

Development stage 1: (trig simplification and equation)

1. Evaluate the following trigonometric ratios using related angles.

(a) $\sin(90^\circ + \theta)$

(b) $\cos(90^\circ + \theta)$

(c) $\tan(90^\circ + \theta)$

2. Solve $3\sec^2\theta + \tan\theta - 5 = 0$ for $-180^\circ \leq \theta \leq 180^\circ$ (Correct to the nearest minute).

Development stage 2: (trig identity)

1. (a) Factorise by difference of two squares: $\sin^4 \theta - \cos^4 \theta$.

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- (b) Hence, prove that $\frac{\sin^3 \theta}{\cos \theta} - \frac{\cos^3 \theta}{\sin \theta} = \tan \theta - \cot \theta$.

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Development stage 1:

1. (a)

$$\begin{aligned}\sin(90^\circ + \theta) &= \cos[90^\circ - (90^\circ + \theta)] \\ &= \cos(-\theta) \\ &= \cos\theta\end{aligned}$$

(b)

$$\begin{aligned}\cos(90^\circ + \theta) &= \sin[90^\circ - (90^\circ + \theta)] \\ &= \sin(-\theta) \\ &= -\sin\theta\end{aligned}$$

(c)

$$\begin{aligned}\tan(90^\circ + \theta) &= \sin(90^\circ + \theta)/\cos(90^\circ + \theta) \\ &= \cos\theta/(-\sin\theta) \\ &= -\cot\theta \text{ or } -1/\tan\theta\end{aligned}$$

2. $3\sec^2\theta + \tan\theta - 5 = 0$

$$3(1 + \tan^2\theta) + \tan\theta - 5 = 0$$

$$3\tan^2\theta + \tan\theta - 2 = 0$$

$$(3\tan\theta - 2)(\tan\theta + 1) = 0$$

$$\tan\theta = 2/3, -1$$

$$\begin{aligned}\theta &= \tan^{-1}2/3, \tan^{-1}2/3 - 180^\circ, -45^\circ, 135^\circ \\ &= 33^\circ 41', -146^\circ 19', -45^\circ, 135^\circ (\text{nearest minute})\end{aligned}$$

Development stage 2: (trig identity)

1. (a)

$$(\sin \theta - \cos \theta)(\sin \theta + \cos \theta) \quad (\text{since } \sin^2 \theta + \cos^2 \theta = 1) \\ = (\sin^2 \theta - \cos^2 \theta)$$

$$(b) \text{ LHS} = (\sin^4 \theta - \cos^4 \theta) / \sin \theta \cos \theta$$

$$= (\sin^2 \theta + \cos^2 \theta)(\sin^2 \theta - \cos^2 \theta) / \sin \theta \cos \theta$$

$$= (\sin^2 \theta - \cos^2 \theta) / \sin \theta \cos \theta$$

$$= \sin \theta / \cos \theta - \cos \theta / \sin \theta$$

$$= \tan \theta - \cot \theta$$

$$= \text{RHS}$$

