

- **Three reciprocal identities:** $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$, $\sec \theta = \frac{1}{\cos \theta}$, $\cot \theta = \frac{1}{\tan \theta}$

1. (a) $(\sec x)$

(b) $(\tan x)$

(c) $1/\sin x \times \sin x = 1$

(d) $\sin x / \cos x \times \cos / \sin x = 1$

2. (a) $\sin x \times \sin x = \sin^2 x$

(b) $\cos x / \sin x \times \sin x = \cos x$

(c) $(\cos x / \sin x)^2 = \cot^2 x$

(d) $(\sin x \times 1 / \sin x)^2 = 1$

3. (a)

$$\text{LHS} = \sec x \operatorname{cosec} x$$

$$= \text{RHS}$$

(b)

$$\text{LHS} = 1/\cos x \times \cos x / \sin x$$

$$= 1/\sin x$$

$$= \operatorname{cosec} x$$

$$= \text{RHS}$$

(c)

$$\text{LHS} = 1/\operatorname{cosec} x + 1/\sec x$$

$$= (\sec x + \operatorname{cosec} x) / \operatorname{cosec} x \sec x$$

$$= \text{RHS}$$

(d)

$$\text{LHS} = \tan x - 1/\tan x$$

$$= (\tan^2 x - 1) / \tan x$$

$$= \text{RHS}$$

- Ratio identities: $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\sec \theta}{\operatorname{cosec} \theta}$, $\cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{\operatorname{cosec} \theta}{\sec \theta}$

4. (a)

$$\begin{aligned}\text{LHS} &= \cos \theta \sin \theta / \cos \theta \\ &= \sin \theta \\ &= \text{RHS}\end{aligned}$$

(b)

$$\begin{aligned}\text{LHS} &= \sin \theta \cos \theta / \sin \theta \\ &= \cos \theta \\ &= \text{RHS}\end{aligned}$$

(c)

$$\begin{aligned}\text{LHS} &= 1/\sin \theta \times \cos \theta \\ &= \cos \theta / \sin \theta \\ &= \cot \theta \\ &= \text{RHS}\end{aligned}$$

(d)

$$\begin{aligned}\text{LHS} &= \tan \theta \times 1/\sin \theta \times \cos \theta \\ &= \sin \theta / \cos \theta \times \cos \theta / \sin \theta \\ &= 1 \\ &= \text{RHS}\end{aligned}$$

- Complementary angle identities:

$$\cos(90^\circ - \theta) = \sin \theta, \quad \cot(90^\circ - \theta) = \tan \theta, \quad \operatorname{cosec}(90^\circ - \theta) = \sec \theta$$

5. (a)

$$\begin{aligned}\sin 70^\circ &= \sin(90^\circ - 20^\circ) \\ &= \cos 20^\circ\end{aligned}$$

(b)

$$\begin{aligned}\cos 36^\circ &= \cos(90^\circ - 54^\circ) \\ &= \sin 54^\circ\end{aligned}$$

(c)

$$\begin{aligned}\tan 20^\circ &= \tan(90^\circ - 70^\circ) \\ &= \cot 70^\circ\end{aligned}$$

(d)

$$\begin{aligned}\operatorname{cosec} 13^\circ &= \operatorname{cosec}(90^\circ - 77^\circ) \\ &= \sec 77^\circ\end{aligned}$$

