Stage 3

1. Differentiate each function by product rule.
   Fully factorize your answers and find the values of x for which the derivative is zero.

   (a) $y = x^3(3x+8)$

   (b) $y = (3x-2)(2x+1)$

   (c) $y = x(x+3)^4$

(d) $y = 3x(x-2)^5$

(e) $y = x(1-x)^6$

(f) $y = x^2(x+1)^4$

(g) $y = x^2(1-4x)^3$

(h) $y = x^3(3x+1)^4$

(i) $y = x^5(1-x)^7$

(j) $y = 9x^3(2x-7)^4$

(k) $y = 3x^4(4-x)^3$

(l) $y = 2x^5(5x+3)^3$

(m) $y=(x-1)(x-2)^3$

(n) $y=(x+2)(x+4)^6$

(o) $y=(x+1)(2x+5)^4$

2. (i) Find the derivative of $y = x^2(2x-1)^4$.

(ii) Hence, find the equation of tangent and normal to the curve at the point $P(1, 1)$.

3. (i) Find the derivative of $y = (2x+3)(3x-1)^5$.

(ii) Hence, find the equation of tangent and normal to the curve at the point $A(0, -3)$.

4. (i) Find the derivative of $y = (2x-1)^3(x-2)^4$.

(ii) Hence, find the equation of tangent and normal to the curve at the point $P(1, 1)$.

5. Differentiate each function by product rule. Fully factorised your answers.

(a) $y = x(1 - x^2)^5$

(b) $y = 2x^2(x^2 + 3)^4$

(c) $y = 4x^4(x^2 + x - 1)^3$

Stage 4

1. Differentiate each function by quotient rule.
   Fully factorize your answers and find the values of x for which the derivative is zero.

   (a) $y = \dfrac{x^3}{x^2 - 4}$

   (b) $y = \dfrac{x}{2x^2 - 1}$

   (c) $y = \dfrac{x+1}{3x^2 - 7}$

(d) $y = \dfrac{x^2 - 4x - 1}{3x + 4}$

(e) $y = \dfrac{4x^2 - 2}{x^2 + 5}$

(f) $y = \dfrac{x^3 + 2x - 1}{x + 3}$ (x values for $y' = 0$ are not required in this question)

(g) $y = \dfrac{x+1}{x^3-1}$ (x values for $y' = 0$ are not required in this question)

(h) $y = \dfrac{x-1}{(7x+2)^4}$

(i) $y = \dfrac{x}{(x^2+1)^2}$

2. (i) Find the derivative of $y=\dfrac{4x+5}{1-2x}$.

(ii) Find the gradient of tangent at point $A\left(2,-\dfrac{13}{3}\right)$ and its angle of inclination.

(iii) Find the equation of tangent and normal at point A.

3. (i) Find the derivative of $y = \frac{x^2 - 1}{x + 3}$.

(ii) Find the gradient of tangent at point $P(-1, 0)$ and its angle of inclination.

(iii) Find the equation of tangent and normal at point A.

4. Let the function $y = \dfrac{x}{x+2}$.

(a) Find the equation of tangent to the curve at the origin O.

(b) (i) Find the equation of tangent to the curve at the point $P(-3, 3)$.

(ii) Find the points A and B where the tangent at P meets at the x and y axis.

(iii) Find the area of triangle OAB.

(c) Find the point where the tangent at O and at P intersect.

