

Stage 1:

1. (2, 2)
(0) (-1)
(1) (-2)
(3) (-3)
2. (2) (-1.5)
(-2) (0.5)
3. (0) (0)
(5) (-7)
(5/2) (-21/4)
4. (0)
(-1)
(-56)
(2/a)
 $Q(S - R)/RS$

Stage 2:

1. (0, 3), (1, 2r2), (-1, 2r2), (2, r5), (2, -r5);
(0) (-r2/4)
(r2/4) (-2/r5)
(2/r5);
(r(9-x²)/x)
(x = -3r2/2)
2. C(0, -2), r = 4
(-r(16 - x²)/x)
(x/r(16 - x²))
(-r15/15)
(r3/3)
(-3r7/7)
(-2r2, 2r2)
3. C(1, 0), r = 5
(r[25 - (x - 1)²]/(x - 1))
(1 - x)/r(25 - (x - 1)²)
(3x + 4y - 28 = 0)
(x = 1 - r5)
4. (5, 0), r = 4, lower semicircle
(x - 5)/r[16 - (x - 5)²]
5. (4, 7), r = 3, upper
(4 - x)/r[9 - (4 - x)²]
6. (-1, 2), r = r3, lower
(1 + x)/r[3 - (1 + x)²]
7. (2, 9), r = 2, lower
(x - 2)/r(4x - x²)

Stage 3:

1. (x = a and x = (m - 3a)/3)
(m = 6a)
(f'(x) = 6x);
(x = m/6)
(f'(x) = 6x);
2. (x = (1 + m)/6)
(f'(x) = 6x - 1)
3. (x = m/2K)
f'(x) = 2Kx
4. (x = (m - B)/2A)
(f'(x) = 2Ax + B)
5. (x = -b/2m)
(b = +- r(-4m))
(-1/x²)

Stage 1:

1. (i) $m = \frac{8}{4} = 2$

$$f'(1) = 2$$

(ii) $f'(0) = 0$

$$f'(0.5) = \frac{3}{3} = 1$$

$$f'(1.5) = \frac{6}{2} = 3$$

2. $f'(0) = \frac{4}{2} = 2$

$$f'(1) = \frac{4}{2} = 2$$

3. (a) $f'(x) = 0$

(c) $f'(x) = 5$

(e) $f'(x) = \frac{5}{2}$

4. (a) $f(x) = 3 - 6x - 2 + 6x = 1$

$$f'(x) = 0$$

(b) $f(x) = \frac{1}{2}(3 - 11x - 5 + 9x) = -1 - x$

$$f'(x) = -1 - 0 = -1$$

(c) $f(x) = 4x(-14) = -56x$

$$f'(x) = -56$$

(d) $f(x) = \frac{2x - b - c}{a}$

$$f'(x) = \frac{2}{a}$$

(e) $f(x) = \frac{PS + SQx + PR - QRx}{RS}$

$$f'(x) = \frac{SQ - QR}{RS}$$

$$f'(-0.5) = \frac{-3}{3} = -1$$

$$f'(-1) = \frac{-4}{2} = -2$$

$$f'(-1.5) = \frac{-6}{2} = -3$$

$$f'(0.5) = \frac{-6}{4} = -1.5$$

$$f'(1.5) = \frac{2}{4} = 0.5$$

(b) $f'(x) = 0$

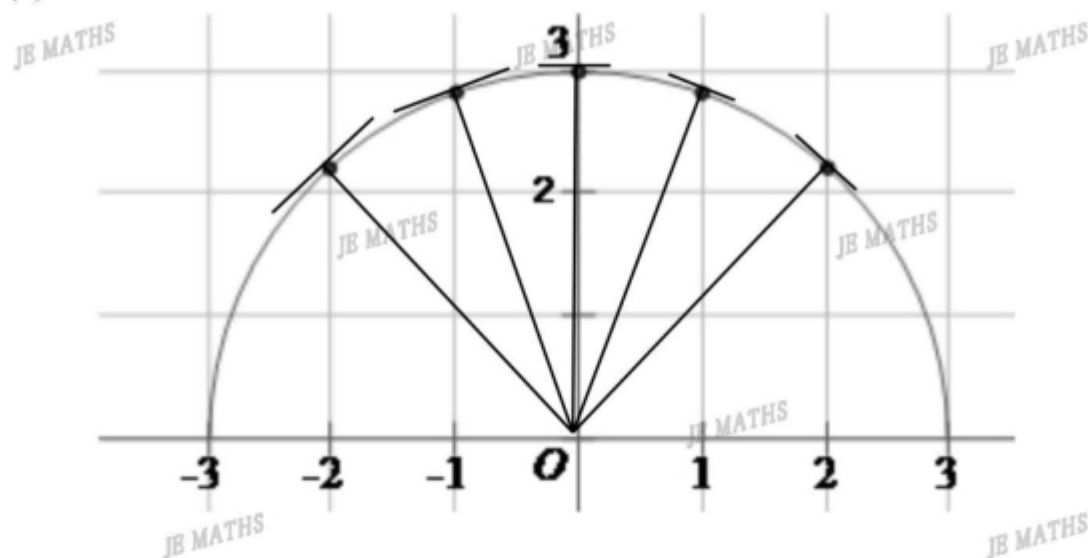
(d) $f'(x) = -7$

(f) $f'(x) = \frac{-21}{4}$

Stage 2:

1. (a) (i) $(-2, \sqrt{5}), (-1, 2\sqrt{2}), (1, 2\sqrt{2}), (0, 3), (2, \sqrt{5})$

(ii)



(iii) $f'(0) = 0$

$$f'(1) = \frac{-1}{2\sqrt{2}} = -\frac{\sqrt{2}}{4}$$

$$f'(-1) = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

$$f'(2) = \frac{-2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

$$f'(-2) = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$f'(-2) = \frac{-2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