6. (a) $\cos x$

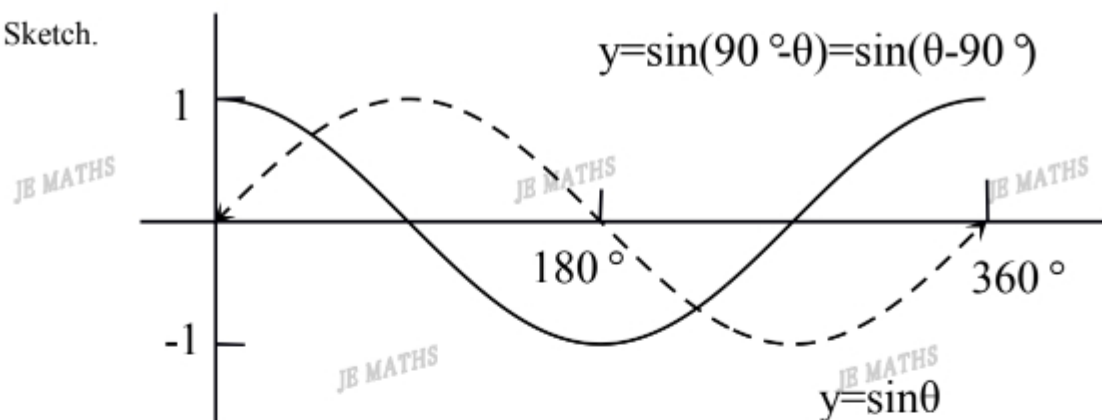
(b) $\operatorname{cosec} x$

(c) $1/\tan x = \cot x$

(d) $\sin x / \cos x = \tan x$

7. (a) $y = \sin(90^\circ - \theta) = -\sin(\theta - 90^\circ)$

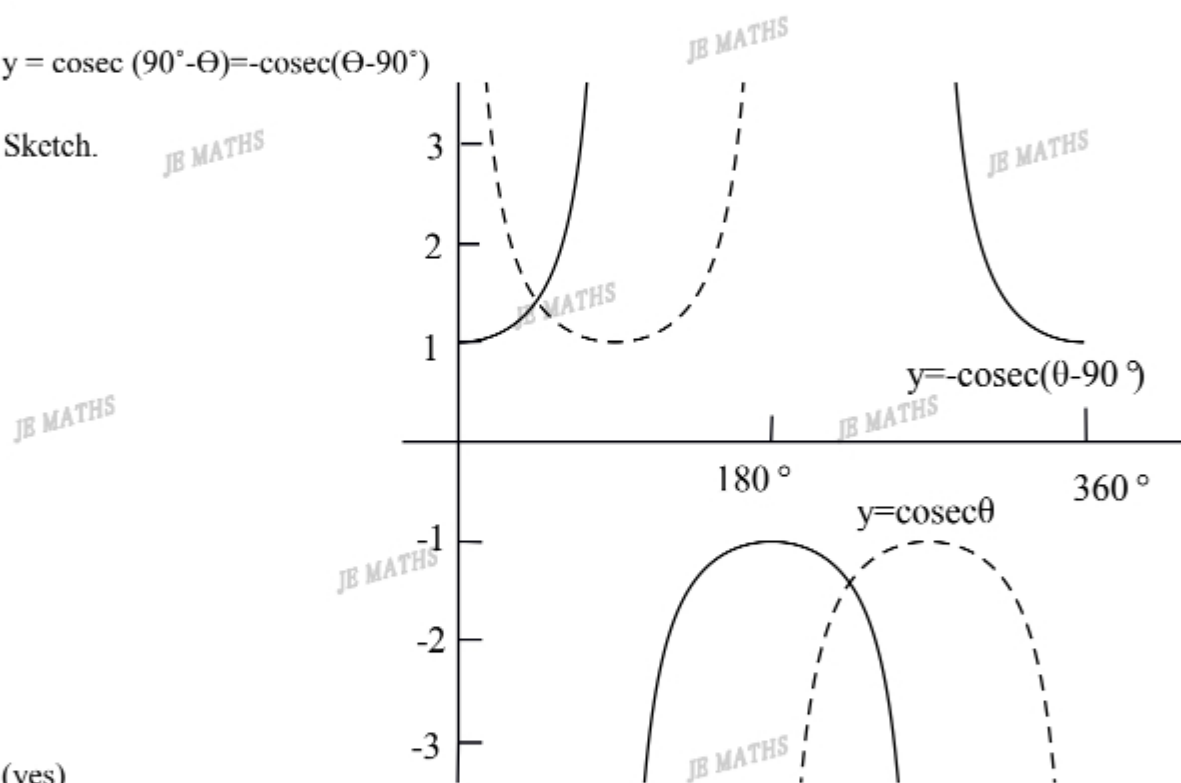
(b) Sketch.



(c) (yes)

8. (a) $y = \operatorname{cosec}(90^\circ - \theta) = -\operatorname{cosec}(\theta - 90^\circ)$

(b) Sketch.



