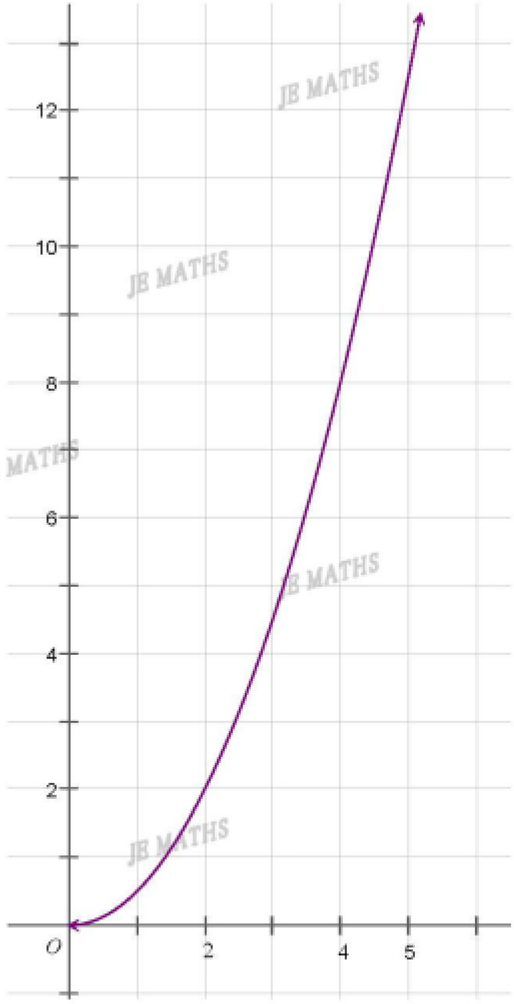
Stage 1

1. Given the graph of a function $d = \frac{t^2}{2}$, where d is the distance in kilometres a car travelled in t minutes.



(i) Find the distance travelled when $t = 1, 2, 3, 4, 5$.

t	1	2	3	4	5
d					

(ii) Mark the points A, B, C, D, E on the graph where $t = 1, 2, 3, 4, 5$ respectively.

(iii) (a) Find the average speed of the car over the first five minutes.

(b) Draw the secant OE. Find the gradient of OE.

(c) What conclusion can be drawn from (a) and (b)?

(iv) (a) Find the average speed of the car over the first three minutes.

(b) Draw the secant OC. Find the gradient of OC.

(c) What conclusion can be drawn from (a) and (b)?

(v) (a) Find the average speed of the car over the period from $t = 2$ to $t = 4$.

(b) Draw the secant BD. Find the gradient of BD.

(c) What conclusion can be drawn from (a) and (b)?

(vi) (a) Draw secant OB. Find the gradient of OB.

(b) What is the geometric significance of gradient of secant OB ?

(vii) (a) Draw secant AC . Find the gradient of AC .

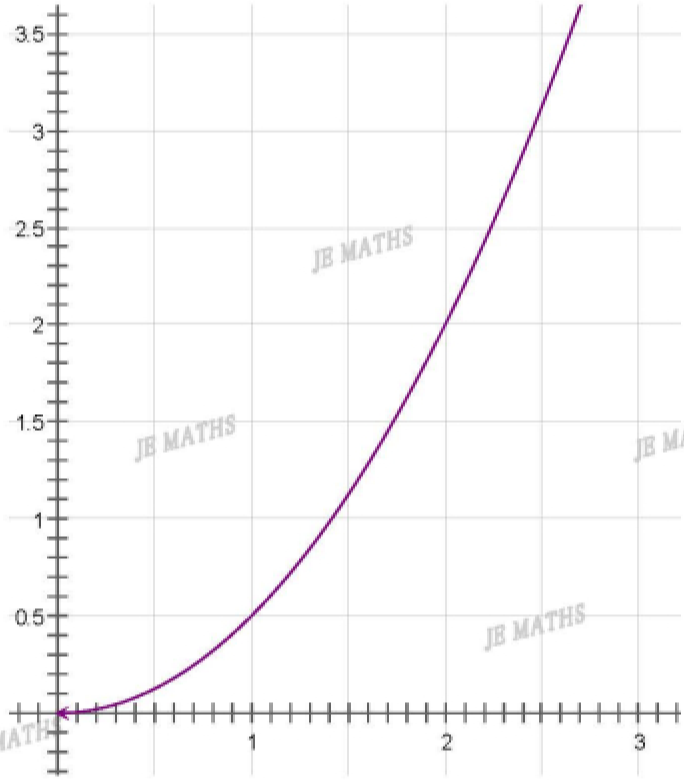
(b) What is the geometric significance of gradient of secant AC ?