(b) (i) $m_{OP} = \frac{\sqrt{9-x^2}-0}{x-0} = \frac{\sqrt{9-x^2}}{x}$

(ii) $m_{PT} \cdot \frac{\sqrt{9-x^2}}{x} = -1$

$$f'(x) = \frac{-x}{\sqrt{9-x^2}}$$

$$(iii) \ 1 = \frac{-x}{\sqrt{9-x^2}}$$

$$\sqrt{9-x^2} = -x$$

$$x^2 = 9 - x^2$$

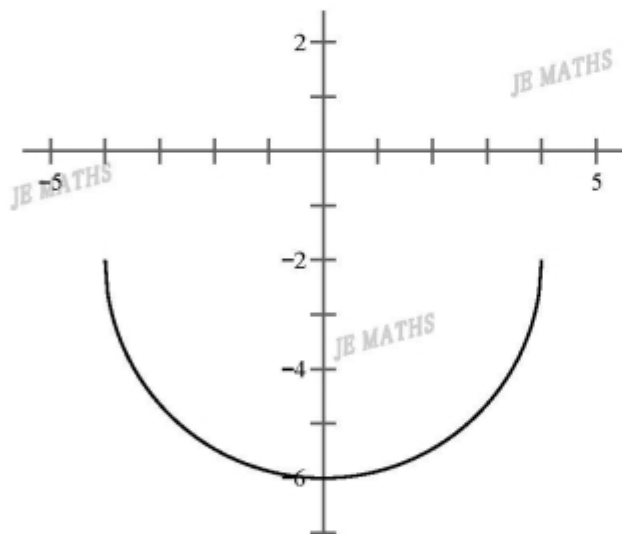
$$2x^2 = 9$$

$$x^2 = \frac{9}{2}$$

$$x = \frac{-3}{\sqrt{2}}$$

$$x = \frac{-3\sqrt{2}}{2}$$

2. (i) $C(0, -2), \ r = 4$



$$(ii) \ m = \frac{-\sqrt{16-x^2}}{x}$$

$$(iii) \ m_{PT} = \frac{x}{\sqrt{16-x^2}}$$

$$f'(x) = \frac{x}{\sqrt{16-x^2}}$$

$$(iv) (a) f'(-1) = \frac{-\sqrt{15}}{15}$$

$$\frac{-1}{\sqrt{16-1^2}} = \frac{-1}{\sqrt{15}}$$

$$(b) f'(2) = \frac{\sqrt{3}}{3}$$

$$\frac{2}{\sqrt{16-2^2}} = \frac{2}{\sqrt{12}} = \frac{2\sqrt{3}}{6}$$

$$(c) f'(-3) = \frac{-3\sqrt{7}}{7}$$

$$\frac{-3}{\sqrt{16-3^2}} = \frac{-3}{\sqrt{7}}$$

$$(v) \frac{x}{\sqrt{16-x^2}} = -1$$

$$x = -\sqrt{16-x^2}$$

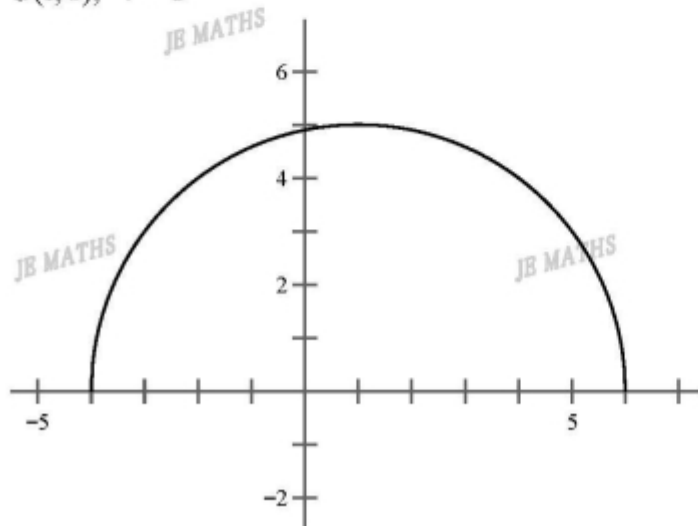
$$x^2 = 16-x^2$$

$$2x^2 = 16$$

$$x^2 = 8$$

$$x = -2\sqrt{2} \quad (\text{omit positive})$$

$$3. (i) C(1,0), r=5$$



$$(ii) \quad m = \frac{\sqrt{25 - (x-1)^2}}{x-1}$$

$$(iii) \quad m_{PT} = \frac{1-x}{\sqrt{25 - (x-1)^2}}$$

$$f'(x) = \frac{1-x}{\sqrt{25 - (x-1)^2}}$$