(c) (yes)

9. (a)

$$\text{LHS} = -\sin \theta \sin \theta$$

$$= -\sin^2 \theta$$

$$= \text{RHS}$$

(b)

$$\text{LHS} = \tan \theta \tan \theta \cos \theta$$

$$= \tan \theta \sin \theta / \cos \theta \times \cos \theta$$

$$= \tan \theta \sin \theta$$

$$= \text{RHS}$$

- **Pythagorean identities:** $\sin^2 \theta + \cos^2 \theta = 1$, $1 + \tan^2 \theta = \sec^2 \theta$, $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

10. (a)

$$(\cos A)$$

(b)

$$(\tan A)$$

(c)

$$(1)$$

(d)

$$(1+1=2)$$

11. (a)

$$1 + \tan^2 A = \sec^2 A$$

(b)

$$\sin \theta \times \operatorname{cosec} \theta = \sin \theta \times 1 / \sin \theta = 1$$

(c)

$$1+1=2$$

(d)

$$\tan A / \tan A = 1$$

12. $1 + \tan^2 A = \sec^2 A$

$$\tan A = -\sqrt{(\sec^2 A - 1)} \quad (A \text{ in 4th Q, } \tan A < 0)$$

$$= -\sqrt{(9-1)}$$

$$= -\sqrt{8}$$

$$= -2\sqrt{2}$$

13. (a)

$$\text{LHS} = \sin \theta + 2\sin \theta \cos \theta + \cos \theta$$

$$= 1 + 2\sin \theta \cos \theta$$

$$= \text{RHS}$$

(b)

$$\text{LHS} = 1 - \sin \theta - \sin \theta$$

$$= 1 - 2\sin \theta$$

$$= \text{RHS}$$

14. (a)

$$\begin{aligned}
 \text{LHS} &= \cos x / \sin x + \sin x / \cos x \\
 &= (\cos^2 x + \sin^2 x) / \sin x \cos x \\
 &= 1 / \sin x \cos x \\
 &= \operatorname{cosec} x \sec x \\
 &= \text{RHS}
 \end{aligned}$$

(b)

$$\begin{aligned}
 \text{LHS} &= 1 / \sin x - \sin x \\
 &= (1 - \sin^2 x) / \sin x \\
 &= \cos^2 x / \sin x \\
 &= \cot x \operatorname{cosec} x \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 15. \text{LHS} &= (\sin \theta - \cos \theta)(\sin \theta + \cos \theta) / \sin \theta (\sin \theta + \cos \theta) \\
 &= (\sin \theta - \cos \theta) / \sin \theta \\
 &= 1 - \cot \theta \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 16. \text{LHS} &= (\sin x + \cos x)(1 / \cos x + 1 / \sin x) \\
 &= (\sin x + \cos x)[(\sin x + \cos x) / \cos x \sin x] \\
 &= (\sin x + \cos x)^2 / \cos x \sin x \\
 &= (\sin^2 x + 2 \sin x \cos x + \cos^2 x) / \cos x \sin x \\
 &= (1 + 2 \sin x \cos x) / \cos x \sin x \\
 &= 1 / \cos x \sin x + 2 \\
 &= \sec x \operatorname{cosec} x + 2 \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 17. \text{LHS} &= (1 / \sin x + \cos x / \sin x) / (1 / \sin x - \cos x / \sin x) \\
 &= (1 + \cos x) / \sin x \div (1 - \cos x) / \sin x \\
 &= (1 + \cos x) / (1 - \cos x) \times (1 - \cos x) / (1 - \cos x) \\
 &= (1 - \cos^2 x) / (1 - \cos x)^2 \\
 &= \sin^2 x / (1 - \cos x)^2 \\
 &= \text{RHS}
 \end{aligned}$$

(reciprocal identities)

(times conjugate terms)

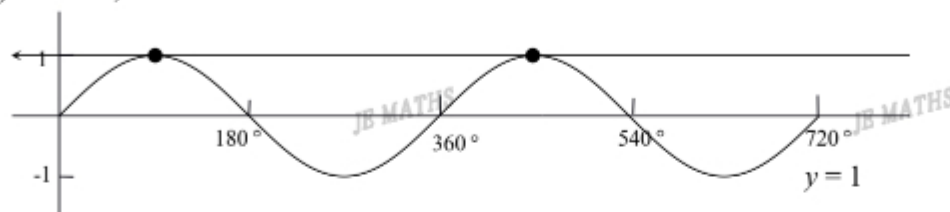
(Pythagorean identities)

