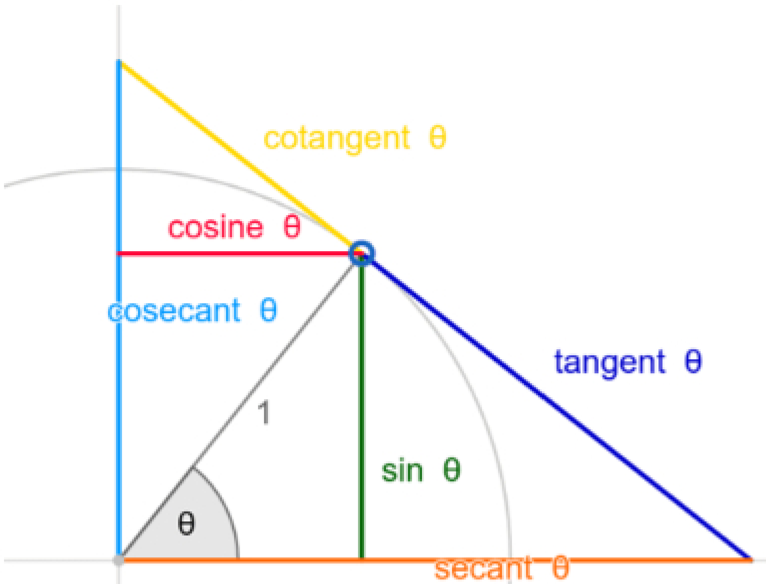
QO = 1/sinΘ = cosecΘ

(vi)

tan(90-Θ) = QT/1 (complementary identities)

cotΘ = QT

(b) Hence, put all 6 trig ratios on the sides of the given graph by using 6 different colours.



- Involving sine rule and the area formula: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ and $A = \frac{1}{2}ab\sin C$

4.

(a)

$x/\sin 60^\circ = 10/\sin 45^\circ$

$x = 10\sin 60^\circ/\sin 45^\circ$

$= 10\times\sqrt{3}/2/(\sqrt{2}/2)$

$= 10\sqrt{3}/\sqrt{2}$

$= 5\sqrt{6}$ cm

(b) Hence, find the area of ΔABC. (2dps)

$\angle A = 180^\circ - 45^\circ - 60^\circ = 75^\circ$

Area $= \frac{1}{2}\times 5\sqrt{6}\times 10\times\sin 75^\circ$

$= 59.15\ldots$

$= 59.2$ cm²

5. $\angle C = 180^\circ - 75^\circ - 45^\circ = 60^\circ$

$BC/\sin 45^\circ = 1/\sin 60^\circ$

$BC = \sin 45^\circ/\sin 60^\circ$

$= (\sqrt{2}/2)/(\sqrt{3}/2)$

$= \sqrt{2}/\sqrt{3}$

$= \sqrt{6}/3$ km

6. Let $\angle BDC = \theta$, $\angle BDA = 180 - \theta$

$\sin\theta/x = \sin 45^\circ/1$

$\sin\theta = x\sin 45^\circ$ 1)

$\sin(180^\circ - \theta)/\sqrt{3} = \sin 60^\circ/\sqrt{2}$

$\sin(180^\circ - \theta) = \sqrt{3}\sin 60^\circ/\sqrt{2}$ 2)

since $\sin\theta = \sin(180^\circ - \theta)$

1) and 2): $x\sin 45^\circ = \sqrt{3}\sin 60^\circ/\sqrt{2}$

$x = \sqrt{3}\sin 60^\circ/\sqrt{2}\sin 45^\circ$

$= \sqrt{3}/\sqrt{2}\times(\sqrt{3}/2)/(\sqrt{2}/2)$

$= 3/2$ cm

7.

(a) $180^\circ \div (1+2+2) = 36^\circ$

ans: 36°, 72, 72°

(b) (i)

$x/\sin72^\circ = 1/\sin36^\circ$

$x = \sin72^\circ/\sin36^\circ$

$= 1.6180\ldots$

$= 1.618$

Notice: x represents the golden ratio.

