Problem Set 5

MaPS Correspondence Program

Instructions

- Some of these problems are based off the notes "*Mathematical Induction*". Some other are revision problems for the previous notes.
- They are in roughly difficulty order and get quite difficult, so you are **not** expected to be able to solve every problem.
- However, please attempt as many questions as you can and submit your solutions to your mentor for marking and feedback.
- You may (and encouraged to) submit incomplete solutions if you can not solve a problem completely.
- You may type your solutions or submit a pdf of a **clear** scan/photo of **legible** written solutions.
- Feel free to discuss these problems with your peers and on the forum but the solutions you submit must be written by yourself.

Problems

1. Prove that for all positive integers n,
$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$
2. Seven points lie inside a regular hexagon of side length 1. Show that there are two of these points which are at most 1 unit apart.
3. Prove that for all positive integers n,
$$7 + 77 + 777 + \cdots + \underbrace{77\ldots77}_{n\text{digits}} = \frac{7(10^{n+1} - 9n - 10)}{81}$$
4. Let $\mathcal{C}_1$ and $\mathcal{C}_2$ be two circles intersecting at A and B. Let P and Q be any two points on $\mathcal{C}_1$ on the same side of AB. Let PA extended and QA extended meet $\mathcal{C}_2$ at R and S respectively.
   (a) Prove that $\triangle QBS$ is similar to $\triangle PBR$.
   (b) Prove that $\triangle QBP$ is similar to $\triangle SBR$
5. Prove that any triangle can be dissected into n isosceles triangles for $n \geq 4$.