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JE
MATHS

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ADV

- Three reciprocal identities: $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$, $\sec\theta = \dfrac{1}{\cos\theta}$, $\cot\theta = \dfrac{1}{\tan\theta}$

1. Simplify:

(a) $\dfrac{1}{\cos x}$

(b) $\dfrac{1}{\cot x}$

(c) $\operatorname{cosec} x \sin x$

(d) $\tan x \cot x$

2. Simplify:

(a) $\dfrac{\sin x}{\operatorname{cosec} x}$

(b) $\dfrac{\cot x}{\operatorname{cosec} x}$

(c) $\dfrac{\cos^2 x}{\sin^2 x}$

(d) $\sin^2 x \operatorname{cosec}^2 x$

3. Prove the following identities:

(a) $\dfrac{1}{\cos x \sin x} = \sec x \operatorname{cosec} x$

LHS =

= RHS

(b) $\sec x \cot x = \operatorname{cosec} x$

LHS =

=

=

= RHS

(c) $\sin x + \cos x = \dfrac{\sec x + \operatorname{cosec} x}{\operatorname{cosec} x + \sec x}$

LHS =

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= RHS

(d) $\tan x - \cot x = \dfrac{\tan^2 x - 1}{\tan x}$

LHS =

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= RHS

- Ratio identities: $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\sec\theta}{\operatorname{cosec}\theta}$, $\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{\operatorname{cosec}\theta}{\sec\theta}$

4. Prove that:

(a) $\cos\theta \tan\theta = \sin\theta$

LHS =

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= RHS

(b) $\sin\theta \cot\theta = \cos\theta$

LHS =

=

= RHS

(c) $\operatorname{cosec}\theta \cos\theta = \cot\theta$

LHS =

=

=

= RHS

(d) $\tan\theta \operatorname{cosec}\theta \cos\theta = 1$

LHS =

=

=

= RHS

- Complementary angle identities:

$\cos(90° - \theta) = \sin\theta$, $\cot(90° - \theta) = \tan\theta$, $\operatorname{cosec}(90° - \theta) = \sec\theta$

5. Use the complementary angle identities to find the missing acute angle inside the bracket:

(a) $\sin 70° = \cos(\_\_\_)$

(b) $\cos 36° = \sin(\_\_\_)$

(c) $\tan 20° = \cot(\_\_\_)$

(d) $\operatorname{cosec} 13° = \sec(\_\_\_)$

6. Simplify:

(a) $\sin(90^o - x)$

(b) $\sec(90^o - x)$

(c) $\dfrac{1}{\cot(90^o - x)}$

(d) $\dfrac{\cos(90^o - x)}{\sin(90^o - x)}$

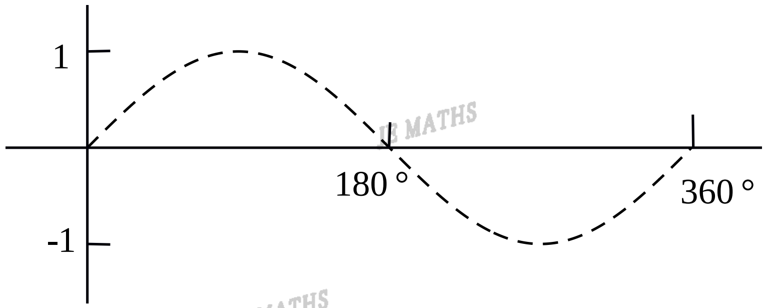
7. Given that $y = \sin(90° - \theta)$.

(a) Use the symmetry property $\sin(x - \theta) = -\sin(\theta - x)$ to rewrite this function.

(b) Hence, sketch $y = \sin(90° - \theta)$

from $y = \sin\theta$ for $0° \le \theta \le 360°$.

(c) Same as the graph for $y = \cos\theta$?



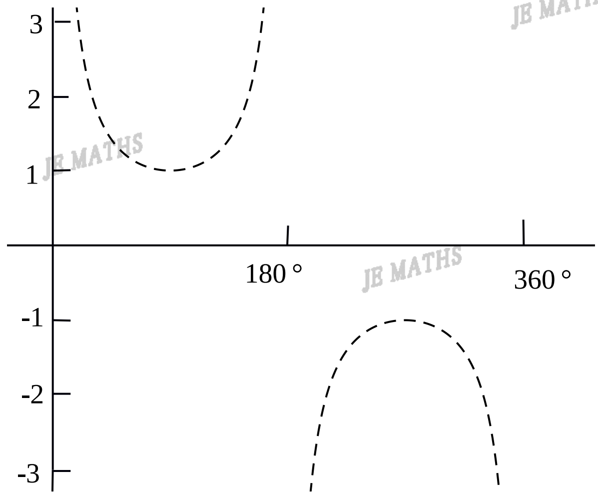
8. Given that $y = \cos ec(90° - \theta)$.