$$(iv) \quad f'(x) = \frac{1-4}{\sqrt{25 - (1-4)^2}} = -\frac{3}{4}$$

$$y = \sqrt{25 - (4-1)^2} = 4$$

$$y-4 = \frac{-3}{4}(x-4)$$

$$4y-16 = -3x+12$$

$$3x+4y-28=0$$

$$(v) \quad \frac{1-x}{\sqrt{25 - (x-1)^2}} = \frac{1}{2}$$

$$2-2x = \sqrt{25 - (x-1)^2}$$

$$4-8x+4x^2 = 25 - x^2 + 2x - 1$$

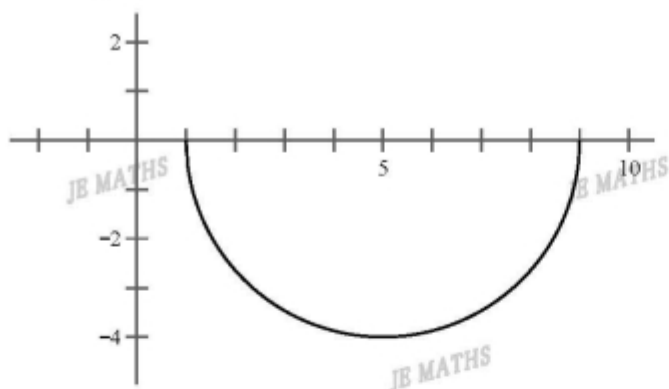
$$0 = 5x^2 - 10x - 20$$

$$0 = x^2 - 2x - 4$$

$$x = \frac{2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot -4}}{2}$$

$$x = 1 \pm \sqrt{5} \quad (\text{omit positive})$$

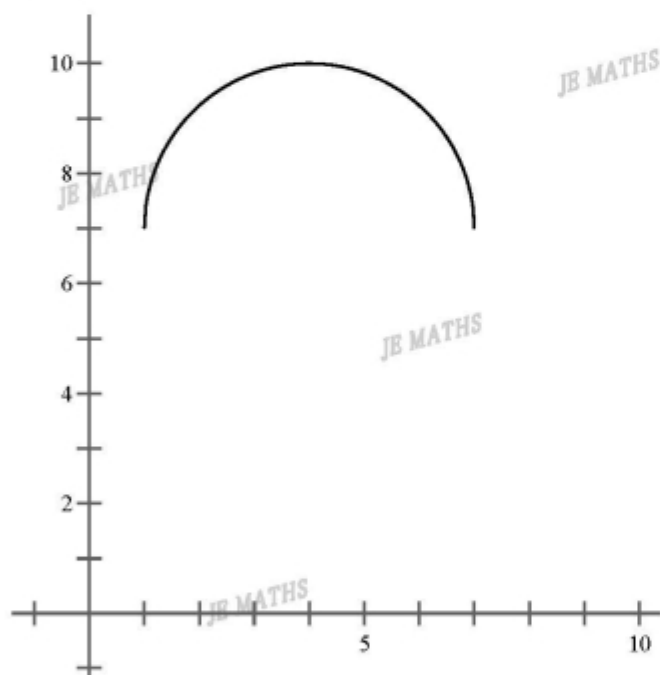
4. (i) $C(0,5)$, $r=4$



(ii) $m = \frac{-\sqrt{16 - (x-5)^2}}{x-5}$

$$f'(x) = \frac{x-5}{\sqrt{16 - (x-5)^2}}$$

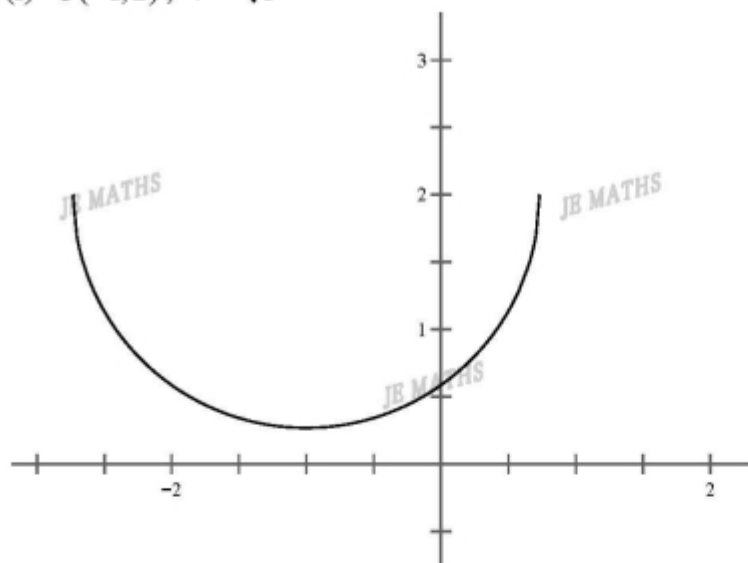
5. (i) $C(4,7)$, $r=3$



(ii) $m = \frac{\sqrt{9 - (4-x)^2}}{x-4}$

$$f'(x) = \frac{4-x}{\sqrt{9 - (4-x)^2}}$$

6. (i) $C(-1, 2)$, $r = \sqrt{3}$

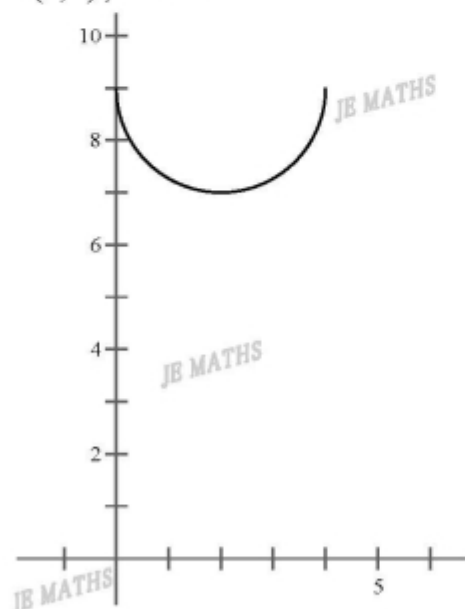


(ii) $m = \frac{-\sqrt{3 - (x+1)^2}}{x+1}$

$$f'(x) = \frac{x+1}{\sqrt{3 - (x+1)^2}}$$

7. (i) $f(x) = 9 - \sqrt{4 - 4 + 4x - x^2} = 9 - \sqrt{4 - (x-2)^2}$

$C(2, 9)$, $r = 2$



(ii) $m = \frac{-\sqrt{4x - x^2}}{x-2}$

$$f'(x) = \frac{x-2}{\sqrt{4x - x^2}}$$

Stage 3:

1. (a) (i) $3x^2 - 3a^2 = mx - ma$

$$3(x+a)(x-a) = m(x-a)$$

$$3x + 3a = m$$

