Extra Enrichment:

1. Given $F(x) = f^2\left(g(x)\right)$, $g(1) = 2$, $g'(1) = 3$, $f(2) = 4$ and $f'(2) = 5$, find $F'(1)$.

2. If $F(t) = f\left(g(t)\right)$, $f'(t) = \dfrac{1}{t^2+1}$, $g'(t) = \dfrac{10}{t^4+1}$ and $g(0) = 3$, what is $F'(0)$?

3. Let $h(x)=g\left(f(x)\right)$ and $j(x)=f\left(g(x)\right)$, where $f(x)$ and $g(x)$ are differentiable functions for all real x values. Fill in the missing entries in the table below.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$	$j(x)$	$j'(x)$
0		-3			1		1	$-\frac{3}{2}$
1	0			$\frac{3}{2}$	0	$\frac{1}{2}$		

Solutions:

1. $F(x)=f^2\left(g(x)\right) \Rightarrow F'(x)=2\times f\left(g(x)\right)\times\dfrac{d}{dx}f\left(g(x)\right)$

$$=2\times f\left(g(x)\right)\times f'\left(g(x)\right)\times g'(x)$$

$$\Rightarrow F'(1)=2\times f\left(g(1)\right)\times f'\left(g(1)\right)\times g'(1)$$

$$=2\times f(2)\times f'(2)\times 3$$
$$=2\times 4\times 5\times 3$$
$$=120$$

2. $F(t)=f\left(g(t)\right) \Rightarrow F'(t)=f'\left(g(t)\right)\times g'(t)$

$$=\frac{1}{\left(g(t)\right)^2+1}\times\frac{10}{t^4+1}$$

$$\Rightarrow F'(0)=\frac{1}{\left(g(0)\right)^2+1}\times\frac{10}{0+1}$$

$$=\frac{1}{9+1}\times 10$$
$$=1$$

3.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$	$j(x)$	$j'(x)$
0	1	-3	0	$\frac{1}{2}$	1	$-\frac{9}{2}$	1	$-\frac{3}{2}$
1	0	1	1	$\frac{3}{2}$	0	$\frac{1}{2}$	0	$\frac{3}{2}$

$$h(x) = g\left(f(x)\right) \Rightarrow h'(x) = g'\left(f(x)\right)\cdot f'(x)$$

$$j(x) = f\left(g(x)\right) \Rightarrow j'(x) = f'\left(g(x)\right)\cdot g'(x)$$

Given: $f(1) = 0$, $f'(0) = -3$

$g'(1) = \frac{3}{2}$

$h(0) = 1$, $h(1) = 0$, $h'(1) = \frac{1}{2}$

$j(0) = 1$, $j'(0) = -\frac{3}{2}$

$$h(1) = 0 \Rightarrow g\left(f(1)\right) = 0$$

$$\Rightarrow g(0) = 0 \quad (\text{as } f(1) = 0)$$

$$j(0) = 1 \Rightarrow f\left(g(0)\right) = 1$$

$$\Rightarrow f(0) = 1 \quad (\text{as } g(0) = 0)$$

$$j'(0) = -\frac{3}{2} \Rightarrow f'\left(g(0)\right)\cdot g'(0) = -\frac{3}{2}$$

$$\Rightarrow f'(0)\cdot g'(0) = -\frac{3}{2} \quad (\text{as } g(0) = 0)$$

$$\Rightarrow -3\times g'(0) = -\frac{3}{2} \quad (\text{as } f'(0) = -3)$$

$$\Rightarrow g'(0) = \frac{1}{2}$$

$$h'(1)=\frac{1}{2} \;\Rightarrow\; g'\left(f(1)\right)\cdot f'(1)=\frac{1}{2}$$

$$\Rightarrow\; g'(0)\cdot f'(1)=\frac{1}{2} \qquad (\text{as } f(1)=0)$$

$$\Rightarrow\; \frac{1}{2}\times f'(1)=\frac{1}{2}$$

$$\Rightarrow\; f'(1)=1$$

$$h(0)=1 \;\Rightarrow\; g\left(f(0)\right)=1$$

$$\Rightarrow\; g(1)=1$$

$$j(1)=f\left(g(1)\right)$$

$$=f(1)$$

$$=0$$

$$h'(0)=g'\left(f(0)\right)\times f'(0)$$

$$=g'(1)\times f'(0)$$

$$=\frac{3}{2}\times(-3)$$

$$=-\frac{9}{2}$$

$$j'(1)=f'\left(g(1)\right)\times g'(1)$$

$$=f'(1)\times g'(1)$$

$$=1\times\frac{3}{2}$$

$$=\frac{3}{2}$$