Stage 3

1. (a) $y' = (x^3)' \cdot (3x+8) + x^3 \cdot (3x+8)'$

$= 3x^2(3x+8) + 3x^3$

$= 3x^2(3x+8+x)$

$= 12x^2(x+2)$

$\therefore \quad y' = 12x^2(x+2)$

Let $y' = 0 \Rightarrow 12x^2(x+2) = 0$

$\Rightarrow x = 0, \; x = -2$

(b) $y' = (3x-2)' \cdot (2x+1) + (3x-2) \cdot (2x+1)'$

$= 3(2x+1) + 2(3x-2)$

$= 12x - 1$

$\therefore \quad y' = 12x - 1$

Let $y' = 0 \Rightarrow 12x - 1 = 0$

$\Rightarrow x = \dfrac{1}{12}$

(c) $y' = x' \cdot (x+3)^4 + x \cdot \left[(x+3)^4\right]'$

$= (x+3)^4 + 4x(x+3)^3 \cdot (x+3)'$

$= (x+3)^4 + 4x(x+3)^3$

$= (x+3)^3(x+3+4x)$

$= (x+3)^3(5x+3)$

$\therefore \quad y' = (x+3)^3(5x+3)$

Let $y'=0 \;\Rightarrow\; (x+3)^3(5x+3)=0$

$\Rightarrow\; x=-3,\; x=-\dfrac{3}{5}$

(d) $y'=(3x)'\cdot(x-2)^5+3x\cdot\left[(x-2)^5\right]'$

$=3(x-2)^5+15x(x-2)^4\cdot(x-2)'$

$=3(x-2)^5+15x(x-2)^4$

$=3(x-2)^4\left[(x-2)+5x\right]$

$=3(x-2)^4(6x-2)$

$=6(x-2)^4(3x-1)$

$\therefore\; y'=6(x-2)^4(3x-1)$

Let $y'=0 \;\Rightarrow\; 6(x-2)^4(3x-1)=0$

$\Rightarrow\; x=2,\; x=\dfrac{1}{3}$

(e) $y'=x'\cdot(1-x)^6+x\cdot\left[(1-x)^6\right]'$

$=(1-x)^6+6x(1-x)^5\cdot(1-x)'$

$=(1-x)^6+6x(1-x)^5\cdot(-1)$

$=(1-x)^6-6x(1-x)^5$

$=(1-x)^5\left[(1-x)-6x\right]$

$=(1-x)^5(1-7x)$

$\therefore\; y'=(1-x)^5(1-7x)$

Let $y'=0 \;\Rightarrow\; (1-x)^5(1-7x)=0$

$\Rightarrow\; x=1,\; x=\dfrac{1}{7}$

(f) $y' = \left(x^2\right)' \cdot (x+1)^4 + x^2 \cdot \left[(x+1)^4\right]'$

$= 2x(x+1)^4 + 4x^2(x+1)^3 \cdot (x+1)'$

$= 2x(x+1)^4 + 4x^2(x+1)^3$

$= 2x(x+1)^3\left[(x+1)+2x\right]$

$= 2x(x+1)^3(3x+1)$

$\therefore \quad y' = 2x(x+1)^3(3x+1)$

Let $y' = 0 \;\Rightarrow\; 2x(x+1)^3(3x+1) = 0$

$\Rightarrow \; x = 0, \; x = -1, \; x = -\dfrac{1}{3}$

(g) $y' = \left(x^2\right)' \cdot (1-4x)^3 + x^2 \cdot \left[(1-4x)^3\right]'$

$= 2x(1-4x)^3 + 3x^2(1-4x)^2 \cdot (1-4x)'$

$= 2x(1-4x)^3 + 3x^2(1-4x)^2 \cdot (-4)$

$= 2x(1-4x)^3 - 12x^2(1-4x)^2$

$= 2x(1-4x)^2\left[(1-4x)-6x\right]$

$= 2x(1-4x)^2(1-10x)$

$\therefore \quad y' = 2x(1-4x)^2(1-10x)$

Let $y' = 0 \;\Rightarrow\; 2x(1-4x)^2(1-10x) = 0$

$\Rightarrow \; x = 0, \; x = \dfrac{1}{4}, \; x = \dfrac{1}{10}$

(h) $y' = \left(x^3\right)' \cdot (3x+1)^4 + x^3 \cdot \left[(3x+1)^4\right]'$