18. (a) $(-120+360=240^\circ)$ (b) $(-78+360=282^\circ)$

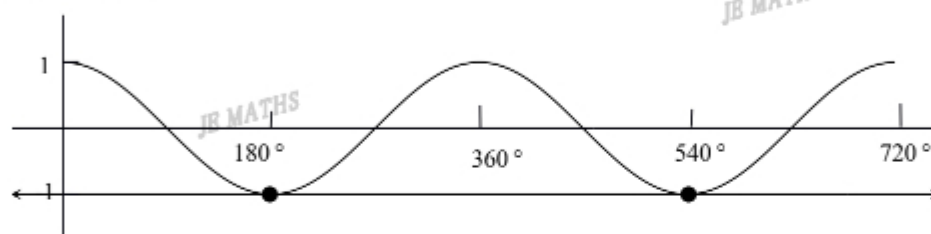
- (c) $(370-360=10^\circ)$ (d) $(432-360=72^\circ)$

- Boundary angles:

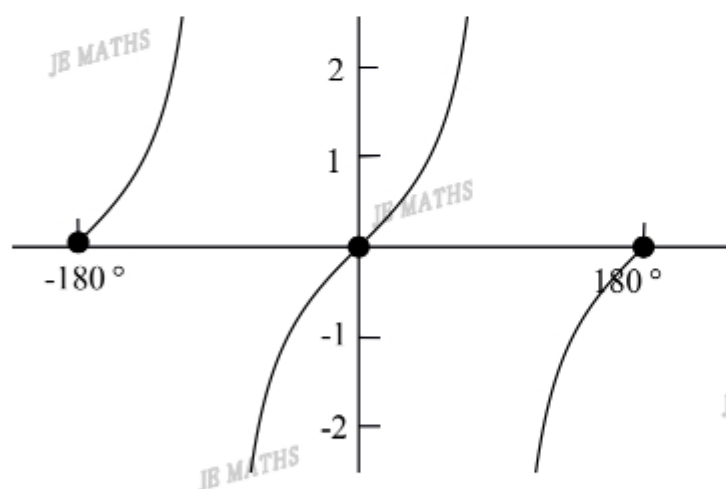
19. (a) $x = 90^\circ, 450^\circ$



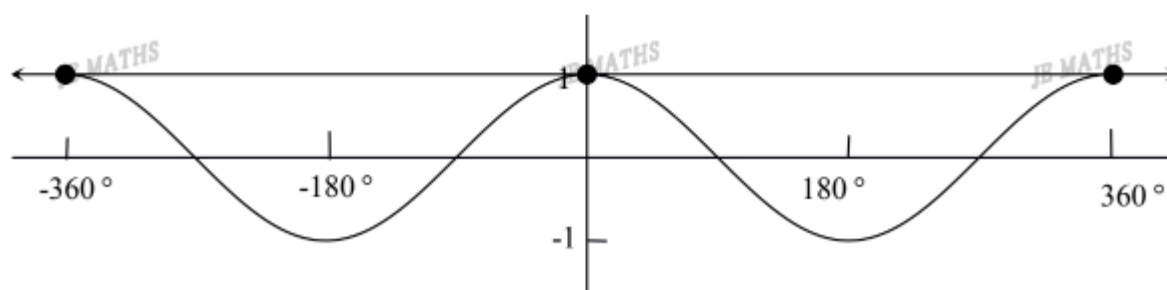
- (b) $x = 180^\circ, 540^\circ$



- (c) $x = -180^\circ, 0, 180^\circ$



- (d) $\cos x = 1$
 $x = -360^\circ, 0^\circ, 360^\circ$

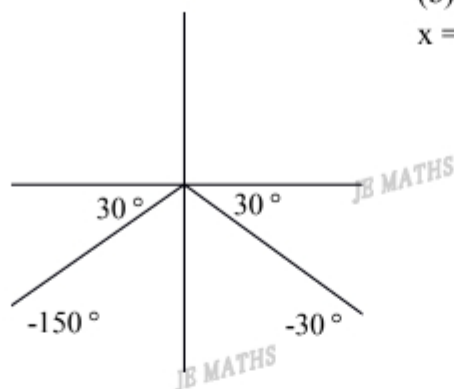


- Use related angles:

20. (a)

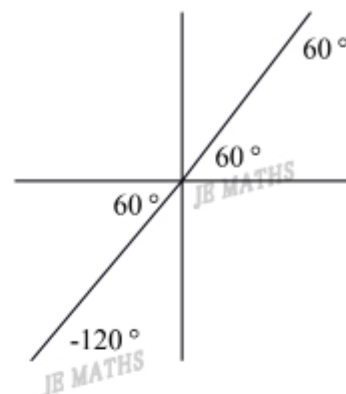
$$x = -150^\circ, -30^\circ$$

JE MATHS



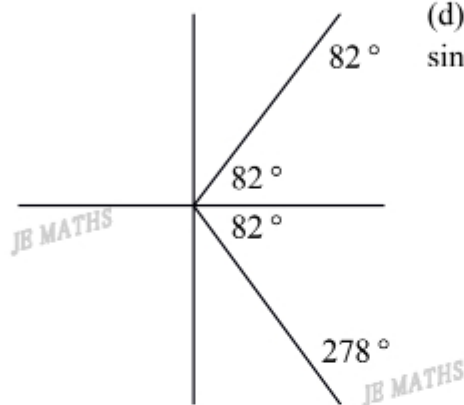
(b)

$$x = -120^\circ, 60^\circ$$



(c)

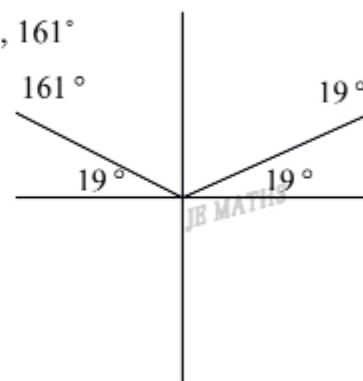
$$x = 82^\circ, 278^\circ$$



(d)

$$\sin x = 1/3, x = 19^\circ, 161^\circ$$

JE MATHS

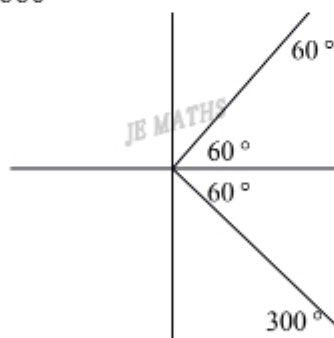


21. (a)

$$\cos x = 1/2$$

$$x = 60^\circ, 300^\circ, 420^\circ, 660^\circ$$

JE MATHS

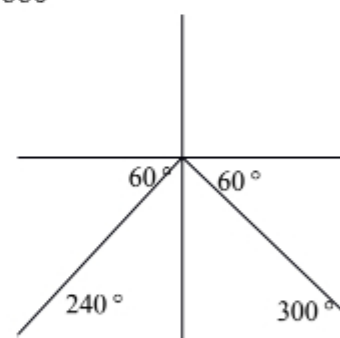


(b)

$$\sin x = -\sqrt{3}/2$$

$$x = 240^\circ, 300^\circ, 600^\circ, 660^\circ$$

JE MATHS

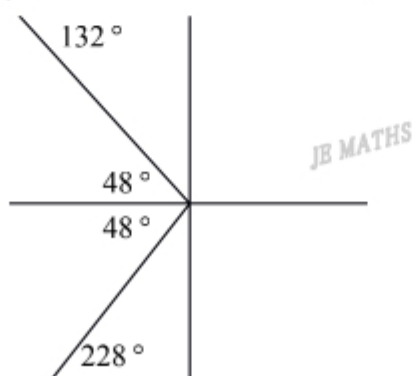


(c)

$$\sec x = -3/2 \rightarrow \cos x = -2/3$$

$$x = 132^\circ, 228^\circ, 492^\circ, 588^\circ$$

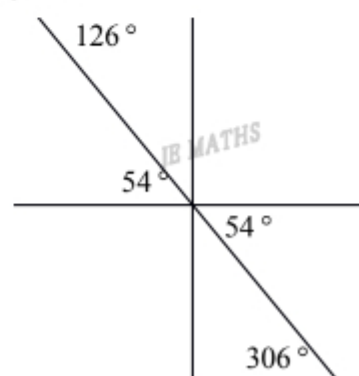
JE MATHS



(d)

$$\cot x = -\sqrt{2}/2, \tan x = -\sqrt{2}$$

$$x = 126^\circ, 306^\circ, 486^\circ, 666^\circ$$



- Compound angle:

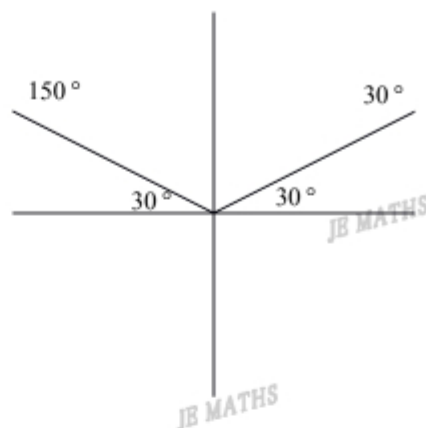
22. (a) Let:
- $\Theta = 2x$
- ,
- $0^\circ \leq x \leq 360^\circ$
- ,
- $0^\circ \leq \Theta \leq 720^\circ$

$$\sin \Theta = 1/2$$

$$\Theta = 30^\circ, 150^\circ, 390^\circ, 510^\circ$$

$$x = \Theta/2$$

$$= 15^\circ, 75^\circ, 195^\circ, 255^\circ$$



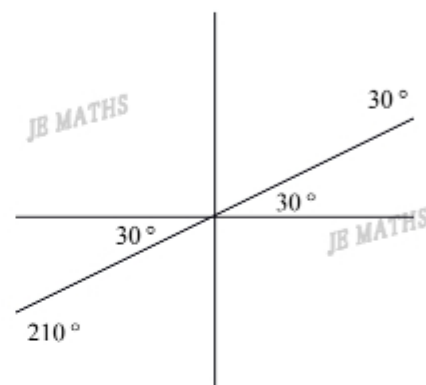
- (b) Let:
- $\Theta = x - 45^\circ$
- ,
- $0^\circ \leq x \leq 360^\circ$
- ,
- $0^\circ \leq \Theta \leq 315^\circ$

$$\tan \Theta = \sqrt{3}/3$$

$$\Theta = 30^\circ, 210^\circ$$

$$x = \Theta + 45^\circ$$

$$= 75^\circ, 255^\circ$$



23. (a)
- $\tan 3x = -\sqrt{3}/3$

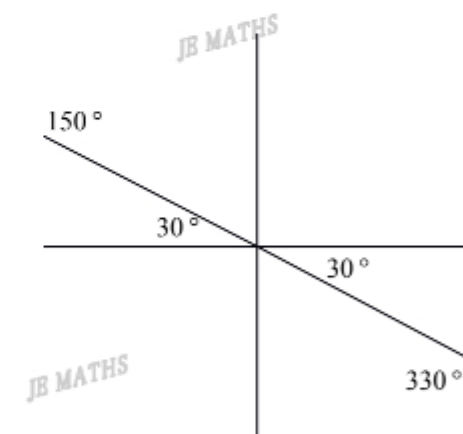
$$\text{Let: } \Theta = 3x, 0^\circ \leq x \leq 360^\circ, 0^\circ \leq \Theta \leq 1080^\circ$$

$$\tan \Theta = -\sqrt{3}/3$$

$$\Theta = 150^\circ, 330^\circ, 510^\circ, 690^\circ, 870^\circ, 1050^\circ$$

$$x = \Theta/3$$

$$= 50^\circ, 110^\circ, 170^\circ, 230^\circ, 290^\circ, 350^\circ$$



- (b)
- $\cos(75^\circ - x) = \cos(x - 75^\circ) = -\sqrt{3}/2$

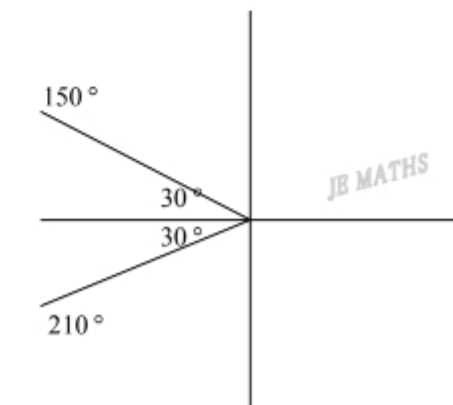
$$\text{Let: } \Theta = x - 75^\circ, 0^\circ \leq x \leq 360^\circ, -75^\circ \leq \Theta \leq 285^\circ$$