2. Given the graph of a function $d = \frac{t^2}{2}$, where d is the distance in kilometres a car travelled in t minutes.

(i) Find the distance travelled at the given times.

t	1	1.125	1.25	1.5	2
d					

(ii) Mark these points A , B_3, B_2 , B_1, B on the graph.



(iii) Find the average speed of the car over the time period:

(a) from $t=1$ to $t=2$

(b) from $t=1$ to $t=1.5$

(c) from $t=1$ to $t=1.25$

(d) from $t=1$ to $t=1.125$

(iv) Draw the corresponding secants AB, AB_1, AB_2, AB_3 whose gradient represents the average speeds in (ii).

(v) What value does the gradient of the secant appear to approach as B get closer to A?

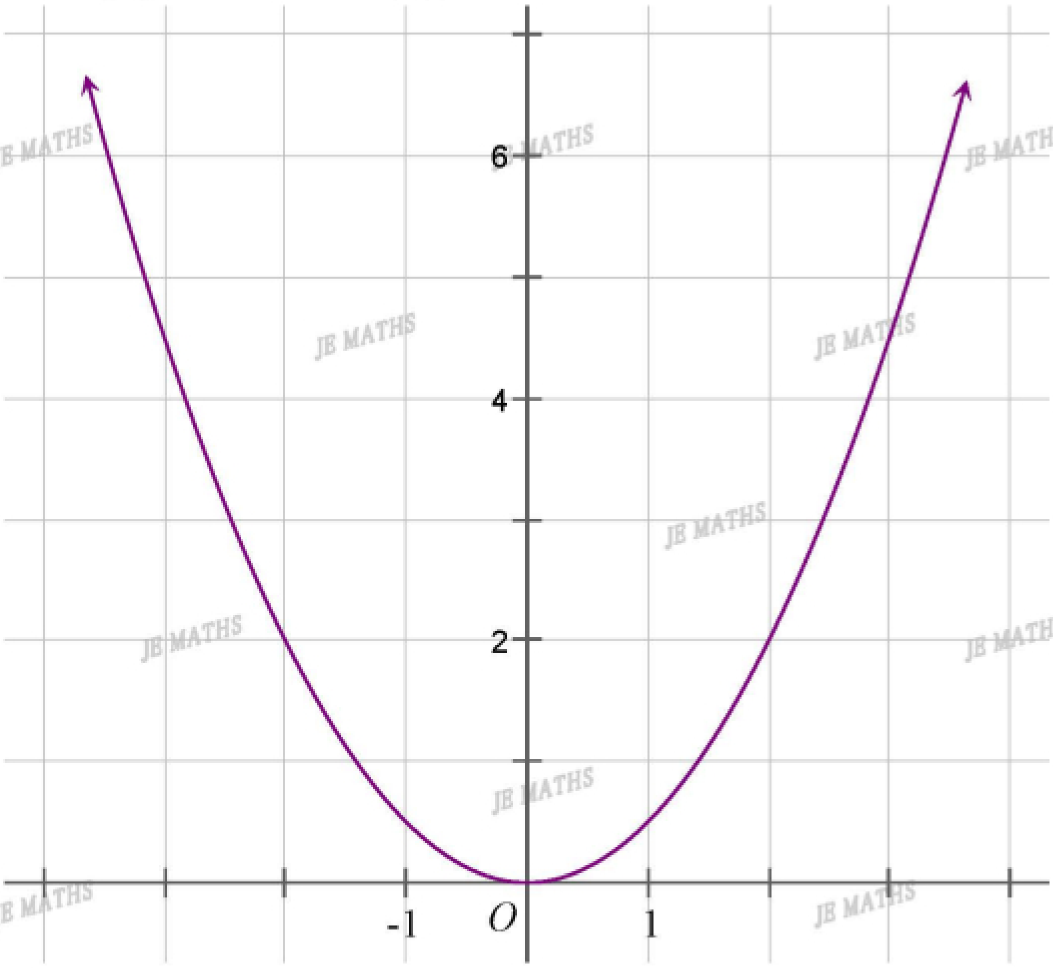
(vi) When the point B coincides with the point A, secant AB becomes the tangent at A.
Gradient of tangent at A is the limit of gradient AB when B get infinitely close to A.
Hence, write down the gradient of tangent to the curve at point A.

