Development stage 1: (logarithm equation)

1. (a) Given that $x^{\sqrt{x}} = \left(\sqrt{x}\right)^{x}$, find the value of x by

   (i) using index laws:

   (ii) taking logarithm on both sides:

   (b) Given that $a^{\sqrt{x}} = \left(\sqrt{x}\right)^{a}$, show that $\log_{x} a = \dfrac{a}{2\sqrt{x}}$.

Development stage 2: (quadratic equation involving logarithm)

1. (a) It is given that $P(x) = x^3 + x - 30$ has a integer root. Factorise $P(x)$.

$P(x) = x^3 + x - 30$

Test $x = 3$

$P(3) = 3^3 + 3 - 30$

$= 0$

$\therefore x = 3$ is the integer root.

$P(x) = x^3 + x - 27 - 3$

$= (x - 3)(x^2 + 3x + 9) + (x - 3)$

$= (x - 3)(x^2 + 3x + 10)$

Alternatively, long division or sum and product of roots can be used to achieve the same result.

(b) Find the value of x if $8^x + 2^x = 30$. Correct your answer to 3 decimal places.

$8^x + 2^x = 30$

$(2^x)^3 + 2^x = 30$

Let $u = 2^x$

$u^3 + u - 30 = 0$

$(u - 3)(u^2 + 3u + 10) = 0$

$u^2 + 3u + 10 = 0$ has a discriminant of -31 which is negative

Hence, $u^2 + 3u + 10$ has no real solutions.

$\therefore u = 3$ only

$2^x = 3$

$\log_2 2^x = \log_2 3$

$x = \log_2 3$

$= \frac{\log 3}{\log 2}$

$= 1.585$ (3dp)

Development stage 1:

1. (a)

(i)

$x^{\sqrt{x}} = (x^{1/2})^x$

$\sqrt{x} = x/2$

$x = x^2/4$

$x^2 - 4x = 0$

$x(x - 4) = 0$

$x = 0$ or 4

$x \neq 0$ as 0^0 is undefined

$x = 4$ only

(ii)

$x^{\sqrt{x}} = (\sqrt{x})^x$

$\log(x^{\sqrt{x}}) = \log[(\sqrt{x})^x]$

$\sqrt{x} \times \log x = x \times \log x^{1/2}$

$\sqrt{x} \times \log x = x \times 1/2 \times \log x$

$\sqrt{x} = x/2$, $\log x > 0$ for all x

$x = x^2/4$

$x^2 - 4x = 0$

$x(x - 4) = 0$

$x = 0$ or 4

$x \neq 0$ as 0^0 is undefined

$x = 4$ only

(b)

$a^{\sqrt{x}} = (\sqrt{x})^a$

$\log(a^{\sqrt{x}}) = \log[(\sqrt{x})^a]$

$\sqrt{x} \times \log a = a \times \log x^{1/2}$

$\sqrt{x} \times \log a = a \times 1/2 \times \log x$

$\log a / \log x = a / 2\sqrt{x}$

$\log_x a = a/2\sqrt{x}$

Development stage 2: (quadratic equation involving logarithm)

1. (a) $P(x) = x^3 + x - 30$

Test $x = 3$

$P(3) = 3^3 + 3 - 30$

$= 0$

$\therefore x = 3$ is the integer root.

$P(x) = x^3 + x - 27 - 3$

$= (x - 3)(x^2 + 3x + 9) + (x - 3)$

$= (x - 3)(x^2 + 3x + 10)$

Alternatively, long division or sum and product of roots can be used to achieve the same result.

(b) $8^x + 2^x = 30$

$(2^x)^3 + 2^x = 30$

Let $u = 2^x$

$u^3 + u - 30 = 0$

$(u - 3)(u^2 + 3u + 10) = 0$

$u^2 + 3u + 10 = 0$ has a discriminant of -31 which is negative

Hence, $u^2 + 3u + 10$ has no real solutions.

$\therefore u = 3$ only

$2^x = 3$

$\log_2 2^x = \log_2 3$

$x = \log_2 3$

$= \log 3/\log 2$

$= 1.585$ (3dp)