Enrichment stage 1: (other special trig)

1. Given that $\sin 15° = \frac{\sqrt{6}-\sqrt{2}}{4}$, find the exact value of:

   (a) $\sin 195°$

   (b) $\cos ec 345°$

   (c) $\cos 75°$

   (d) $\cos 15°$ by using Pythagorean identity.

   (e) $\cos ec^2 15° - \cot^2 15°$.

Enrichment stage 2: (trig expression and equation)

1. If $a = \tan\theta + \cot\theta$ and $b = \sin\theta + \cos\theta$, prove that $a(b^2 - 1) = 2$.

2. If $a = \sec\theta + \tan\theta$, find $\sin\theta$ in terms of a.

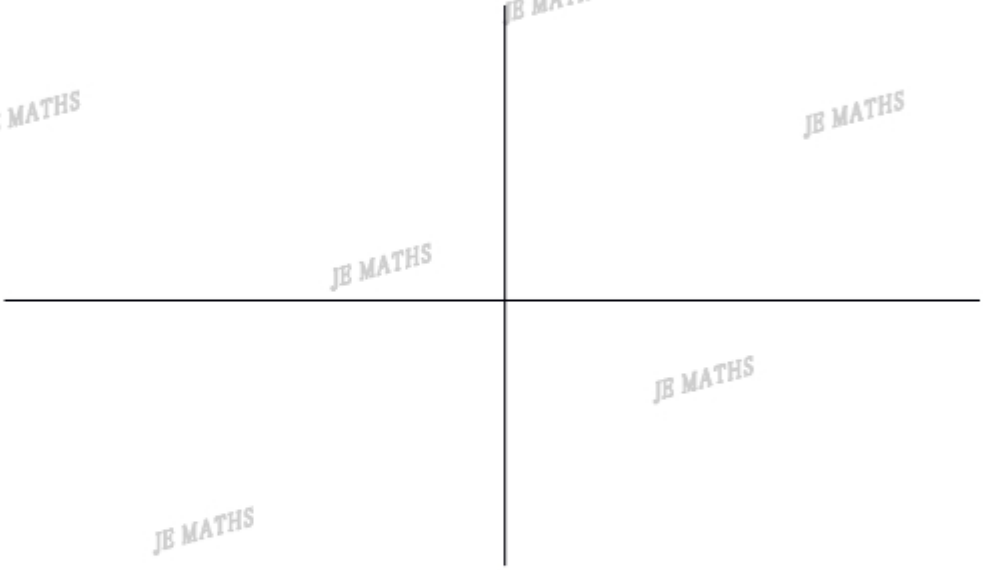
3. Solve for x in $81^{\cos^2 x} + 81^{\sin^2 x} = 30$ for $0° \leq x \leq 180°$.

Enrichment stage 3: (trig function)

1. Given that $f(\tan x) = \cos^2 x$.

   (a) Find $f(x)$.

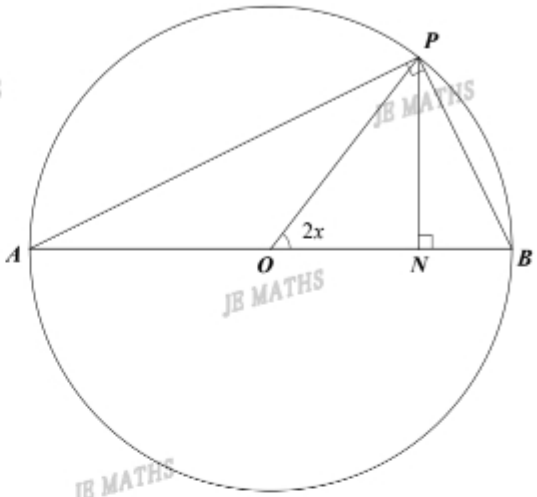
   (b) Sketch $y = f(x)$.



   (c) Hence, find the exact value(s) of $f^{-1}(\frac{1}{4})$.

Enrichment stage 4: (trig application)

1. The diagram below shows a circle with centre O and diameter AB. P is the point on the circumference of the circle. PN is drawn perpendicular to AB and AP is perpendicular to PB.
   Let $\angle POB = 2x$.



(a) Explain why $\triangle APO$ is isosceles.

(b) Explain why $\angle OAP = \angle OPA = x$.

(c) Show that $\sin 2x = \dfrac{2PN}{AB}$.