(vii) Gradient of tangent at point A represent the instantaneous speed of the car at $t=1$.
Hence, write down the instantaneous speed of the car at $t=1$.

(viii) Similarly, state the geometric significance of the gradient of tangent at $t=3.5$.

Stage 2

1. Consider the graph of the function $f(x)$.



(i) Draw tangent lines at the points where $t = -3, -2, -1, 0, 1, 2, 3$.

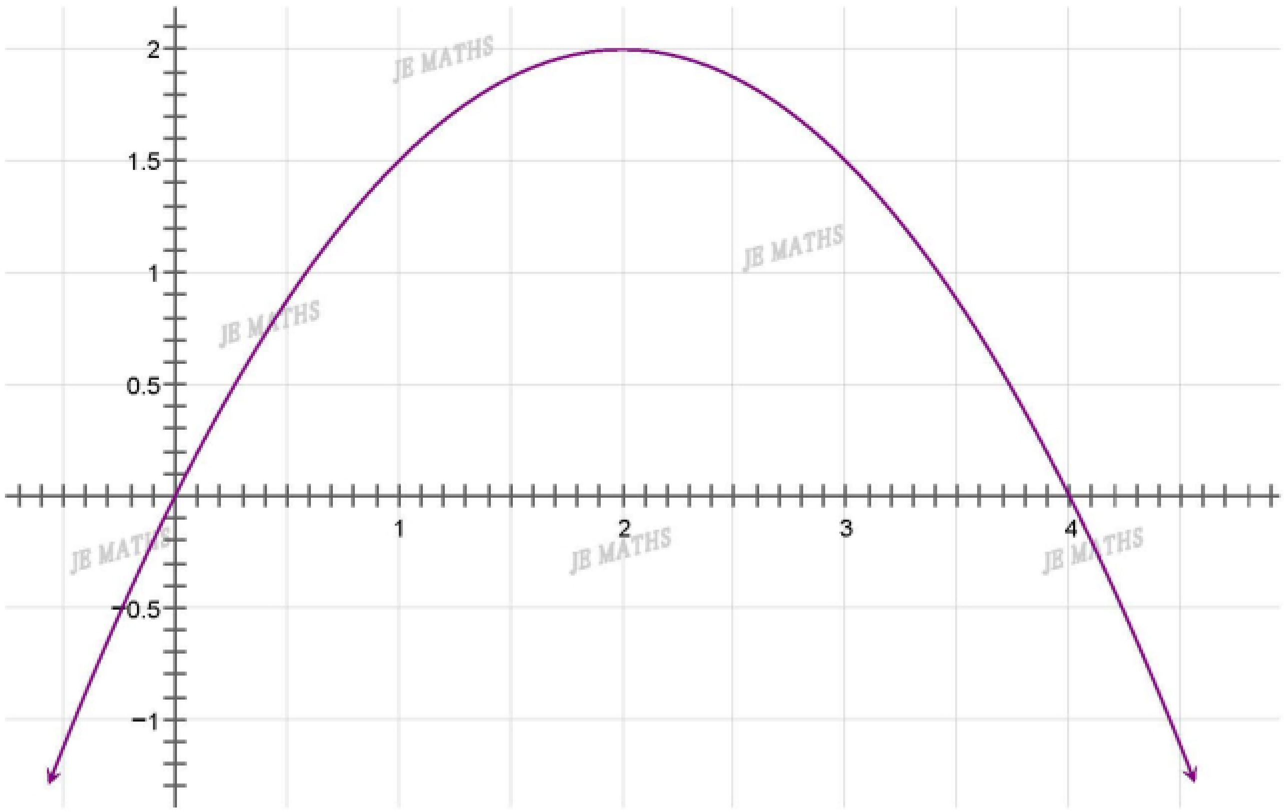
(ii) Use the definition $gradient = \dfrac{rise}{run}$ to measure the gradient of tangent at the given x values.

x	-3	-2	-1	0	1	2	3
m							

(iii) Plot these points in part (ii) on a number plane and sketch the derive function $f'(x)$.

(iv) Hence, make a reasonable guess to the equation of the derivative $f'(x)$.

2. Consider the graph of the function $f(x)$.



(i) Draw tangent lines at the points where $t = 0, 1, 2, 3, 4$.

