Stage 1:

1. (a)

$\lim_{x\to 3} f(x) = 1$

$\lim_{x\to 6} f(x) = 0$

$\lim_{x\to 0} f(x) = 2$

(b)

$\lim_{x\to -3} f(x) = 0$

$\lim_{x\to 1} f(x) = -\frac{3}{2}$

$\lim_{x\to -2} f(x) = 3$

(c)

$\lim_{x\to -1} f(x) = 1$

$\lim_{x\to -4} f(x) = -\frac{1}{2}$

(d)

$\lim_{x\to 5} f(x) = 0$

$\lim_{x\to 2} f(x) = 2$

$\lim_{x\to 0} f(x) = 1$

$\lim_{x\to -2} f(x) = -2$

2. (a) $\lim_{x\to 4}(5-2x) = 5-2(4) = 5-8 = -3$

(b) $\lim_{x\to -2}(x^2-2x+3) = (-2)^2 - 2(-2) + 3 = 4+4+3 = 11$

(c) $\lim_{x\to 3} x(x-1)(x+2) = 3(3-1)(3+2) = 3\times 2\times 5 = 30$

(d) $\lim_{t\to -2}(t+1)^3(t^2-1) = (-1)^3(3) = -3$

(e) $\lim_{x\to 2}\frac{5}{2x+3}=\frac{5}{2(2)+3}=\frac{5}{7}$

(f) $\lim_{x\to -1}\frac{x-2}{x^2+4x-3}=\frac{(-1)-2}{(-1)^2+4(-1)-3}=\frac{-3}{-6}=\frac{1}{2}$

(g) $\lim_{x\to 3}\frac{x^3-2x}{x-4}=\frac{(3)^3-2(3)}{3-4}=\frac{27-6}{-1}=-21$

(h) $\lim_{x\to -1}\frac{x^4+x^2-6}{x^2+2x-3}=\frac{(-1)^4+(-1)^2-6}{(-1)^2+2(-1)-3}=\frac{-4}{-4}=1$

(i) $\lim_{u\to -2}\sqrt{16-x^2}=\sqrt{16-(-2)^2}=\sqrt{16-4}=2\sqrt{3}$

(j) $\lim_{u\to -2}\sqrt{u^4+3u+6}=\sqrt{(-2)^4+3(-2)+6}=\sqrt{16-6+6}=4$

(k) $\lim_{x\to 0}\frac{1}{\sqrt{x+9}}=\frac{1}{\sqrt{+9}}=\frac{1}{3}$

(l) $\lim_{x\to -2}\frac{1}{\sqrt[3]{(-2)^2-2(-2)}}=\frac{1}{\sqrt[3]{4+4}}=\frac{1}{2}$

(m) $\lim_{\theta\to \frac{\pi}{4}}\sin 2\theta=\sin\frac{\pi}{2}=1$

(n) $\lim_{x\to 1}\ln(2x-1)=\ln(2(1)-1)=\ln 1=0$

Stage 2:

1. (i) When x getting closer and closer to 1, f(x) getting closer and closer to 5.
   We say that the limit of f(x) is 5 when x approaches 1.

   (ii) Yes. Limit of f(x) is the value that f(x) approaches to.
   It is the tendency of f(x) when x approaches 1, which may not be the same as f(1).

2. (a) $\lim_{x \to 4} f(x) = 3$

   $\lim_{x \to -3} f(x) = -2$

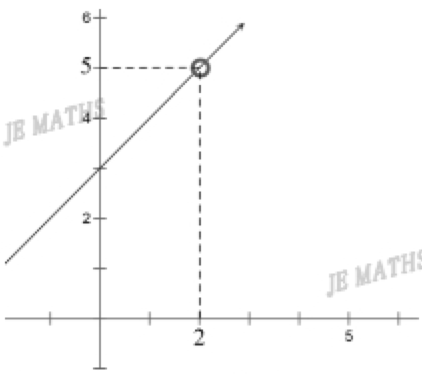
   (b) $\lim_{x \to -1} f(x) = -0.8$

   $\lim_{x \to 2} f(x) = 0.8$

   $\lim_{x \to -4} f(x) = -3$

3. (i) different domains hence different functions

   (ii)



   (iii) The behaviour of f(x) and g(x) around x=2 is the same. Therefore, the limit or to say the tendency of both function when x approaches 2 are the same.

