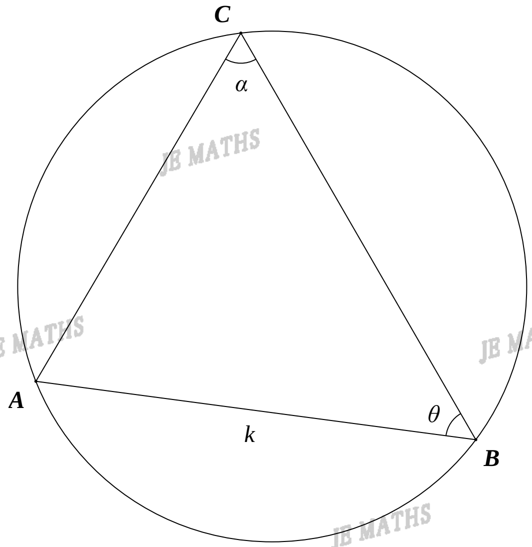
Enrichment stage 1:

1. In the diagram below, points A, B and C lie on a circle. The length of the chord AB is a constant, k.



Let ∠ACB = α and ∠ABC = θ respectively.

(a) Explain why α is a constant?

The length of a chord AB is a constant, k. (Given)

Angles subtended at the circumference on the same side of the circle by equal chords are equal.

∴α is a constant.

(b) If S is the sum of the lengths of the chords AC and BC, show that S is given by $S = \dfrac{k}{\sin\alpha}\left[\sin\theta + \sin(\theta+\alpha)\right]$.

CA/sinθ = k/sinα

CA = ksinθ/sinα

CB/sin[180° - (θ + α)] = k/sinα

CB/sin(θ + α) = k/sinα

CB = ksin(θ + α)/sinα

S = CA + CB

S = k/sinα × [sinθ + sin(θ + α)]

(c) Find the expression for S, in simplified form when $\theta = \left(90^\circ - \dfrac{\alpha}{2}\right)$.

When θ = 90° - α/2

S = k/sinα × [sin(90° - α/2) + sin(90° + α/2)]

= k/sinα × [sin(90° - α/2) + sin(180° - 90° - α/2)]

= k/sinα × [sin(90° - α/2) + sin(90° - α/2)]

= 2k/sinα × cosα/2

Enrichment stage 1:

1. (a)

The length of a chord AB is a constant, k. (Given)

Angles subtended at the circumference on the same side of the circle by equal chords are equal.

$\therefore \alpha$ is a constant.

(b)

$CA/\sin\theta = k/\sin\alpha$

$CA = k\sin\theta/\sin\alpha$

$CB/\sin[180° - (\theta + \alpha)] = k/\sin\alpha$

$CB/\sin(\theta + \alpha) = k/\sin\alpha$

$CB = k\sin(\theta + \alpha)/\sin\alpha$

$S = CA + CB$

$S = k/\sin\alpha \times [\sin\theta + \sin(\theta + \alpha)]$

(c)

When $\theta = 90° - \alpha/2$

$S = k/\sin\alpha \times [\sin(90° - \alpha/2) + \sin(90° + \alpha/2)]$

$\quad = k/\sin\alpha \times [\sin(90° - \alpha/2) + \sin(180° - 90° - \alpha/2)]$

$\quad = k/\sin\alpha \times [\sin(90° - \alpha/2) + \sin(90° - \alpha/2)]$

$\quad = 2k/\sin\alpha \times \cos\alpha/2$