$= 3x^2(3x+1)^4 + 4x^3(3x+1)^3 \cdot (3x+1)'$

$= 3x^2(3x+1)^4 + 4x^3(3x+1)^3 \cdot 3$

$= 3x^2(3x+1)^4 + 12x^3(3x+1)^3$

$= 3x^2(3x+1)^3\left[(3x+1)+4x\right]$

$= 3x^2(3x+1)^3(7x+1)$

$\therefore \quad y' = 3x^2(3x+1)^3(7x+1)$

Let $y'=0 \Rightarrow 3x^2(3x+1)^3(7x+1)=0$

$\Rightarrow x=0,\ x=-\dfrac{1}{3},\ x=-\dfrac{1}{7}$

(i) $y' = \left(x^5\right)' \cdot (1-x)^7 + x^5 \cdot \left[(1-x)^7\right]'$

$= 5x^4(1-x)^7 + 7x^5(1-x)^6 \cdot (1-x)'$

$= 5x^4(1-x)^7 + 7x^5(1-x)^6 \cdot (-1)$

$= 5x^4(1-x)^7 - 7x^5(1-x)^6$

$= x^4(1-x)^6\left[5(1-x)-7x\right]$

$= x^4(1-x)^6(5-12x)$

$\therefore \quad y' = x^4(1-x)^6(5-12x)$

Let $y'=0 \Rightarrow x^4(1-x)^6(5-12x)=0$

$\Rightarrow x=0,\ x=1,\ x=\dfrac{5}{12}$

(j) $y' = \left(9x^3\right)' \cdot (2x-7)^4 + 9x^3 \cdot \left[(2x-7)^4\right]'$

$= 27x^2(2x-7)^4 + 9x^3 \cdot 4(2x-7)^3 \cdot (2x-7)'$

$= 27x^2(2x-7)^4 + 9x^3 \cdot 4(2x-7)^3 \cdot 2$

$= 27x^2(2x-7)^4 + 72x^3(2x-7)^3$

$= 9x^2(2x-7)^3\left[3(2x-7)+8x\right]$

$= 9x^2(2x-7)^3(14x-21)$

$= 63x^2(2x-7)^3(2x-3)$

$\therefore \quad y' = 63x^2(2x-7)^3(2x-3)$

Let $y' = 0 \Rightarrow 63x^2(2x-7)^3(2x-3) = 0$

$\Rightarrow \quad x = 0, \quad x = \frac{7}{2}, \quad x = \frac{3}{2}$

(k) $y' = \left(3x^4\right)' \cdot (4-x)^3 + 3x^4 \cdot \left[(4-x)^3\right]'$

$= 12x^3(4-x)^3 + 3x^4 \cdot 3(4-x)^2 \cdot (4-x)'$

$= 12x^3(4-x)^3 + 3x^4 \cdot 3(4-x)^2 \cdot (-1)$

$= 12x^3(4-x)^3 - 9x^4(4-x)^2$

$= 3x^3(4-x)^2\left[4(4-x)-3x\right]$

$= 3x^3(4-x)^2(16-7x)$

$\therefore \quad y' = 3x^3(4-x)^2(16-7x)$

Let $y' = 0 \Rightarrow 3x^3(4-x)^2(16-7x) = 0$

$\Rightarrow \quad x = 0, \quad x = 4, \quad x = \frac{16}{7}$

(l) $y' = \left(2x^5\right)' \cdot (5x+3)^3 + 2x^5 \cdot \left[(5x+3)^3\right]'$

$= 10x^4(5x+3)^3 + 2x^5 \cdot 3(5x+3)^2 \cdot (5x+3)'$

$= 10x^4(5x+3)^3 + 2x^5 \cdot 3(5x+3)^2 \cdot 5$

$= 10x^4(5x+3)^3 + 30x^5(5x+3)^2$

$= 10x^4(5x+3)^2\left[(5x+3)+3x\right]$

$= 10x^4(5x+3)^2(8x+3)$

$\therefore \quad y' = 10x^4(5x+3)^2(8x+3)$

Let $y' = 0 \Rightarrow 10x^4(5x+3)^2(8x+3) = 0$

$\Rightarrow x = 0, \; x = -\frac{3}{5}, \; x = -\frac{3}{8}$

(m) $y' = (x-1)' \cdot (x-2)^3 + (x-1) \cdot \left[(x-2)^3\right]'$

$= (x-2)^3 + (x-1) \cdot 3(x-2)^2 \cdot (x-2)'$

