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JE MATHS

10 Adv

Stage 1:

1. Differentiate each function by product rule and leave your answer in the fully factored form. Hence, find the value(s) of x where the curve has a horizontal tangent.

(a) $y = x(x-10)^4$

(b) $y = x^4(x-1)^3$

(c) $y = 2x(x+5)^3$

(d) $y = x^2(2x-4)^3$

(e) $y = x^3(1-3x^2)^4$

(f) $y = x^4(x^2+x)^3$

(g) $y=(x-2)(2x+1)^3$

(h) $y=(3x+1)^2(4x-5)^3$

(i) $y=-2\pi x^2(x^2-3x+1)^3$

(j) $y = 2x\sqrt{x+2}$

(k) $y = -3x^2\sqrt{3-x}$

(l) $y = x^2\sqrt{1-x^2}$

(m) $y=(3-2x^2)\sqrt{4-x^2}$

2. Consider the curve $y=x^m(a-x)^n$, where $m>1$, $n>1$ and $a>0$.

(i) Show that the curve has a horizontal tangent at a point T whose $x-$coordinate lies between 0 and a. Hence, find the coordinate of point T.

(ii) Find the coordinate of point T if $m = n$.

3. (a) Derive the product rule for a product of three: $y = uvw$, where u, v and w are functions of x.

(b) Hence, differentiate $y = x(x-1)^4\sqrt{2x+1}$.

State value(s) of x when the derivative is zero.

Stage 2:

1. Differentiate each function by quotient rule and leave your answer in a single fraction. Hence, find any value(s) of x that the tangent is horizontal.

(a) $y=\frac{4x-1}{2x+1}$

(b) $y=\frac{2x}{x^2-2}$

(c) $y=\frac{x^2-4}{x^2+1}$

(d) $y = \dfrac{3x^2}{x^3 - 2}$

(e) $y = \dfrac{(x-1)^2}{x^2 + 2x}$

(f) $y = \dfrac{(3x-1)^2}{x^2 - x}$

(g) $y = \dfrac{3x-4}{\sqrt{2-x}}$

(h) $y = \dfrac{x+1}{\sqrt{x^2+1}}$

(i) $y = \sqrt{\dfrac{3-x}{x-2}}$

2. Consider the hyperbola $y=\dfrac{x}{x-2}$.

(i) Sketch the graph of the hyperbola, showing the vertical and horizontal asymptotes, and state the domain and range.

(ii) Show that the tangent at point P where $x=p$ is $2x+(p-2)^2y-p^2=0$.

(iii) Suppose the tangent at P meets $x-$axis at point A and the vertical line through P meets $x-$axis at point B. Find the value(s) of p such that $OA=2OB$, where O is the origin.

(iv) Suppose the tangent at point P passes through a fixed point $C(0, c)$.

(a) Show that $p = \dfrac{2\sqrt{c}}{\sqrt{c}-1}$ or $p = \dfrac{2\sqrt{c}}{\sqrt{c}+1}$.

(b) State the values of c if such tangent exists.

(c) State the values of c if only one such tangent exists.

(d) Suppose there are two such tangents exist.

(1) Show that the tangent at $p=\dfrac{2\sqrt{c}}{\sqrt{c}+1}$ is always to the left branch of the hyperbola regardless the values of c.

(2) Find values of c if the tangent at $p=\dfrac{2\sqrt{c}}{\sqrt{c}-1}$ is to the right branch of the hyperbola.

(3) Hence, state the values of c if both tangents are to the same branch of the hyperbola.

3. Suppose that $y=\frac{u}{x}$, where u is a function of x. Show that $\frac{du}{dx}=y+x\frac{dy}{dx}$.

4. (i) Sketch a point T on a curve $y=f(x)$ where x, $f(x)$ and $f'(x)$ are all positive. Suppose the tangent, normal and vertical at T meet $x-$axis at point A, B and C respectively. Let the angle of inclination of the tangent be acute and denoted as $\theta=\angle TAB$ so that $y'=\tan\theta$.

(ii) Use trigonometry to show that:

(a) $AC=\frac{y}{y'}$

(b) $BC = yy'$

(c) $\sec\theta = \sqrt{1+(y')^2}$

(d) $\text{cosec} = \dfrac{\sqrt{1+(y')^2}}{y'}$

(e) $AT = \dfrac{y\sqrt{1+(y')^2}}{y'}$

(f) $BT = y\sqrt{1+(y')^2}$

(iii) Hence, if $f(x) = \dfrac{2x-3}{x+1}$ and $x = 4$, find the length of AC, BC, AT and BT.

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