Stage 1:

1. Suppose F(x) and G(x) satisfy the following properties:

$$F(3) = 2$$
, $G(3) = 4$, $G(0) = 3$

 $_{\text{IS MATHS}} F'(3) = -1, G'(3) = 0, G'(0) = \#3^{\text{HS}}$

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(a) If $K(x) = F(x) \cdot G(x)$, find K'(3).

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(b) If $S(x) = \frac{F(x)}{G(x)}$, find S'(3).

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(c) If T(x) = F(G(x)), find T'(0).

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2. Given that
$$y = \frac{\sqrt{1+2x}\sqrt[4]{1+4x}\sqrt[6]{1+6x}\cdots^{100}\sqrt{1+100x}}{\sqrt[3]{1+3x}\sqrt[4]{1+5x}\sqrt[3]{1+7x}\cdots^{10}\sqrt[4]{1+101x}}$$
, find $\frac{dy}{dx}$ at $x = 0$.

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3. Given that function $y = \frac{x}{x + \frac{x}{x + \frac{x}{x + \cdots}}}$, find y'(1).

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Solutions:

1. (a)
$$K(x) = F(x) \cdot G(x) \implies K'(x) = F'(x) \cdot G(x) + F(x) \cdot G'(x)$$

$$\implies K'(3) = F'(3) \cdot G(3) + F(3) \cdot G'(3)$$

$$= (-1) \cdot A + 2 \cdot 0$$

$$= -4$$

(b)
$$S(x) = \frac{F(x)}{G(x)} \implies S'(x) = \frac{F'(x) \cdot G(x) - F(x) \cdot G'(x)}{[G(x)]^2}$$

$$\implies S'(3) = \frac{F'(3) \cdot G(3) - F(3) \cdot G'(3)}{[G(3)]^2}$$

$$= \frac{-1 \cdot 4 - 2 \cdot 0}{16}$$

$$= -\frac{1}{4}$$

B MATHS
$$= -\frac{1}{4}$$

B MATHS

(c)
$$T(x) = F(G(x)) \Rightarrow T'(x) = F'(G(x)) \times G'(x)$$

$$\Rightarrow T'(0) = F'(G(0)) \times G'(0)$$

$$= F'(3) \times (-3)$$

$$= 3$$

2.
$$y = \frac{\sqrt{1+2x}}{\sqrt[3]{1+3x}} \cdot \frac{\sqrt[4]{1+6x}}{\sqrt[4]{1+5x}} \cdot \frac{\sqrt[4]{1+6x}}{\sqrt[4]{1+7x}} \cdot \frac{\sqrt{1+100x}}{\sqrt[10]{1+101x}}$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{\sqrt{1+2x}}{\sqrt[3]{1+3x}}\right)' \cdot \frac{\sqrt[4]{1+4x}}{\sqrt[4]{1+5x}} \cdot \frac{\sqrt[4]{1+6x}}{\sqrt[4]{1+7x}} \cdot \frac{\sqrt[100]{1+100x}}{\sqrt[100]{1+101x}}$$

$$+ \frac{\sqrt{1+2x}}{\sqrt[4]{1+3x}} \cdot \left(\frac{\sqrt[4]{1+4x}}{\sqrt[4]{1+5x}}\right)' \cdot \frac{\sqrt[4]{1+6x}}{\sqrt[4]{1+7x}} \cdot \frac{\sqrt[100]{1+100x}}{\sqrt[100]{1+101x}}$$

$$+ \frac{\sqrt{1+2x}}{\sqrt[4]{1+3x}} \cdot \frac{\sqrt[4]{1+4x}}{\sqrt[4]{1+5x}} \cdot \left(\frac{\sqrt[4]{1+6x}}{\sqrt[4]{1+7x}}\right)' \cdot \dots \cdot \frac{\sqrt[100]{1+100x}}{\sqrt[100]{1+101x}}$$

$$+ \frac{\sqrt{1+2x}}{\sqrt[4]{1+3x}} \cdot \frac{\sqrt[4]{1+4x}}{\sqrt[4]{1+5x}} \cdot \left(\frac{\sqrt[4]{1+6x}}{\sqrt[4]{1+7x}}\right)' \cdot \dots \cdot \frac{\sqrt[100]{1+100x}}{\sqrt[100]{1+101x}}$$

$$+\frac{\sqrt{1+2x}}{\sqrt[3]{1+3x}} \cdot \frac{\sqrt[4]{1+4x}}{\sqrt[3]{1+5x}} \cdot \frac{\sqrt[6]{1+6x}}{\sqrt[3]{1+7x}} \cdot \dots \cdot \left(\frac{\sqrt[100]{1+100x}}{\sqrt[100]{1+101x}}\right)'$$
 (by product rule)