(a) Use the symmetry property $\cos ec(x - \theta) = -\cos ec(\theta - x)$ to rewrite this function.

(b) Hence, sketch $y = \cos ec(90° - \theta)$

from $y = \cos ec\theta$ for $0° \le \theta \le 360°$.

(c) Same as the graph of $y = \sec\theta$?



9. Prove that:

(a) $\sin(180^o + \theta)\cos(90^o - \theta) = -\sin^2\theta$

LHS =

=

= RHS

(b) $\cot(90^o - \theta)\tan(180^o + \theta)\sec(90 - \theta) = \tan\theta\sec\theta$

LHS =

=

=

= RHS

- Pythagorean identities: $\sin^2\theta+\cos^2\theta=1,\ 1+\tan^2\theta=\sec^2\theta,\ 1+\cot^2\theta=\operatorname{cosec}^2\theta$

10. Simplify:

(a) $1-\sin^2 A$

(b) $\sec^2 A-1$

$1+\tan^2 x=\sec^2 x$
1ts=1tsp

(c) $\operatorname{cosec}^2 A-\cot^2 A$

(d) $\sin^2 A+1+\cos^2 A$

11. Simplify:

(a) $1+\dfrac{\sin^2 A}{\cos^2 A}$

(b) $\sin^2\theta(\cot^2\theta+1)$

$1+\cot^2 x=\operatorname{cosec}^2 x$
1cc=1mL

(c) $1+\sec^2 A-\tan^2 A$

(d) $\dfrac{\sec^2 A-1}{\tan^2 A}$

12. If $\sec A=\dfrac{1}{3}$, and A is in 4th quadrant, find tan$A$ by using Pythagorean identities.

13. Prove the following by using Pythagorean identities:

(a) $(\sin\theta+\cos\theta)^2=1+2\sin\theta\cos\theta$

LHS =

=

= RHS

(b) $\cos^2\theta-\sin^2\theta=1-2\sin^2\theta$

LHS =

=

= RHS

14. Prove the following by using any trig identities:

(a) $\cot x + \tan x = \cos ecx \sec x$

LHS =

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=

=

= RHS

(b) $\cos ecx - \dfrac{1}{\cos ecx} = \cot x \cos x$

LHS =

=

=

=

= RHS

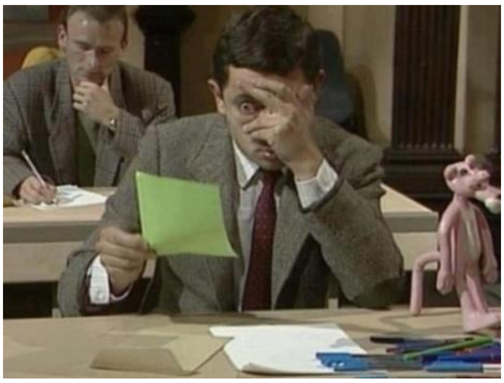
15. Using algebraic factorisation method to prove that:

$$\frac{\sin^2\theta - \cos^2\theta}{\sin^2\theta + \sin\theta\cos\theta} = 1 + \cot\theta$$

16. By breaking into sine and cosine and using the Pythagorean identity to prove that:
$(\sin x + \cos x)(\sec x + \cos ecx) = \sec x \cos ecx + 2$

17. Using any trig identity and times the conjugate terms (ie. 1-cosx) to prove that:

$$\frac{\cos ecx + \cot x}{\cos ecx - \cot x} = \frac{\sin^2 x}{(1-\cos x)^2}$$



- Boundary angles:

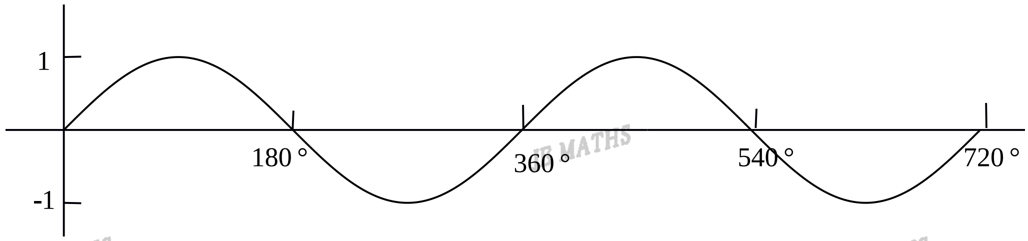
18. Change the following angles into angles in revolution.