$= (x-2)^3 + 3(x-1)(x-2)^2$

$= (x-2)^2\left[(x-2)+3(x-1)\right]$

$= (x-2)^2(4x-5)$

$\therefore \quad y' = (x-2)^2(4x-5)$

Let $y' = 0 \Rightarrow (x-2)^2(4x-5) = 0$

$\Rightarrow x = 2, \; x = \frac{5}{4}$

(n) $y' = (x+2)' \cdot (x+4)^6 + (x+2) \cdot \left[(x+4)^6\right]'$

$= (x+4)^6 + (x+2) \cdot 6(x+4)^5 \cdot (x+4)'$

$= (x+4)^6 + 6(x+2)(x+4)^5$

$= (x+4)^5 \left[(x+4) + 6(x+2)\right]$

$= (x+4)^5 (7x+16)$

$\therefore \quad y' = (x+4)^5 (7x+16)$

Let $y' = 0 \Rightarrow (x+4)^5 (7x+16) = 0$

$\Rightarrow x = -4, \ x = -\dfrac{16}{7}$

(o) $y' = (x+1)' \cdot (2x+5)^4 + (x+1) \cdot \left[(2x+5)^4\right]'$

$= (2x+5)^4 + (x+1) \cdot 4(2x+5)^3 \cdot (2x+5)'$

$= (2x+5)^4 + (x+1) \cdot 4(2x+5)^3 \cdot 2$

$= (2x+5)^4 + 8(x+1)(2x+5)^3$

$= (2x+5)^3 \left[(2x+5) + 8(x+1)\right]$

$= (2x+5)^3 (10x+13)$

$\therefore \quad y' = (2x+5)^3 (10x+13)$

Let $y' = 0 \Rightarrow (2x+5)^3 (10x+13) = 0$

$\Rightarrow x = -\dfrac{5}{2}, \ x = -\dfrac{13}{10}$

2. (i) $y' = \left(x^2\right)' \cdot (2x-1)^4 + x^2 \cdot \left[(2x-1)^4\right]'$

$= 2x(2x-1)^4 + x^2 \cdot 4(2x-1)^3 \cdot (2x-1)'$

$= 2x(2x-1)^4 + x^2 \cdot 4(2x-1)^3 \cdot 2$

$= 2x(2x-1)^4 + 8x^2(2x-1)^3$

$= 2x(2x-1)^3\left[(2x-1)+4x\right]$

$= 2x(2x-1)^3(6x-1)$

$\therefore \quad y' = 2x(2x-1)^3(6x-1)$

(ii) $y' = 2x(2x-1)^3(6x-1) \Rightarrow y'(1) = 10$

$\Rightarrow m_T = 10, \; m_N = -\frac{1}{10}$ at point $P(1,1)$

$l_T: \quad y-1 = 10(x-1) \Rightarrow 10x - y - 9 = 0$

$l_N: \quad y-1 = -\frac{1}{10}(x-1) \Rightarrow x + 10y - 11 = 0$

3. (i) $y' = (2x+3)' \cdot (3x-1)^5 + (2x+3) \cdot \left[(3x-1)^5\right]'$

$= 2(3x-1)^5 + (2x+3) \cdot 5(3x-1)^4 \cdot (3x-1)'$

$= 2(3x-1)^5 + (2x+3) \cdot 5(3x-1)^4 \cdot 3$

$= 2(3x-1)^5 + 15(2x+3)(3x-1)^4$

$= (3x-1)^4\left[2(3x-1) + 15(2x+3)\right]$

$= (3x-1)^4(36x+43)$

$\therefore \quad y' = (3x-1)^4(36x+43)$

(ii) $y' = (3x-1)^4(36x+43) \Rightarrow y'(0) = 43$

$$\Rightarrow m_T = 43,\ m_N = -\frac{1}{43} \text{ at point } A(0,-3)$$

$l_T:\ y+3 = 43x \Rightarrow 43x - y - 3 = 0$

$l_N:\ y+3 = -\frac{1}{43}x \Rightarrow x + 43y + 129 = 0$

4. (i) $y' = \left[(2x-1)^3\right]' \cdot (x-2)^4 + (2x-1)^3 \cdot \left[(x-2)^4\right]'$