$$\cos \Theta = -\sqrt{3}/2$$

$$\Theta = 150^\circ, 210^\circ$$

$$x = \Theta + 75^\circ$$

$$= 225^\circ, 285^\circ$$



- Substitution:24. (a) Let $t = \cos x$

$$4t^2 = t$$

$$4t^2 - t = 0$$

$$t(4t-1) = 0$$

1) $t = 0 = \cos x$,

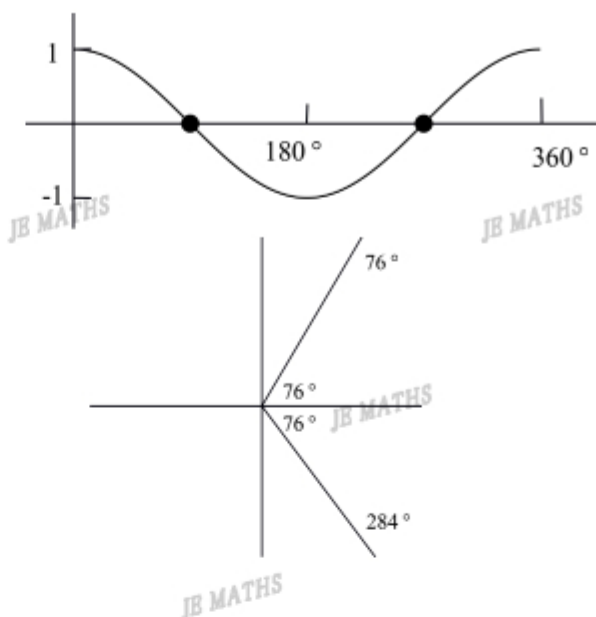
$$x = 90^\circ, 270^\circ$$

2) $4t - 1 = 0$

$$t = 1/4 = \cos x$$

$$x = 76^\circ, 284^\circ$$

1) 2): ans: $76^\circ, 90^\circ, 270^\circ, 284^\circ$



(b) $5/\sin x - \sin x = 0$

Let $t = \sin x$

$$5/t - t = 0$$

$$5 - t^2 = 0$$

$$t^2 - 5 = 0$$

$$(t - \sqrt{5})(t + \sqrt{5}) = 0$$

1) $t - \sqrt{5} = 0$

$$t = \sqrt{5} = \sin x$$

$$x = \text{no solution}$$

2) $t + \sqrt{5} = 0$

$$t = -\sqrt{5} = \sin x$$

$$x = \text{no solution}$$

1) 2): ans: no solution

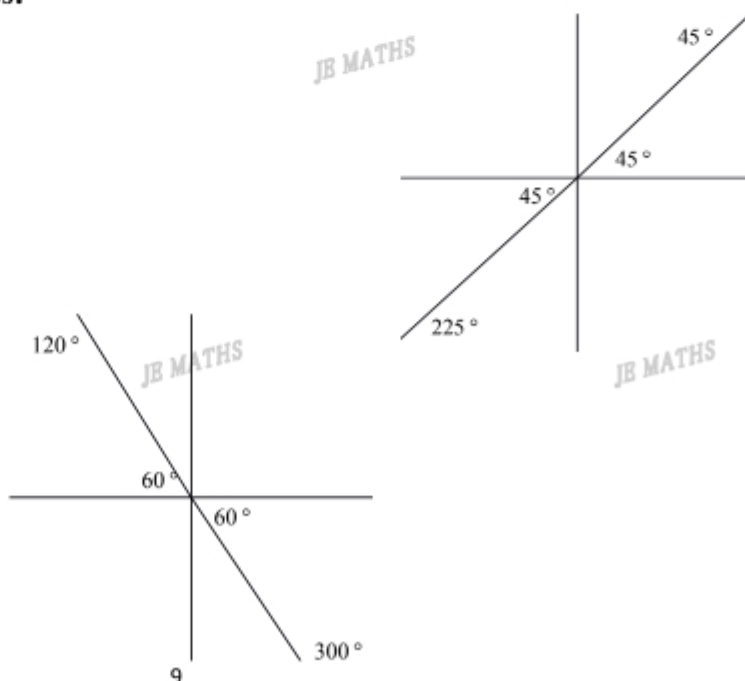
- Using trigonometric identities:25. (a) $\tan x = 1, 0^\circ \leq x \leq 360^\circ$