(a) -120˚ (b) -78˚

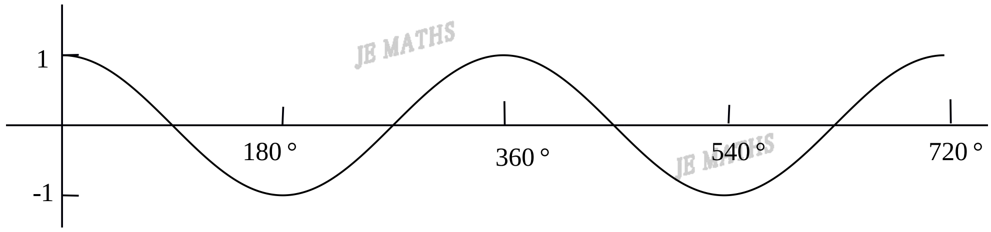
(c) 370˚ (d) 432˚

19. Solve trig equations with boundary angles and trig graph:

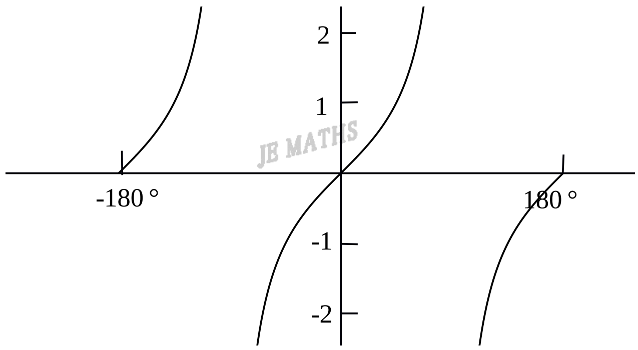
(a) $\sin x = 1$ for $0^o \le x \le 720^o$



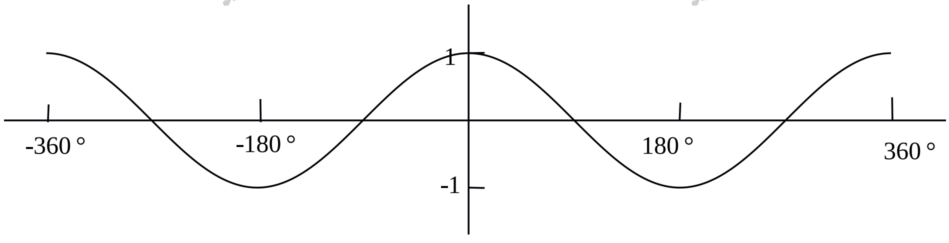
(b) $\cos x = -1$ for $0^o \le x \le 720^o$



(c) $\tan x = 0$ for $-180^o \le x \le 180^o$



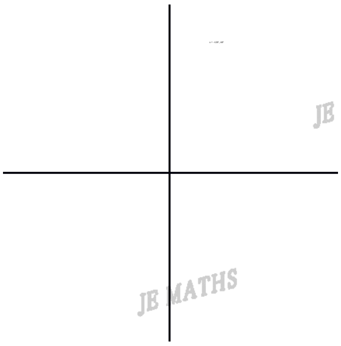
(d) $\sec x = 1$ for $-360^o \le x \le 360^o$



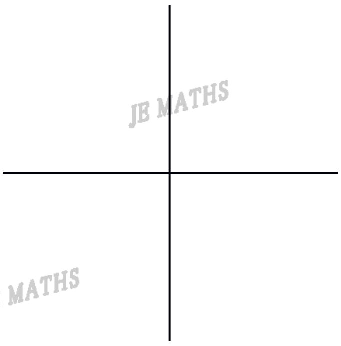
- Use related angles:

20. Solve trig equations by using related angles:

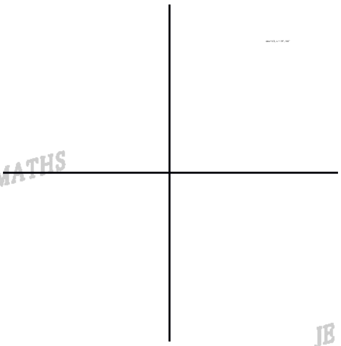
(a) $\sin x = -\dfrac{1}{2}$ for $-180^o \le x \le 180^o$