   (iv) $\lim_{x \to 2} \frac{(x+3)(x-2)}{(x-2)} = \lim_{x \to 2}(x+3) = 2+3 = 5$

4. (a) $\lim_{x \to 1} \frac{(x+1)(x-1)}{(x-1)} = \lim_{x \to 1}(x+1) = 1+1 = 2$

   (b) $\lim_{x \to -3} \frac{(x-4)(x+3)}{(x+3)} = \lim_{x \to -3}(x-4) = -3-4 = -7$

(c) $\lim\limits_{x\to1}\dfrac{(x+2)(x-1)}{(x-2)(x-1)}=\lim\limits_{x\to1}\dfrac{(x+2)}{(x-2)}=\dfrac{1+2}{1-2}=-3$

(d) $\lim\limits_{x\to1}\dfrac{(\sqrt[3]{x}-1)}{(\sqrt[3]{x}-1)(x^{\frac{2}{3}}+x^{\frac{1}{3}}+1)}=\lim\limits_{x\to1}\dfrac{1}{(x^{\frac{2}{3}}+x^{\frac{1}{3}}+1)}=\dfrac{1}{3}$

(e) $\lim\limits_{h\to0}\dfrac{h^2-10h+25-25}{h}=\lim\limits_{h\to0}\dfrac{h(h-10)}{h}=\lim\limits_{h\to0}(h-10)=-10$

(f) $\lim\limits_{h\to0}\dfrac{h^3+2h^2+8h+4h^2+8+4h-8}{h}=\lim\limits_{h\to0}\dfrac{h^3+6h^2+12h}{h}$

$$=\lim_{h\to0}\frac{h(h^2+6h+12)}{h}$$

$$=\lim_{h\to0}(h^2+6h+12)=12$$

(g) $\lim\limits_{x\to2}\dfrac{\frac{2-x}{2x}}{x-2}=\lim\limits_{x\to2}\dfrac{2-x}{2x(x-2)}=\lim\limits_{x\to2}\dfrac{-(x-2)}{2x(x-2)}=\lim\limits_{x\to2}\dfrac{-1}{2x}=-\dfrac{1}{4}$

(h) $\lim\limits_{h\to0}\dfrac{\frac{3-(3+h)}{9+3h}}{h}=\lim\limits_{h\to0}\dfrac{-h}{9h+3h^2}=\lim\limits_{h\to0}\dfrac{-h}{h(9+3h)}=\lim\limits_{h\to0}\dfrac{-1}{9+3h}=\dfrac{-1}{9+3(0)}=-\dfrac{1}{9}$

(i) $\lim\limits_{x\to0}\dfrac{x(\sqrt{1+3x}+1)}{(\sqrt{1+3x}-1)(\sqrt{1+3x}+1)}=\lim\limits_{x\to0}\dfrac{x(1+3x-1)}{3x(\sqrt{1+3x}-1)}$

$$=\lim_{x\to0}\frac{x(\sqrt{1+3x}+1)}{3x}$$

$$=\lim_{x\to0}\frac{(\sqrt{1+3x}+1)}{3}$$

$$=\frac{\sqrt{1+3(0)}+1}{3}=\frac{2}{3}$$

(j) $\lim_{h\to 0}\frac{(\sqrt{3-h}-\sqrt{3})(\sqrt{3-h}+\sqrt{3})}{h(\sqrt{3-h}+\sqrt{3})}=\lim_{h\to 0}\frac{3-h-3}{h(\sqrt{3-h}+\sqrt{3})}$

$$=\lim_{h\to 0}\frac{h}{h(\sqrt{3-h}+\sqrt{3})}$$

$$=\lim_{h\to 0}\frac{-1}{\sqrt{3-h}+\sqrt{3}}$$

$$=\frac{-1}{\sqrt{3-0}+\sqrt{3}}=\frac{-1}{2\sqrt{3}}=-\frac{\sqrt{3}}{6}$$

(k) $\lim_{t\to 0}\frac{(\sqrt{t^2+9}-3)(\sqrt{t^2+9}+3)}{t^2(\sqrt{t^2+9}+3)}=\lim_{t\to 0}\frac{(t^2+9-9)}{t^2(\sqrt{t^2+9}+3)}$