(ii)

$x = \sin72^\circ/\sin36^\circ$

$= \sqrt{(10+2\sqrt{5})}/\sqrt{(10-2\sqrt{5})}$

$= \sqrt{[(10+2\sqrt{5})/(10-2\sqrt{5})]}$

$= \sqrt{[(5+\sqrt{5})/(5-\sqrt{5})]}$

$= \sqrt{[(5+\sqrt{5})^2/20]}$

$= \sqrt{[(3+\sqrt{5})/2]}$

$= \sqrt{[(6+2\sqrt{5})/4]}$

$= \sqrt{[(1+\sqrt{5})^2/4]}$

$= (1+\sqrt{5})/2$

(iii)

$A = 1/2 \times [(1+\sqrt{5})/2]^2 \times \sin36^\circ$

$= 1/2 \times (1+\sqrt{5})^2/4 \times \sqrt{(10-2\sqrt{5})}/4$

$= 1/32 \times (6+2\sqrt{5}) \times \sqrt{(10-2\sqrt{5})}$

$= 1/16 \times (3+\sqrt{5}) \times \sqrt{(10-2\sqrt{5})}$

$= 1/16 \times \sqrt{[(3+\sqrt{5})^2(10-2\sqrt{5})]}$

Since $(3+\sqrt{5})^2(10-2\sqrt{5}) = 80+32\sqrt{5}$

$= 1/16 \times \sqrt{(80+32\sqrt{5})}$

$= 1/16 \times 4\sqrt{(5+2\sqrt{5})}$

$= \sqrt{(5+2\sqrt{5})}/4$

- Involving cosine rule: $\cos A = \dfrac{b^2+c^2-a^2}{2bc}$.

8. $x^2 = 2^2+(\sqrt{3})^2-2\times2\times\sqrt{3}\times\cos150^\circ$

$= 4+3-4\sqrt{3}\cos 150^{\circ}$ $\quad (\cos 150^{\circ}=-\cos 30^{\circ}= -\sqrt{3}/2)$

$= 7-4\sqrt{3}\times(-\sqrt{3}/2)$

$= 7+ 6$

$x^2= 13$

$x = \sqrt{13}$ cm (omit -)

9. $4^2=2^2+x^2-2\times 2\times x\times \cos 45^{\circ}$

$16=4+x^2-4x\times\sqrt{2}/2$

$12=x^2-2\sqrt{2}x$

$x^2-2\sqrt{2}x-12=0$

$\Delta=(2\sqrt{2})^2-4\times 1\times(-12)=56$

$x = (2\sqrt{2}\pm\sqrt{56})/2$

$= (2\sqrt{2}\pm 2\sqrt{14})/2$

$= (\sqrt{2}+\sqrt{14})$ cm (omit -)

10. (a)

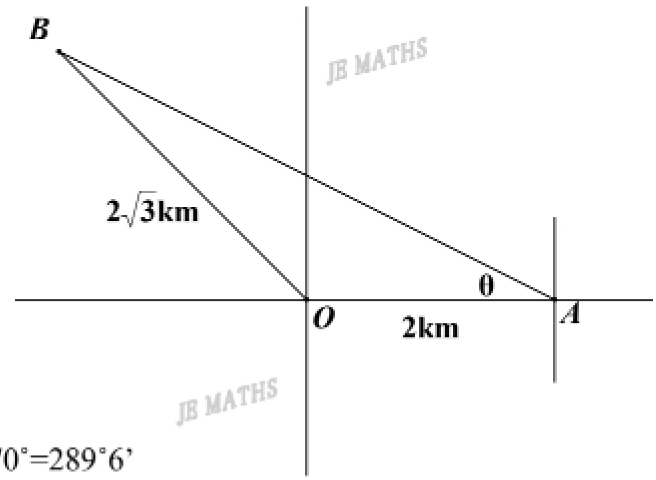
$\angle ABO = (360-300)+90 = 150^{\circ}$

