# Problem Set 3

MaPS Correspondence Program

## Instructions

- Some of these problems are based off the notes "*The Pigeonhole Principle*".
- They are in roughly difficulty order and get quite difficult, so you are **not** expected to be able to solve every problem.
- However, please attempt as many questions as you can and submit your solutions to your mentor for marking and feedback.
- You may (and encouraged to) submit incomplete solutions if you can not solve a problem completely.
- You may type your solutions or submit a pdf of a **clear** scan/photo of **legible** written solutions.
- Feel free to discuss these problems with your peers and on the forum but the solutions you submit must be written by yourself.

## Problems

1. Prove the **Pigeonhole Principle**. Extend your proof to the **Generalised Pigeonhole Principle** and the **Infinite Pigeonhole Principle**.
2. Suppose that we have a set of three numbers with the property that the sum is 8 times the smallest number and 3 times one of the other two numbers. What is the ratio of the sum of the three numbers to the remaining number?
3. Five points are chosen on the Cartesian plane such that their coordinates are both integers. Prove that there exists a pair of them whose midpoint also has integer coordinates.
4. Suppose that $p$ is a prime number with the property that there exists some larger prime $q$ such that both $2q - p$ and $2q + p$ are prime. Prove that there is only one such $p$.
5. Prove that if every point in the Cartesian Plane is coloured one of 2022 colours, there must exist a rectangle whose vertices are all the same colour.