$$x = \frac{m-3a}{3} \text{ or } x = a$$

(ii) $a = \frac{m-3a}{3}$

$$3a = m - 3a$$

$$m = 6a$$

(iii) $f'(x) = m_{PT} = 6x$

(b) (i) $3x^2 = mx + b$

$$0 = 3x^2 - mx - b$$

$$x = \frac{m \pm 0}{2 \cdot 3} = \frac{m}{6}$$

(ii) $m = 6x$

$$f'(x) = m = 6x$$

2. (i) $3x^2 - x + 2 = mx + b$

$$0 = 3x^2 - x - mx + 2 - b$$

$$x = \frac{m+1 \pm 0}{2 \cdot 3} = \frac{m+1}{6}$$

(ii) $m+1 = 6x$

$$m = 6x - 1$$

$$f'(x) = m = 6x - 1$$

3. (i) $kx^2 = mx + b$

$$0 = kx^2 - mx - b$$

$$x = \frac{m \pm 0}{2k} = \frac{m}{2k}$$

(ii) $2kx = m$

$$f'(x) = m = 2kx$$

4. (i) $Ax^2 + Bx + c = mx + b$

$$0 = Ax^2 + (B - m)x + C - b$$

$$x = \frac{m - B \pm 0}{2A}$$

(ii) $2Ax = m - B$

$$m = 2Ax + B$$

$$f'(x) = m = 2Ax + B$$

5. (i) $\frac{1}{x} = mx + b$

$$1 = mx^2 + bx$$

$$0 = mx^2 + bx - 1$$

$$x = \frac{-b \pm 0}{2m} = \frac{-b}{2m}$$

(ii) $1 = m \left(\frac{-b}{2m} \right)^2 + b \left(\frac{-b}{2m} \right)$

$$1 = \frac{mb^2}{4m^2} + \frac{-b^2}{2m}$$

$$4m = b^2 - 2b^2$$

$$b^2 = -4m$$

$$b = \pm \sqrt{-4m}$$

$$(iii) \frac{-b}{2m} = x$$

$$\pm\sqrt{-4m} = 2mx$$

$$-4m = 4m^2x^2$$

$$0 = m^2x^2 + m$$

$$0 = m(mx^2 + 1)$$

$$0 = mx^2 + 1$$

$$m = \frac{-1}{x^2}, \text{ as } m \neq 0$$

$$f'(x) = -\frac{1}{x^2}$$

