# Problem Set 1

## MaPS Correspondence Program

## Instructions

- This problem set is based off the notes *"Introduction to Proof"*.

- They are in roughly difficulty order and get quite difficult, so you are **not** expected to be able to solve every problem.

- However, please attempt as many questions as you can and submit your solutions to your mentor for marking and feedback.

- You may (and encouraged to) submit incomplete solutions if you can not solve a problem completely.

- You may type your solutions or take a **clear** scan/photo of **legible** written solutions.

- Feel free to discuss these problems with your peers and on the forum but the solutions you submit must be written by yourself.

## Problems

1. Prove that if $a$ and $b$ are integers, then $a - b$ is even if and only if $a^2 - b^2$ is even.

2. Prove that the cube of any integer $n$ will leave a remainder of 0, 1 or 6 when divided by 7.

3. (a) Use a proof by contrapositive to show that for an integer $x$, if $2x - 4$ is odd, then $x$ is even.

   (b) Use a proof by contrapositive to show that for an integer $x$, if $2x - 4$ is odd, then $x$ is odd.

   (c) Does it seem like (a) and (b) contradict each other? What has happened here? You may want to look into *"vacuous truths"*.

4. Prove that for any positive integer $k$, the number $\sqrt{k}$ is either an integer, or irrational.

5. There are 10 black points and 10 white points in the plane, no three of which are collinear. Prove that it is possible to join every black point with a white point such that no two lines intersect.