$$x = 45^\circ, 225^\circ$$

(b) $\sqrt{3}\sin x = -\cos x$

$$\tan x = -\sqrt{3}, 0^\circ \leq x \leq 360^\circ$$

$$x = 120^\circ, 300^\circ$$



26. (a) $3\sin x + 2(1 - \sin x) = 0$

$$3\sin x + 2 - 2\sin x = 0$$

$$2\sin x - 3\sin x - 2 = 0$$

$$(2\sin x + 1)(\sin x - 2) = 0$$

1) $2\sin x = -1$

$$\sin x = -1/2$$

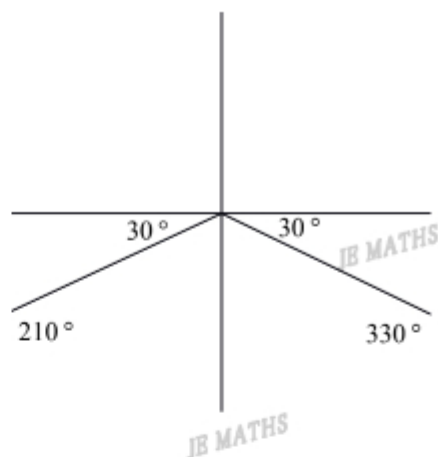
$$x = 210^\circ, 330^\circ$$

2) $\sin x - 2 = 0$

$$\sin x = 2$$

$$x = \text{no solution}$$

1) 2): ans: $210^\circ, 330^\circ$



(b) $1 + \tan x = 4 + 2\tan x$

$$\tan x - 2\tan x - 3 = 0$$

$$(\tan x - 3)(\tan x + 1) = 0$$

1) $\tan x - 3 = 0$

$$\tan x = 3$$

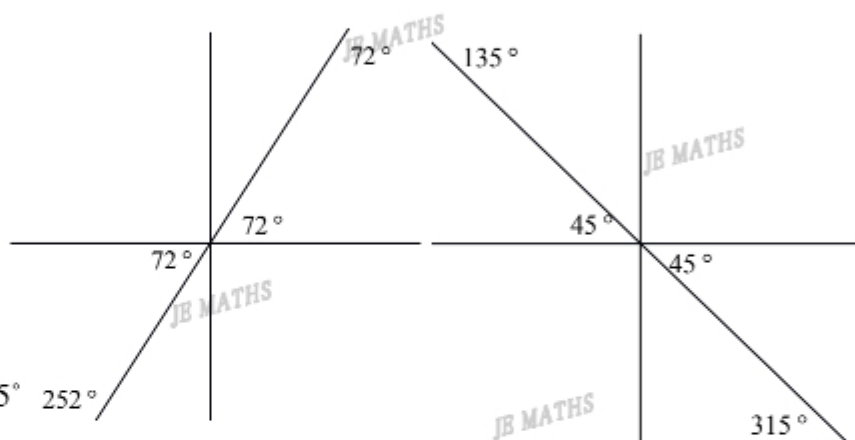
$$x = 72^\circ, 252^\circ$$

2) $\tan x + 1 = 0$

$$\tan x = -1$$

$$x = 135^\circ, 315^\circ$$

1) 2): ans: $72^\circ, 135^\circ, 252^\circ, 315^\circ$



- Homogeneous equation: Sum of indices of $\sin x$ and $\cos x$ in each term is the same.

27. $\tan x + \tan x - 2 = 0$ ($\div \cos x$)

$$(\tan x - 1)(\tan x + 2) = 0$$

1) $\tan x - 1 = 0$

$$\tan x = 1$$

$$x = 45^\circ, 225^\circ$$

2) $\tan x + 2 = 0$

$$\tan x = -2$$

$$x = 116^\circ, 296^\circ$$

1) 2): ans: $x = 45^\circ, 116^\circ, 225^\circ, 296^\circ$