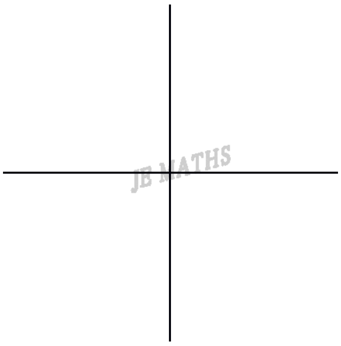
(b) $\tan x = \sqrt{3}$ for $-180^o \le x \le 180^o$



(c) $\cos x = \dfrac{1}{7}$ for $0^o \le x \le 360^o$ (0dp)

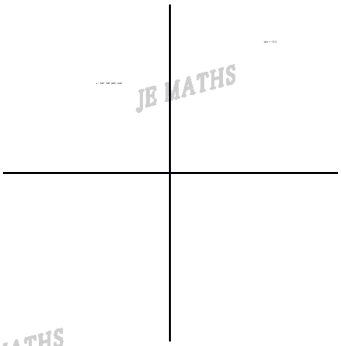


(d) $\operatorname{cosec} x = 3$ for $0^o \le x \le 360^o$ (0dp)

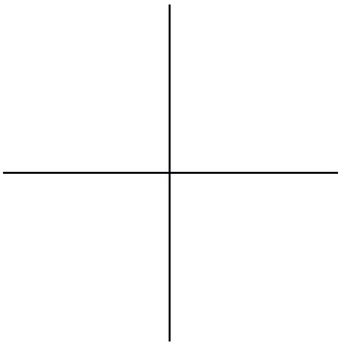


21. Solve the following trig equations for $0^o \le x \le 720^o$.

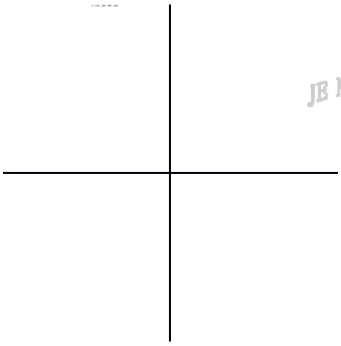
(a) $2\cos x - 1 = 0$



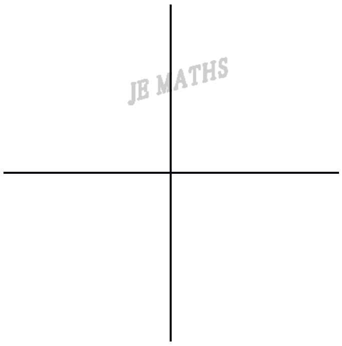
(b) $2\sin x + \sqrt{3} = 0$



(c) $2\sec x + 3 = 0$ (to the nearest degree)



(d) $2\cot x + \sqrt{2} = 0$ (to the nearest degree)



- Compound angle:

22. Solve the following equations with compound angle.

(a) $\sin 2x = \frac{1}{2}$, $\quad 0^o \le x \le 360^o$

(b) $\tan(x - 45^o) = \frac{\sqrt{3}}{3}$, $\quad 0^o \le x \le 360^o$

23. Solve the following compound angle trig equations for $0^o \le x \le 360^o$.

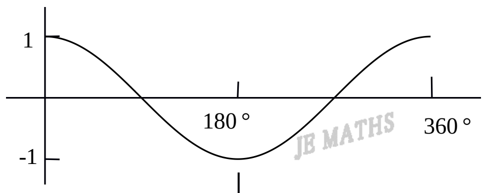
(a) $\cot 3x = -\sqrt{3}$

(b) $\sec(75^o - x) = -\frac{2\sqrt{3}}{3}$

- Substitution:

24. Solve trig equations by using **substitution**, to the nearest degree:

(a) $4\cos^2 x = \cos x$, $0^o \le x \le 360^o$.



(b) $5\cos ecx - \sin x = 0$, $-180^o \le x \le 180^o$

- Using trigonometric identities:

25. Solve trig equations by using **ratio identities**, for $0^o \le x \le 360^o$:

(a) $\sin x = \cos x$

(b) $\sin x + \sqrt{3}\cos x = 0$

26. Solve trig equations by using **Pythagorean identities**, for $0^o \le x \le 360^o$:

(a) $3\sin x + 2\cos^2 x = 0$

(b) $\sec^2 x = 4 + 2\tan x$, to the nearest degree

- Homogeneous equation: Sum of indices of $\sin x$ and $\cos x$ in each term is the same.

27. Solve the following **homogeneous equation** by dividing $\cos^2 x$ on both side.

$\sin^2 x + \sin x\cos x - 2\cos^2 x = 0$, $0^o \le x \le 360^o$, to the nearest degree.

homogeneous

adjective | hoh-muh-JEEN-yus

of the same or a similar kind or nature

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