$$\Rightarrow \frac{dy}{dx}\Big|_{x=0}^{ATHS} \left(\frac{\sqrt{1+2x}}{\sqrt[3]{1+3x}}\right)'_{x=0} + \left(\frac{\sqrt[4]{1+4x}}{\sqrt[3]{1+5x}}\right)'_{x=0} + \left(\frac{\sqrt[4]{1+6x}}{\sqrt[4]{1+7x}}\right)'_{x=0} + \dots + \left(\frac{\sqrt[100]{1+100x}}{\sqrt[100]{1+101x}}\right)'_{x=0} + \dots + \left(\frac{\sqrt[100]{1+100x}}{\sqrt[100]{1+101x}}\right)'_{x=0} + \dots + 0$$

$$= 0$$

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$$= 0$$

Check the derivative of the general term $\left(\frac{\sqrt[k]{1+kx}}{\sqrt[k+1]{1+(k+1)x}}\right)'$, where k=2,3,4,...100.

$$\left(\frac{\sqrt[k]{1+kx}}{\sqrt[k+1]{1+(k+1)x}}\right)' = \left((1+kx)^{\frac{1}{k}} \times \left[1+(k+1)x\right]^{\frac{1}{k+1}}\right)'$$

$$= \frac{1}{k} (1 + kx)^{\frac{1}{k}-1} \cdot (1 + kx)' \times \left[1 + (k+1)x\right]^{-\frac{1}{k+1}}$$

$$+ (1 + kx)^{\frac{1}{k}} \times \left(-\frac{1}{k+1}\right) \left[1 + (k+1)x\right]^{-\frac{1}{k+1}-1} \cdot \left[1 + (k+1)x\right]'_{JB \text{ MATHS}}$$

(by product rule and chain rule)

$$= \frac{1}{k} (1 + kx)^{\frac{1-k}{k}} \cdot k \times \left[1 + (k+1)x\right]^{\frac{1}{k+1}}$$

$$+ (1 + kx)^{\frac{1}{k}} \times \left(-\frac{1}{k+1}\right) \left[1 + (k+1)x\right]^{\frac{k+2}{k+1}} \cdot (k+1)$$

$$= (1 + kx)^{\frac{1-k}{k}} \times \left[1 + (k+1)x\right]^{\frac{1}{k+1}} - (1 + kx)^{\frac{1}{k}} \times \left[1 + (k+1)x\right]^{\frac{k+2}{k+1}}$$

$$= (1 + kx)^{\frac{1-k}{k}} \times \left[1 + (k+1)x\right]^{\frac{1}{k+1}} - (1 + kx)^{\frac{1}{k}} \times \left[1 + (k+1)x\right]^{\frac{k+2}{k+1}}$$

$$\therefore \left(\frac{\sqrt[k]{1+kx}}{\sqrt[k+1]{1+(k+1)x}}\right)' = (1+kx)^{\frac{1-k}{k}} \times \left[1+(k+1)x\right]^{-\frac{1}{k+1}} - (1+kx)^{\frac{1}{k}} \times \left[1+(k+1)x\right]^{-\frac{k+2}{k+1}}$$

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$$\therefore \left(\frac{\sqrt[k]{1+kx}}{\sqrt[k+1]{1+(k+1)x}} \right)_{x=0}^{r} = 1-1=0$$

3.
$$y = \frac{x}{x + \frac{x}{x + \dots}}$$
 $\Rightarrow y = \frac{x}{x + y}$

$$\Rightarrow xy + y^2 = x$$

$$\Rightarrow (xy + y^2)' = (x)'$$

$$\Rightarrow y + x \times \frac{dy}{dx} + 2y \times \frac{dy}{dx} = 1$$

$$\Rightarrow (x + 2y) \times \frac{dy}{dx} = 1 - y$$

$$\Rightarrow \frac{dy}{dx} = \frac{1 - y}{x + 2y}$$

$$\Rightarrow y = \frac{x}{x + y}$$

$$y = \frac{x}{x + \frac{x}{x + \dots}}$$

$$y = \frac{x}{x + \frac{x}{x + \dots}}$$

$$y = \frac{1}{1 + y}$$

$$\Rightarrow y = \frac{1}{1 + y}$$

$$\Rightarrow y = \frac{1}{1 + y}$$
(Let $x = 1$)
$$\Rightarrow y^2 + y - 1 = 0$$

$$\Rightarrow y_{1,2} = \frac{-1 \pm \sqrt{5}}{2}$$

$$\Rightarrow y = \frac{-1 \pm \sqrt{5}}{2}$$
(omit $y = \frac{-1 - \sqrt{5}}{2} \times 0$, as when $x = 1, y > 0$)

$$\frac{dy}{dx} = \frac{1 - y}{\sum_{\substack{x + 2y \\ j \in MATHS}}} \implies y'(1) = \frac{1 - \frac{-1 + \sqrt{5}}{2}}{1 + (-1 + \sqrt{5})} = \frac{3\sqrt{5} - 5}{\sum_{\substack{j \in MATHS}}}$$

$$y'(1) = \frac{3\sqrt{5}-5}{10}$$