$AB^2 = 2^2 + (2\sqrt{3})^2 - 2\times 2\times 2\sqrt{3}\times\cos 150^{\circ}$

$= 16- 8\sqrt{3}\times(-\sqrt{3}/2)$ $\quad (\cos 150^{\circ}=-\cos 30^{\circ}=-\sqrt{3}/2)$

$= 28$

$AB = \sqrt{28}=2\sqrt{7}$km



(b)

Let $\angle BAO = \theta$

$\cos\theta=[2^2+(2\sqrt{7})^2-(2\sqrt{3})^2]/(2\times 2\times 2\sqrt{7})$

$= 20/(8\sqrt{7})$

$= 5/(2\sqrt{7})$

$\theta=19^{\circ}6'23.78''$

$= 19^{\circ}6'$

The bearing of B from A is about $19^{\circ}6'+270^{\circ}=289^{\circ}6'$

11.

(a)

Let BD = x

$\cos\angle C=(5^2+6^2-x^2)/(2\times 5\times 6)$

$=(61-x^2)/60$ $\qquad$ 1)

$\cos\angle A=(5^2+1^2-x^2)/(2\times5\times1)$

$= (26-x^2)/10$ 2)

since $\cos\angle C = \cos(180^\circ-\angle A) = -\cos\angle A$

(opposite angles in cyclic quadrilateral)

1) and 2: $(61-x^2)/60 = -(26-x^2)/10$

$(61-x^2)/6 = -(26-x^2)$

$61-x^2= -6(26-x^2)$

$61-x^2= -156+6x^2$

$217 = 7x^2$

$31 = x^2$

$x = \sqrt{31}$ (omit -)

(b)

$\cos\angle C=(61-x^2)/60$

$=(61-31)/60$

$= 30/60$

$= 1/2$

$\angle C = 60^\circ$

(c)

$\angle A= 180^\circ-\angle C = 180^\circ-60^\circ = 120^\circ$

Area (ABCD) = Area (ΔBCD) + Area (ΔBAD)

$= 1/2\times5\times6\times\sin60^\circ+1/2\times5\times1\times\sin120^\circ$ ($\sin120^\circ = \sin(180^\circ-60^\circ)=\sin60^\circ=\sqrt{3}/2$)

$= 1/2\times5\times6\times\sqrt{3}/2+1/2\times5\times1\times\sqrt{3}/2$

$= 1/2\times5\times7\times\sqrt{3}/2$

$= 35\sqrt{3}/4$ cm²

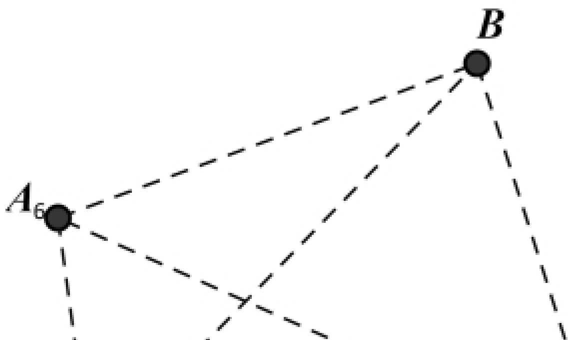
12.

(a) In ΔAPQ,

$\angle PAQ=180^\circ-100^\circ-40^\circ = 40^\circ$

ΔAPQ is an isosceles Δ.

AP = PQ = 1000m



(b) In ΔBPQ,

$\angle PBP = 180^\circ - 60^\circ - 75^\circ = 45^\circ$

By using the sine rule,

$PB/\sin\angle PQB = PQ/\sin\angle PBQ$

$PB/\sin75^\circ = 1000/\sin45^\circ$

$PB = 1000\sin75^\circ/\sin45^\circ$

$= 1366.025404\ldots$

$= 1366.03m$

(c) In ΔAPB,

$\angle APB = 100^\circ - 60^\circ = 40^\circ$

By using the cosine rule,

$AB^2 = AP^2 + PB^2 - 2AP \times PB\cos\angle APB$

$= 1000^2 + 1366.03^2 - 2(1000)(1366.03)\cos40^\circ$

$AB = 879.29\ldots$

$= 879.3m$

- 3D trig application:

13. (a)