(d) Use $\triangle APN$ and $\triangle PAB$ to show that $2\sin x \cos x = \sin 2x$.

2. The horizontal range (R m) of a rocket varies directly with the square of the velocity of projection (v m/s) and the sine of twice the angle of elevation (A) for $0° \leq A \leq 90°$.
That is: $R \propto v^2 \sin(2A)$.

When a rocket is projected with an angle of elevation of 15 ° and a velocity of 10m/s it reaches a range of 5m.

(a) Write a formula for the range of the rocket in terms of R, v and A.

(b) Calculate the angle of elevation required for a rocket projected at 12 m/s to reach a range of 10 m. Correct your answer to the nearest degree.

Enrichment stage 1: (other special trig)

1. (a)

$\sin 195^\circ = \sin(180^\circ+15^\circ)$
$= -\sin 15^\circ$
$= -(\sqrt{6}-\sqrt{2})/4$
$= (\sqrt{2}-\sqrt{6})/4$

(b)

$\operatorname{cosec} 345^\circ = \operatorname{cosec}(360^\circ-15^\circ)$
$= -\operatorname{cosec} 15^\circ$
$= -1/\sin 15^\circ$
$= -4/(\sqrt{6}-\sqrt{2})$
$= -4\times(\sqrt{6}+\sqrt{2})/4$
$= -\sqrt{6}-\sqrt{2}$

(c)

$\cos 75^\circ = \sin(90^\circ-15^\circ)$
$= \sin 15^\circ$
$= (\sqrt{6}-\sqrt{2})/4$

(d)

$\sqrt{(1-\sin^2 15)} = \sqrt{[1-(\sqrt{6}-\sqrt{2})^2/4^2]}$
$= \sqrt{\{[16-(\sqrt{6}-\sqrt{2})^2]/4^2\}}$
$= \sqrt{(6+2\sqrt{12}+2)/4}$
$= \sqrt{(\sqrt{6}+\sqrt{2})^2/4}$
$= (\sqrt{6}+\sqrt{2})/4$

(e)

since $1+\cot^2\theta = \operatorname{cosec}^2\theta$,
then $\operatorname{cosec}^2\theta - \cot^2\theta = 1$
ie, ans is 1.

Enrichment stage 2:

1. If $a = \tan\theta + \cot\theta$ and $b = \sin\theta + \cos\theta$, prove that $a(b^2 - 1) = 2$.

$a(b^2 - 1) = (\tan\theta + \cot\theta)[(\sin\theta + \cos\theta)^2 - 1]$

$= (\tan\theta + \cot\theta)(\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta - 1)$

$= 2\sin\theta\cos\theta(\sin\theta/\cos\theta + \cos\theta/\sin\theta)$

$= 2(\sin^2\theta + \cos^2\theta)$

$= 2$

2. If $a = \sec\theta + \tan\theta$, find $\sin\theta$ in terms of a.

$\sec\theta + \tan\theta = a$ (1)

$\sec^2\theta - \tan^2\theta = 1$

$(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$\sec\theta - \tan\theta = 1/a$ (2)

(1) + (2): $2\sec\theta = a + 1/a$

$\sec\theta = (a + 1/a)/2$

$\cos\theta = 2/(a + 1/a)$ (3)

(1) - (2): $2\tan\theta = a - 1/a$

$\tan\theta = (a - 1/a)/2$ (4)

(3)×(4): $\sin\theta = (a - 1/a)/(a + 1/a)$

$= (a^2 - 1)/(a^2 + 1)$

3. $81^{\cos^2 x} + 81^{1 - \cos^2 x} = 30$

$81^{\cos^2 x} + 81 / 81^{\cos^2 x} = 30$

Let $u = 81^{\cos^2 x}$.