(ii) Use the definition $gradient = \frac{rise}{run}$ to measure the gradient of tangent at the given x values.

x	0	1	2	3	4
m					

(iii) Plot these points in part (ii) on a number plane and sketch the derive function $f'(x)$.

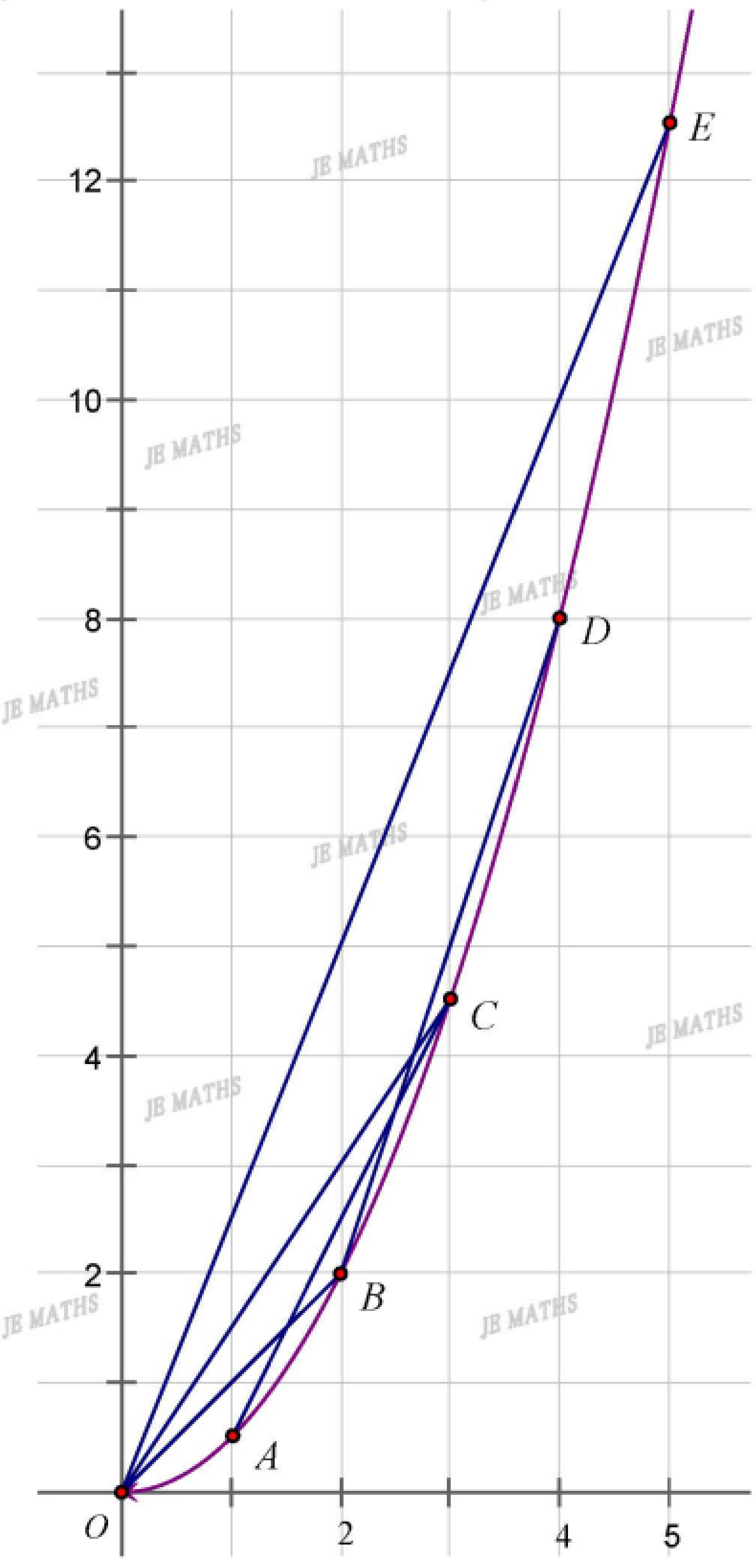
(iv) Hence, make a reasonable guess to the equation of the derivative $f'(x)$.