$$=\lim_{t\to 0}\frac{t^2}{t^2(\sqrt{t^2+9}+3)}$$

$$=\lim_{t\to 0}\frac{1}{\sqrt{t^2+9}+3}$$

$$=\frac{1}{\sqrt{0^2+9}+3}$$

$$=\frac{1}{6}$$

(l) $\lim_{h\to 0}\frac{\frac{1}{\sqrt{x+h+2}}-\frac{1}{\sqrt{x+2}}}{h}=\lim_{h\to 0}\frac{\frac{\sqrt{x+2}-\sqrt{x+h+2}}{\sqrt{x+h+2}\sqrt{x+2}}}{h}=\lim_{h\to 0}\frac{\sqrt{x+2}-\sqrt{x+h+2}}{h\sqrt{x+h+2}\sqrt{x+2}}$

$$=\lim_{h\to 0}\frac{(\sqrt{x+2}-\sqrt{x+h+2})(\sqrt{x+2}+\sqrt{x+h+2})}{h\sqrt{x+h+2}\sqrt{x+2}(\sqrt{x+2}+\sqrt{x+h+2})}$$

$$=\lim_{h\to 0}\frac{x+2-(x+h+2)}{h\sqrt{x+h+2}\sqrt{x+2}(\sqrt{x+2}+\sqrt{x+h+2})}$$

$$=\lim_{h\to 0}\frac{x+2-x-h-2}{h\sqrt{x+h+2}\sqrt{x+2}(\sqrt{x+2}+\sqrt{x+h+2})}$$

$$=\lim_{h\to 0}\frac{-h}{h\sqrt{x+h+2}\sqrt{x+2}(\sqrt{x+2}+\sqrt{x+h+2})}$$

$$= \lim_{h \to 0} \frac{-1}{\sqrt{x+h+2}\sqrt{x+2}(\sqrt{x+2}+\sqrt{x+h+2})}$$

$$= \frac{-1}{\sqrt{x+0+2}\sqrt{x+2}(\sqrt{x+2}+\sqrt{x+0+2})}$$

$$= \frac{-1}{\sqrt{x+2}\sqrt{x+2}(\sqrt{x+2}+\sqrt{x+2})}$$

$$= \frac{-1}{(x+2)(2\sqrt{x+2})}$$

$$= \frac{-1}{2(x+2)^{\frac{3}{2}}}$$

Stage 3:

1. (i) When x approaches to 1 from below, f(x) getting closer and closer to the value of 3.
   When x approaches to 1 from above, f(x) getting closer and closer to the value of 7.

   (ii) no, as left-sided limit =/ right-sided limit

2. (a) $\lim_{x\to 2^-} f(x) = 3$

   $\lim_{x\to 2^+} f(x) = -1$

   $\lim_{x\to 2} f(x) =$ not exist

   (b) $\lim_{x\to 1^-} f(x) = 0.5$

   $\lim_{x\to 1^+} f(x) = 2.5$

   $\lim_{x\to 2^-} f(x) = 1.5$

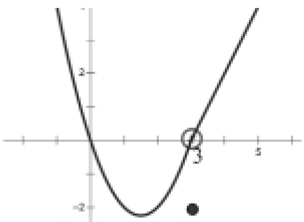
   $\lim_{x\to 2^+} f(x) = 3.5$

   $\lim_{x\to 3^-} f(x) = 2.5$

3. (i) $\lim_{x\to 3^+} f(x) = \lim_{x\to 3}(2x-6) = 6-6 = 0$

   (ii) Yes, as the limit from the left is equal to the limit from the right.