$= 3(2x-1)^2 \cdot (2x-1)' \cdot (x-2)^4 + (2x-1)^3 \cdot 4(x-2)^3 \cdot (x-2)'$

$= 3(2x-1)^2 \cdot 2 \cdot (x-2)^4 + (2x-1)^3 \cdot 4(x-2)^3 \cdot 1$

$= 6(2x-1)^2(x-2)^4 + 4(2x-1)^3(x-2)^3$

$= (2x-1)^2(x-2)^3\left[6(x-2) + 4(2x-1)\right]$

$= (2x-1)^2(x-2)^3(14x-16)$

$= 2(2x-1)^2(x-2)^3(7x-8)$

$\therefore\ y' = 2(2x-1)^2(x-2)^3(7x-8)$

(ii) $y' = 2(2x-1)^2(x-2)^3(7x-8) \Rightarrow y'(1) = 2$

$$\Rightarrow m_T = 2,\ m_N = -\frac{1}{2} \text{ at point } P(1,1)$$

$l_T:\ y-1 = 2(x-1) \Rightarrow 2x - y - 1 = 0$

$l_N:\ y-1 = -\frac{1}{2}(x-1) \Rightarrow x + 2y - 3 = 0$

5. (a) $y' = \left(x\right)' \cdot (1-x^2)^5 + x\left[(1-x^2)^5\right]'$

$= (1-x^2)^5 + x \cdot 5(1-x^2)^4 \cdot (1-x^2)'$

$= (1-x^2)^5 + x \cdot 5(1-x^2)^4 \cdot (-2x)$

$= (1-x^2)^5 - 10x^2(1-x^2)^4$

$= (1-x^2)^4\left[(1-x^2) - 10x^2\right]$

$= (1-x^2)^4(1-11x^2)$

$\therefore \quad y' = (1-x^2)^4(1-11x^2)$

(b) $y' = \left(2x^2\right)' \cdot (x^2+3)^4 + 2x^2\left[(x^2+3)^4\right]'$

$= 4x(x^2+3)^4 + 2x^2 \cdot 4(x^2+3)^3 \cdot (x^2+3)'$

$= 4x(x^2+3)^4 + 2x^2 \cdot 4(x^2+3)^3 \cdot 2x$

$= 4x(x^2+3)^4 + 16x^3(x^2+3)^3$

$= 4x(x^2+3)^3\left[(x^2+3) + 4x^2\right]$

$= 4x(x^2+3)^3(5x^2+3)$

$\therefore \quad y' = 4x(x^2+3)^3(5x^2+3)$

(c) $y' = \left(4x^4\right)' \cdot (x^2+x-1)^3 + 4x^4\left[(x^2+x-1)^3\right]'$

$= 16x^3(x^2+x-1)^3 + 4x^4 \cdot 3(x^2+x-1)^2 \cdot (x^2+x-1)'$

$= 16x^3(x^2+x-1)^3 + 12x^4(x^2+x-1)^2 \cdot (2x+1)$

$= 4x^3(x^2+x-1)^2\left[4(x^2+x-1) + 3x(2x+1)\right]$

$= 4x^3(x^2+x-1)^2(10x^2+7x-4)$

$\therefore \quad y' = 4x^3(x^2+x-1)^2(10x^2+7x-4)$

Stage 4

1. (a) $y' = \dfrac{(x^3)' \cdot (x^2-4) - x^3 \cdot (x^2-4)'}{(x^2-4)^2}$

$= \dfrac{3x^2(x^2-4) - x^3 \cdot 2x}{(x^2-4)^2}$

$= \dfrac{x^2\left[3(x^2-4) - 2x^2\right]}{(x^2-4)^2}$

$= \dfrac{x^2(x^2-12)}{(x^2-4)^2}$

Let $y' = 0 \Rightarrow \dfrac{x^2(x^2-12)}{(x^2-4)^2} = 0$

$\Rightarrow x^2(x^2-12) = 0$

$\Rightarrow x = 0,\ x = 2\sqrt{3},\ x = -2\sqrt{3}$

(b) $y' = \dfrac{x' \cdot (2x^2-1) - x \cdot (2x^2-1)'}{(2x^2-1)^2}$

$= \dfrac{(2x^2-1) - x \cdot 4x}{(2x^2-1)^2}$

$= \dfrac{-2x^2-1}{(2x^2-1)^2}$

Let $y' = 0 \Rightarrow \dfrac{-2x^2-1}{(2x^2-1)^2} = 0$