Stage 1:

1. (i)

t	1	2	3	4	5
d	0.5	2	4.5	8	12.5

(ii)



(iii) (a) $\frac{12.5}{5} = 2.5\,\text{km/min}$

(b) $m_{OE} = \frac{12.5-0}{5-0} = 2.5$

(c) (the gradient of secant OE in the graph represents
the average speed of the car over the first five minutes)

(iv) (a) $\frac{4.5}{3} = 1.5\,\text{km/min}$

(b) $m_{OC} = \frac{4.5-0}{3-0} = 1.5$

(c) (the average speed of the car over the first three minutes
represented by the gradient of secant OC in the graph)

(v) (a) $\frac{8-2}{4-2} = 3\,\text{km/min}$

(b) $m_{BD} = \frac{8-2}{4-2} = 3$

(c) (the average speed of the car over the period from t = 2 tot t = 4 represented by the gradient of secant BD in the graph)

(vi) (a) $m_{OB} = \frac{2-0}{2-0} = 1$

(b) (the average speed of car over the first two minutes is 1km/min)

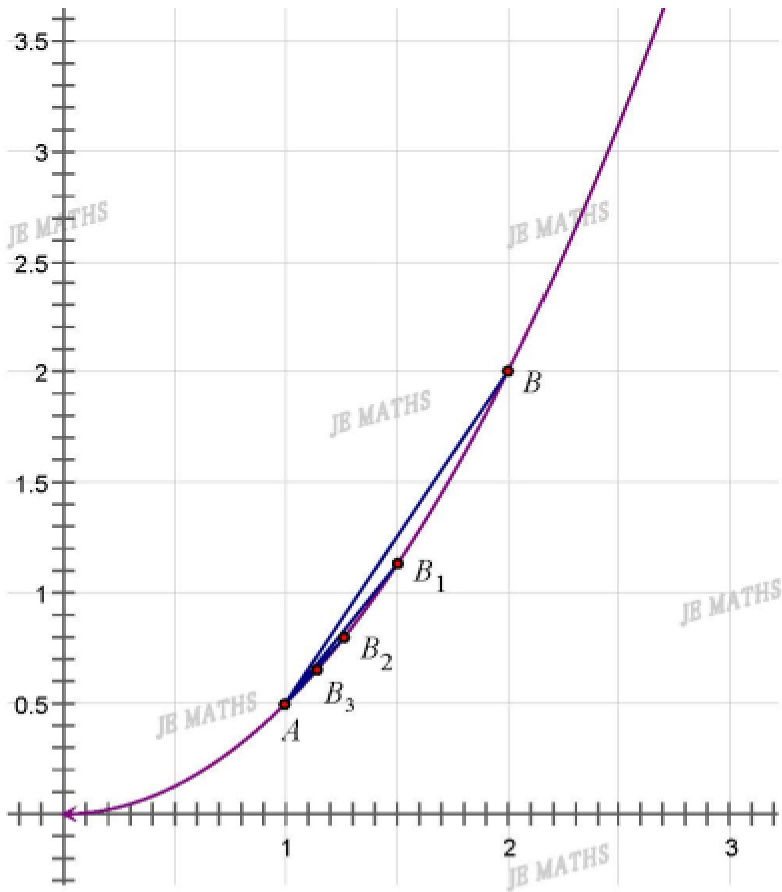
(vii) (a) $m_{AC} = \frac{4.5-0.5}{3-1} = 2$

(b) (the average speed of car over the time from t = 1 to t = 3 is 2km/min)

2. (i)

t	1	1.125	1.25	1.5	2
d	0.5	0.6328125	0.78125	1.125	2

(ii)



(iii) (a) $\dfrac{2-0.5}{2-1} = 1.5\,\text{km/min}$

(b) $\dfrac{1.125-0.5}{1.5-1} = 1.25\,\text{km/min}$

(c) $\dfrac{0.78125-0.5}{1.25-1} = 1.125\,\text{km/min}$

(d) $\dfrac{0.6328125-0.5}{1.125-1} = 1.0625\,\text{km/min}$

(iv)

(v) As point B approaches to point A, gradient of scant approaches to 1.