$u + 81/u = 30$

$u^2 + 81 = 30u$

$u^2 - 30u + 81 = 0$

$(u - 27)(u - 3) = 0$

$u = 3, 27$

$81^{\cos^2 x} = 3 = 81^{1/4}$

$\cos^2 x = 1/4$

$\cos x = \pm 1/2$

$x = 60°$ or $120°$

$81^{\cos^2 x} = 27 = 81^{3/4}$

$\cos^2 x = 3/4$

$\cos x = \pm\sqrt{3}/2$

$x = 30°$ or $150°$

$\therefore x = 30°, 60°, 120°, 150°$

Enrichment stage 3:

1. (a)

cos²x=1/sec²x

=1/(1+tan²x)

f(tanx)=1/(1+tan²x)

tanx→x, for x∈R

∴f(x)=1/(1+x²), for x∈R

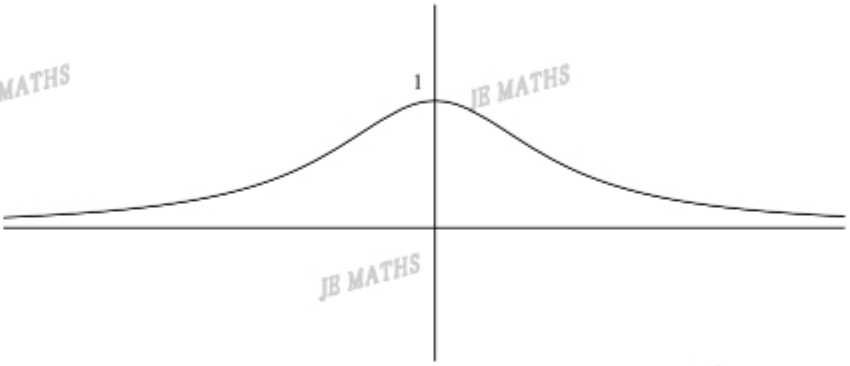
(b)

when x→-∞, y→0

ie, when x=-99, y=0.0001

when x→+∞, y→0

ie, when x=99, y=0.0001



(c)

f(tanx)=cos²x

f^{-1}(cos²x)=tanx

cos²x=1/4

cosx=1/2

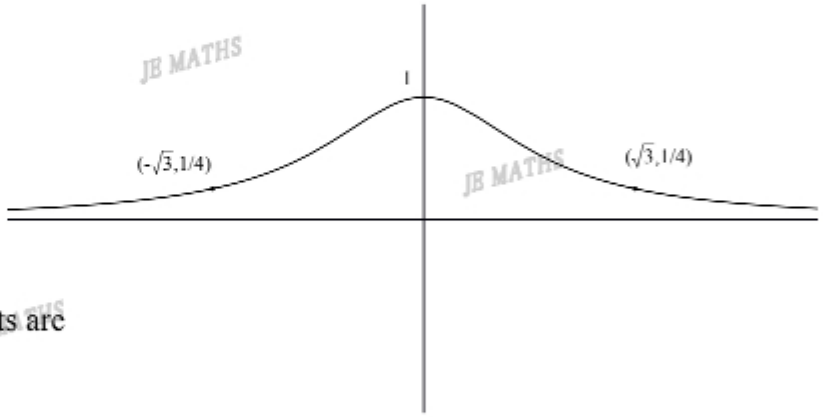
x=-60°, 60°

when x = -60°, tan(-60°)=-√3

when x = 60°, tan(60°)=√3

ans: $f^{-1}(1/4) = \pm\sqrt{3}$

Notice that the original two points are (-√3, ¼) and (√3, ¼).



Enrichment stage 4:

1. (a)

OA = OP (Both are radii)

△PAO is isosceles (Two equal sides)

(b)

∠OAP = ∠OPA (Equal sides are opposite equal sides in △APO)

∠POB = ∠OAP + ∠OPA (Exterior angle equals sum of two interior opposite angles)

$2x = 2\angle OAP$

$\angle OAP = x$

$\therefore \angle OAP = \angle OPA = x$

(c)

OP = AB/2 (Diameter is twice the radius)

In △PON,

$\sin 2x = PN/OP$

$= PN/(AB/2)$

$= 2PN/AB$

(d)

In △APN,

$\sin x = PN/AP$

In △PAB,

$\cos x = AP/AB$

$2\sin x\cos x = 2PN/AP \times AP/AB$

$= 2PN/AB$

$= \sin 2x$

2. (a)

$R = kv^3\sin(2A)$

$5 = k \times 10^2 \times \sin(2 \times 15^\circ)$

$5 = 100k \times 1/2$

$k = 1/10$

$R = v^3\sin(2A)/10$

(b)

$10 = 1/10 \times 12^2 \times \sin(2A)$

$\sin(2A) = 25/36$

$2A = 43.982^\circ$ or 136.017°

$A = 22^\circ$ or 68° (nearest degree)