$\Rightarrow 2x^2+1 = 0$

$\Rightarrow$ no solutions for x

(c) $y' = \dfrac{(x+1)' \cdot (3x^2-7) - (x+1) \cdot (3x^2-7)'}{(3x^2-7)^2}$

$$= \frac{(3x^2-7) - (x+1) \cdot 6x}{(3x^2-7)^2}$$

$$= \frac{-3x^2-7-6x}{(2x^2-1)^2}$$

Let $y' = 0 \;\Rightarrow\; \dfrac{-3x^2-7-6x}{(2x^2-1)^2} = 0$

$\Rightarrow\; 3x^2+6x+7=0$

$\Rightarrow$ no solutions for x

(d) $y' = \dfrac{(x^2-4x-1)' \cdot (3x+4) - (x^2-4x-1) \cdot (3x+4)'}{(3x+4)^2}$

$$= \frac{(2x-4) \cdot (3x+4) - (x^2-4x-1) \cdot 3}{(3x+4)^2}$$

$$= \frac{(6x^2-4x-16) - (3x^2-12x-3)}{(3x+4)^2}$$

$$= \frac{3x^2+8x-13}{(3x+4)^2}$$

Let $y' = 0 \;\Rightarrow\; \dfrac{3x^2+8x-13}{(3x+4)^2} = 0$

$\Rightarrow\; 3x^2+8x-13=0$

$\Rightarrow\; x = \dfrac{-8 \pm \sqrt{220}}{6}$

$\Rightarrow\; x = \dfrac{-4 \pm \sqrt{55}}{3}$

(e) $$y' = \frac{(4x^2-2)' \cdot (x^2+5) - (4x^2-2) \cdot (x^2+5)'}{(x^2+5)^2}$$

$$= \frac{8x \cdot (x^2+5) - (4x^2-2) \cdot 2x}{(x^2+5)^2}$$

$$= \frac{8x^3+40x-8x^3+4x}{(x^2+5)^2}$$

$$= \frac{44x}{(x^2+5)^2}$$

Let $y' = 0 \Rightarrow \dfrac{44x}{(x^2+5)^2} = 0$

$$\Rightarrow 44x = 0$$
$$\Rightarrow x = 0$$

(f) $$y' = \frac{(x^3+2x-1)' \cdot (x+3) - (x^3+2x-1) \cdot (x+3)'}{(x+3)^2}$$

$$= \frac{(3x^2+2) \cdot (x+3) - (x^3+2x-1)}{(x+3)^2}$$

$$= \frac{(3x^3+9x^2+2x+6) - (x^3+2x-1)}{(x+3)^2}$$

$$= \frac{2x^3+9x^2+7}{(x+3)^2}$$ (x values for $y'=0$ are not required in this question)

(g) $$y' = \frac{(x+1)' \cdot (x^3-1) - (x+1) \cdot (x^3-1)'}{(x^3-1)^2}$$

$$= \frac{(x^3-1) - (x+1) \cdot 3x^2}{(x^3-1)^2}$$

$$= \frac{-2x^3-3x^2-1}{(x^3-1)^2}$$ (x values for $y'=0$ are not required in this question)

(h) $$y' = \frac{(x-1)' \cdot (7x+2)^4 - (x-1) \cdot \left[(7x+2)^4\right]'}{(7x+2)^8}$$

$$= \frac{(7x+2)^4 - (x-1) \cdot 4(7x+2)^3 \cdot (7x+2)'}{(7x+2)^8}$$

$$= \frac{(7x+2)^4 - (x-1) \cdot 4(7x+2)^3 \cdot 7}{(7x+2)^8}$$

$$= \frac{(7x+2)^4 - 28(x-1)(7x+2)^3}{(7x+2)^8}$$

$$= \frac{(7x+2)^3\left[(7x+2) - 28(x-1)\right]}{(7x+2)^8}$$

$$= \frac{30-21x}{(7x+2)^5}$$

Let $y' = 0 \Rightarrow \dfrac{30-21x}{(7x+2)^5} = 0$

$$\Rightarrow x = \frac{10}{7}$$

(i) $$y' = \frac{x' \cdot (x^2+1)^2 - x \cdot \left[(x^2+1)^2\right]'}{(x^2+1)^4}$$