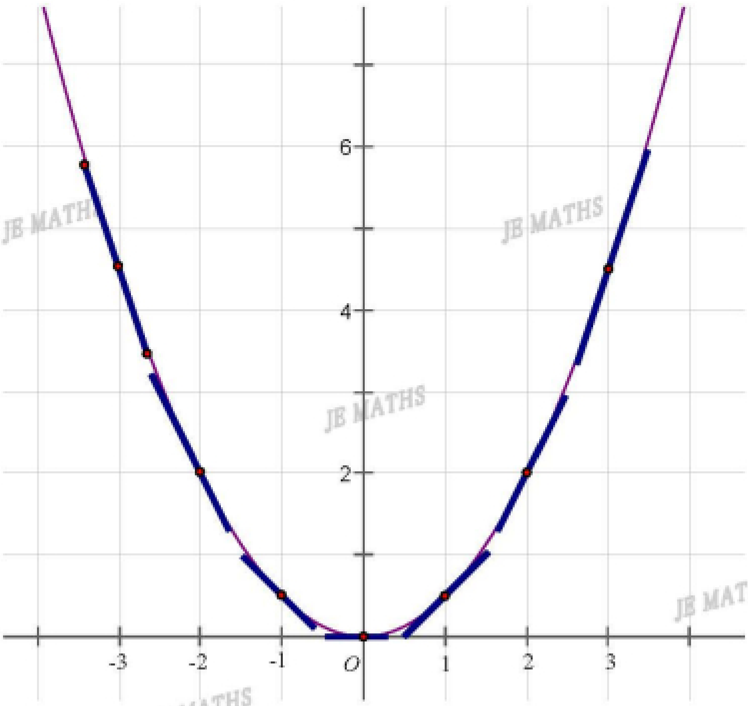
(vi) gradient of tangent at A = 1

(vii) 1 km/min

(viii) the instantaneous speed of the car at t = 3.5 represented by the gradient of tangent at $t = 3.5$

Stage 2

1. (i)

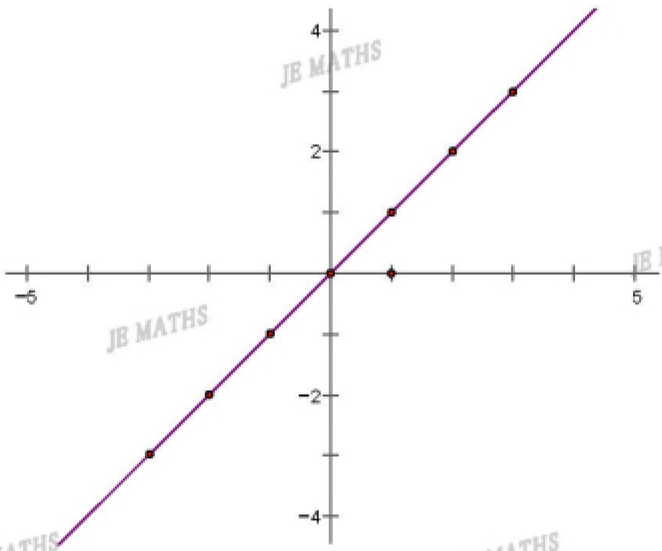


(ii)

x	-3	-2	-1	0	1	2	3
m	-3	-2	-1	0	1	2	3

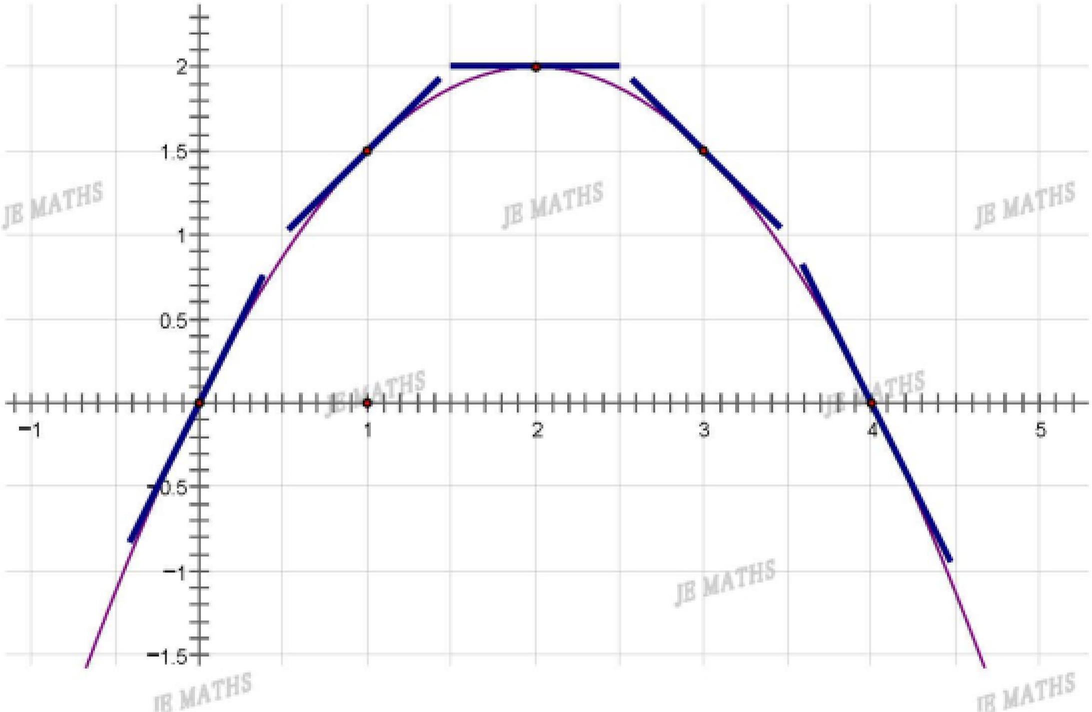
(answers of m that close to those values shown above are acceptable)