$\cos A = (4^2 + 3^2 - 2^2)/(2 \times 4 \times 3)$

$\cos A = 21/24 = 7/8$

$\angle A = \cos^{-1}(7/8)$

(b)

$\sin\angle B/3=\sin\angle A/2$

$\sin\angle B=3\sin\angle A/2$

$= 3\sin(\cos^{-1}(7/8))/2$

$\angle B=46°34'2.87''=46°34'$

14. (a)

(i) $EG = \sqrt{(2^2+1^2)}=\sqrt{5}$cm

(ii) $ED= \sqrt{(2^2+1^2)}=\sqrt{5}$cm

(iii) $DG = \sqrt{(2^2+2^2)}=\sqrt{8}=2\sqrt{2}$cm

(b)

$\cos\angle EDG =[(\sqrt{5})^2+(2\sqrt{2})^2-(\sqrt{5})^2]/[2\times(\sqrt{5})\times(2\sqrt{2})]$

$= (5+8-5)/4\sqrt{10}$

$= 2/\sqrt{10}$

$= \sqrt{10}/5$

$\angle EDG =\cos^{-1}(\sqrt{10}/5) = 50°46'6.53''=50°46'$

(c)

ΔEDG is an isosceles triangle.

$\angle EDG = \angle EGD$ (equal sides opposite to equal angles)

$\angle DEG = 180° - 2\angle EDG$

$= 180° - 2\times[\ \cos^{-1}(\sqrt{10}/5)\ ]$

$= 78°27'46.95''$

$= 78°28'$

(d)

In ΔDEI,

$\sin 78°28' = DI/\sqrt{5}$

$DI=\sqrt{5}\sin 78°28'$

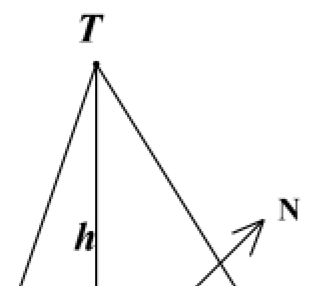
$= 2.190918525$

$= 2.2$ cm

15. (a)

$\tan 44° = h/AB$

$AB = h/\tan 44° = h\cot 44°$



(b)

$\tan 28^\circ = h/AC$

$AC = h/\tan 28^\circ = h\cot 28^\circ$

(c)

Since $BC^2 = AB^2 + AC^2 - 2AB \times AC\cos 95^\circ$

$121 = h^2\cot^2 44^\circ + h^2\cot^2 28^\circ - 2\,h\cot 44^\circ \times h\cot 28^\circ\cos 95^\circ$

$121 = h^2(\cot^2 44^\circ + \cot^2 28^\circ - 2\cot 44^\circ\cot 28^\circ\cos 95^\circ)$

$h^2 = 121/(\cot^2 44^\circ + \cot^2 28^\circ - 2\cot 44^\circ\cot 28^\circ\cos 95^\circ)$

(d)

$h = 4.944664142$

$= 4.94\text{m} = AT$

ans: the height of this mast *AT* is 4.94m.

(e)

$AB = 4.94/\tan 44^\circ = 5.115\ldots = 5.12$

$AC = 4.94/\tan 28^\circ = 9.290\ldots = 9.29$

$BC = 11$

$\cos\angle ABC = (AB^2 + BC^2 - AC^2)/(2 \times AB \times BC)$

$= (5.12^2 + 11^2 - 9.29^2)/(2 \times 5.12 \times 11)$

$\angle ABC = 57.26\ldots = 57.3^\circ$

ans: the bearing of *C* from *B* is 57.3˚.

(f) (i)

$\sin 57.3^\circ = AD/AB$

$\sin 57.3^\circ = AD/5.12$

$AD = 5.12\sin 57.3^\circ$

$= 4.308\ldots$

$= 4.3$ m

(ii)

$\tan\angle ADT = AT/AD$

$= 4.94/(5.12\sin 57.3^\circ)$

$\angle ADT = 48.90593776 = 49^\circ$

ans: the greatest angle of elevation of *T* from any point along *BC* is 49˚.