$$= \frac{x' \cdot (x^2+1)^2 - x \cdot 2(x^2+1) \cdot (x^2+1)'}{(x^2+1)^4}$$

$$= \frac{(x^2+1)^2 - x \cdot 2(x^2+1) \cdot 2x}{(x^2+1)^4}$$

$$= \frac{(x^2+1)\left[(x^2+1) - 4x^2\right]}{(x^2+1)^4}$$

$$= \frac{1-3x^2}{(x^2+1)^3}$$

Let $y'=0 \Rightarrow \dfrac{1-3x^2}{(x^2+1)^3}=0$

$\Rightarrow 1-3x^2=0$

$\Rightarrow x=\dfrac{\sqrt{3}}{3},\ x=-\dfrac{\sqrt{3}}{3}$

2. (i) $y'=\dfrac{(4x+5)'\cdot(1-2x)-(4x+5)\cdot(1-2x)'}{(1-2x)^2}$

$=\dfrac{4(1-2x)-(4x+5)\cdot(-2)}{(1-2x)^2}$

$=\dfrac{4(1-2x)+2(4x+5)}{(1-2x)^2}$

$=\dfrac{14}{(1-2x)^2}$

(ii) $y'=\dfrac{14}{(1-2x)^2} \Rightarrow y'(2)=\dfrac{14}{9}$

$\Rightarrow m_T=\dfrac{14}{9}$ at point $A\left(2,-\dfrac{13}{3}\right)$

Let $\tan\theta=\dfrac{14}{9} \Rightarrow \theta=57^\circ 16'$

(iii) $m_T=\dfrac{14}{9},\ m_N=-\dfrac{9}{14}$ at point $A\left(2,-\dfrac{13}{3}\right)$

$l_T:\ y+\dfrac{13}{3}=\dfrac{14}{9}(x-2) \Rightarrow 14x-9y-67=0$

$l_N:\ y+\dfrac{13}{3}=-\dfrac{9}{14}(x-2) \Rightarrow 27x+42y+128=0$

3. (i) $y' = \dfrac{(x^2-1)' \cdot (x+3) - (x^2-1) \cdot (x+3)'}{(x+3)^2}$

$= \dfrac{2x(x+3) - (x^2-1)}{(x+3)^2}$

$= \dfrac{x^2+6x+1}{(x+3)^2}$

(ii) $y' = \dfrac{x^2+6x+1}{(x+3)^2} \Rightarrow y'(-1) = -1$

$\Rightarrow m_T = -1$ at point $P(-1, 0)$

Let $\tan\theta = -1 \Rightarrow \theta = 135^\circ$

(iii) $m_T = -1$, $m_N = 1$ at point $P(-1, 0)$

l_T: $y = -1(x+1) \Rightarrow x + y + 1 = 0$

l_N: $y = x + 1 \Rightarrow x - y + 1 = 0$

4. (a) $y' = \dfrac{x' \cdot (x+2) - x \cdot (x+2)'}{(x+2)^2}$

$= \dfrac{(x+2)-x}{(x+2)^2}$

$= \dfrac{2}{(x+2)^2}$

$y' = \dfrac{2}{(x+2)^2} \Rightarrow y'(0) = \dfrac{1}{2}$

$\Rightarrow m_T = \dfrac{1}{2}$ at the origin

$l_T:\ y = \dfrac{1}{2}x \Rightarrow x - 2y = 0$

(b) (i) $y' = \dfrac{2}{(x+2)^2} \Rightarrow y'(-3) = 2$

$\Rightarrow m_T = 2$ at point $P(-3, 3)$

$l_T:\ y - 3 = 2(x+3) \Rightarrow 2x - y + 9 = 0$

(ii) $l_T:\ 2x - y + 9 = 0 \Rightarrow A\left(-\dfrac{9}{2}, 0\right),\ B(0, 9)$

(iii) $Area_{\Delta OAB} = \dfrac{1}{2} \times \dfrac{9}{2} \times 9 = \dfrac{81}{4}$

(c) $\begin{cases} x - 2y = 0 \\ 2x - y + 9 = 0 \end{cases} \Rightarrow x = -6,\ y = -3$

$\therefore$ point of intersection $(-6, -3)$