(iii)



(iv) $f'(x) = x$

2. (i)

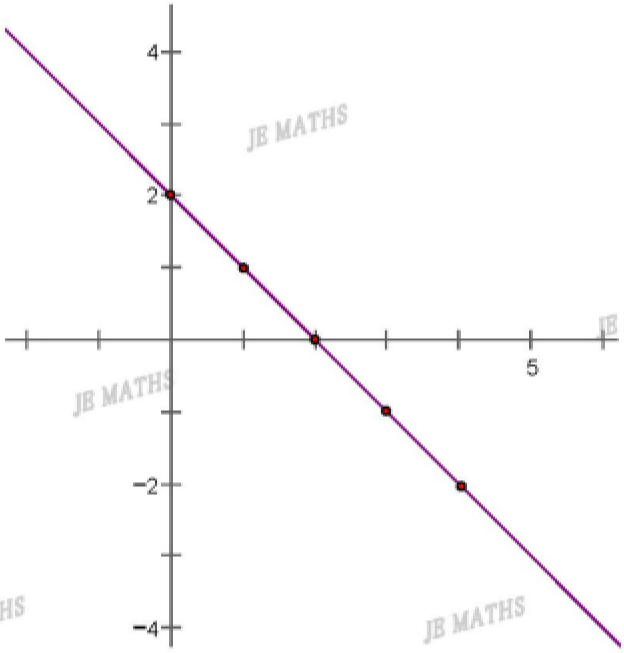


(ii)

x	0	1	2	3	4
m	2	1	0	-1	-2

(answers of m that close to those values shown above are acceptable)

(iii)



(iv) $f'(x) = 2 - x$

3. $f(-2)=3$, $f'(-2)=0$

$f(0)=3$, $f'(0)=0$

$f\left(\frac{1}{4}\right)=3$, $f'\left(\frac{1}{4}\right)=0$

$f'(x)=0$

4. $f(-1)=-3$, $f'(-1)=2$

$f(0)=-1$, $f'(0)=2$

$f(2)=3$, $f'(2)=2$

$f'(x)=2$

5. $f(0)=2$, $f'(0)=-\frac{1}{4}$

$f(4)=1$, $f'(4)=-\frac{1}{4}$

$f(8)=0$, $f'(8)=-\frac{1}{4}$

$f'(x)=-\frac{1}{4}$

6. (a)~(f) $f'(x)=0$

7. (a) $f'(x)=1$ (b) $f'(x)=2$

(c) $f'(x)=11$ (d) $f'(x)=\frac{3}{4}$

(e) $f'(x)=-1$ (f) $f'(x)=-3$

(g) $f'(x)=-\frac{5}{2}$ (h) $f'(x)=\sqrt{3}$

8. (a) $f'(x)=1$ (b) $f'(x)=1$

(c) $f'(x)=8$ (d) $f'(x)=6$

(e) $f'(x)=-4$ (f) $f'(x)=-15$

(g) $f'(x)=\frac{1}{2}$ (h) $f'(x)=-\frac{13}{4}$

(i) $f'(x)=-\frac{1}{3}$ (j) $f'(x)=-\frac{5}{7}$

(k) $f'(x)=10$ (l) $f'(x)=-\frac{7}{3}$

(m) $f'(x)=-\frac{1}{5}$ (n) $f'(x)=-\frac{3}{8}$