



Stage 1:

1. Use Parametric Differentiation to find $\frac{dy}{dx}$ in terms of the parameter t.

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(a) x = 2t + 1, $y = t^2 - 4t$

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(b) $x = t^2 - 3$, $y = \frac{1}{t}$

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(c) x=t+4, $y=\frac{1}{t^3}$

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(d) $x = \sqrt{t}$, $y = 2t^3 + 5$

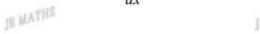
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(e) $x = \frac{1}{t^2}$, $y = \frac{1}{\sqrt{t}}$

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- 2. A curve is defined by $x = \frac{(t+1)^2}{2}$, $y = \frac{(t+1)^3}{3}$.
 - (i) Find the derivative $\frac{dy}{dx}$ of the curve in terms of t. JE MATHS









(ii) Find the gradient of the tangent and hence the point of contact if:

(a)
$$t = -3$$
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(b) t = 0

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(c) $t = \frac{1}{2}$

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- (iii) Find where on the curve the tangent has:
 - (a) gradient 5

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(b) an angle of inclination of 135°.

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JE MATHS (iv) Find the equation of tangent at the point where $x = \frac{(t+1)^2}{2}$.

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(v) Find the area of the triangle formed by the tangent in (iv) and xy axis.

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3. (i) Show that the rectangular hyperbola $xy = c^2$ can be described by the two parametric

equation x = ct and $y = \frac{c}{t}$.

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(ii) Use parametric differentiation to find the equation of tangent at $T\left(ct, \frac{c}{t}\right)$.

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(iii) Let tangent at T meet axis at point A and B. Show that the area of triangle OAB is constant as point T varies.

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(iv) Show that point T bisects AB. Hence, Show that AB = 2OT.

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Stage 2:

- 1. Use $\frac{dy}{dx} = \frac{1}{dx/dy}$ to differentiate:
 - (a) $y = \sqrt{4-x}$

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(b) $y = \frac{1}{\sqrt[5]{x}}$ JE MATHS

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(c) $y = \frac{1}{\sqrt[3]{x+1}}$

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2. (i) Use $\frac{dy}{dx} = \frac{1}{dx/dy}$ to differentiate $y = x^{\frac{1}{k}}$ for non-zero integer k.

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(ii) Use chain rule $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ to prove the Power Rule for rational indices:

 $\frac{d}{dx}\left(x^{\frac{n}{k}}\right) = \frac{n}{k} \times x^{\frac{n}{k}-1}, \text{ where } n, k \text{ are integers.}$

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Stage 3:

1. Find where on the curve the tangent is horizontal.

(a)
$$y = \frac{-3}{x^4 - 2x^2 - 1}$$

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(b) $y = \sqrt{x^2 - 4x}$

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2. Find the equations of the tangent and normal to the curve:

(a)
$$y = \frac{1}{1+x^2}$$
 at $x = 1$

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(b) $y = \sqrt{8-x}$ at x = 4

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- 3. Find the value of k if:
 - (a) the tangent to $y = \frac{1}{x^2 + k}$ at x = 1 has a gradient $-\frac{1}{2}$

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(b) the normal to $y = \sqrt{x+k}$ at x = 4 has gradient -6_B MATHS

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- 4. Let the function $y = \frac{1}{8-x}$.
 - (i) Find the equation of tangent at the point where x = p.

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- (ii) Hence, find the equation of tangents passing through:
 - (a) the origin

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(b) (10, 0)

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Stage 4:

1. Consider the semi circle $y = \sqrt{16 - x^2}$ and semi ellipse $y = k\sqrt{16 - x^2}$ (k > 0) and $k \ne 1$.

(i) Find the equation of tangent to the semi circle and semi ellipse at x = t, where 0 < t < 4, respectively.

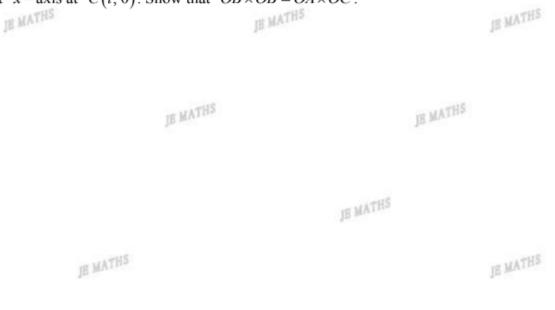


(ii) Find the x-intercept of these two tangents in part (i). Hence, show that these two tangents in part (i) meet each other at x-axis.



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(iii) Suppose the tangent to the semi ellipse at point T where x = t (0 < t < 4) meet x - axis at point A, the semi ellipse meet x -axis at B(4, 0) and the vertical line through T meet x -axis at C(t, 0). Show that $OB \times OB = OA \times OC$.



2. Consider the parabola $y = a(x - h)^2 + k$.

(i) Find point P and Q on the curve where the tangent has gradient m and -m.



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(ii) Show that the area of the quadrilateral formed by the tangents and normals at P and Q

is
$$\frac{m(1+m^2)}{4a^2}$$
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- 3. Consider the parabola $y = a(x-h)^2 + k$.
 - (i) Show that the equation of tangent at point T where x = t is $y = 2a(t-h)x at^2 + ah^2 + k$.

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(ii) Find the x-coordinate of the point(s) on the curve where the tangent passes through the origin.



(iii) Let the vertical line through the vertex V(h, k) meet the tangent at T at point P.

Find the length of VP. Hence, show that VP is proportional to the square of the horizontal distance from point T to the axis of symmetry of the parabola.



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