   (iii)

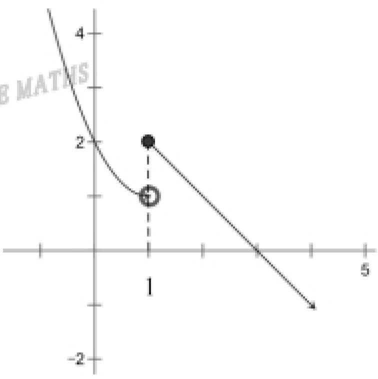


4. (i) $\lim_{x\to 1^+} f(x) = \lim_{x\to 1}(3-x) = 3-1 = 2$

   $\lim_{x\to 1^-} f(x) = \lim_{x\to 1}(x^2-2x+2) = 1$

   (ii) No, as the limit from the left is not equal to the limit from the right.

   (iii)



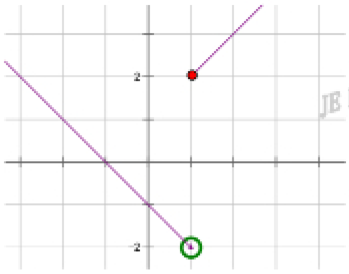
5. (i) $f(x)=\begin{cases} x+1, & x>1 \\ -x-1, & x<1 \end{cases}$

(ii) $\lim\limits_{x\to 1^+} f(x)=\lim\limits_{x\to 1^+}(x+1)=1+1=2$

$\lim\limits_{x\to 1^+} f(x)=\lim\limits_{x\to 1^+}(-x-1)=-1-1=-2$

(iii) No, as the limit from the left is not equal to the limit from the right.

(iv)



6. (a) $\lim\limits_{x\to -3^+}\sqrt{9-x^2}=\sqrt{9-(-3)^2}=0$

(b) $\lim\limits_{x\to -3^-}\sqrt{x^2-9}=\sqrt{(-3)^2-9}=0$

(c) $\lim\limits_{x\to -4^+}\dfrac{|x+4|}{x+4}=\lim\limits_{x\to -4^+}1=1$

(d) $\lim\limits_{x\to -4^-}\dfrac{|x+4|}{x+4}=\lim\limits_{x\to -4^-}-1=-1$

Stage 4:

1. (a) $\lim\limits_{x\to\infty} f(x) = 0$ (b) $\lim\limits_{x\to\infty} f(x) = -1$

2. (a) $\lim\limits_{x\to\infty} \dfrac{2}{x-5} = 0$

(b) $\lim\limits_{x\to\infty} \dfrac{1}{x^2+x-1} = 0$

(c) $\lim\limits_{x\to\infty} \dfrac{x+1}{5-2x} = \lim\limits_{x\to\infty} \dfrac{1+\dfrac{1}{x}}{\dfrac{5}{x}-2} = \dfrac{-1}{2}$

(d) $\lim\limits_{t\to\infty} \dfrac{2+\dfrac{7}{t}+\dfrac{3}{t^2}}{1-\dfrac{1}{t}-\dfrac{12}{t^2}} = 2$

(e) $\lim\limits_{x\to\infty} \dfrac{x^2-1}{x+1} = \lim\limits_{x\to\infty} \dfrac{(x-1)(x+1)}{(x+1)} = \lim\limits_{x\to\infty}(x-1) = \infty$

(f) $\lim\limits_{x\to\infty}(x^2-1)(x-4) = \lim\limits_{x\to\infty}(x-1)(x+1)(x-4) = \infty$

(g) $\lim\limits_{x\to\infty}\left(\sqrt{x}-\sqrt{x-4}\right) = \lim\limits_{x\to\infty} \dfrac{\left(\sqrt{x}-\sqrt{x-4}\right)\left(\sqrt{x}+\sqrt{x-4}\right)}{\left(\sqrt{x}+\sqrt{x-4}\right)}$

$$= \lim_{x\to\infty} \frac{x-(x-4)}{\left(\sqrt{x}+\sqrt{x-4}\right)}$$

$$= \lim_{x\to\infty} \frac{4}{\left(\sqrt{x}+\sqrt{x-4}\right)}$$

$$= 0$$

(h) $\lim\limits_{x\to\infty}\left(\sqrt{x+h}-\sqrt{x}\right) = \lim\limits_{x\to\infty} \dfrac{\left(\sqrt{x+h}-\sqrt{x}\right)\left(\sqrt{x+h}+\sqrt{x}\right)}{\left(\sqrt{x+h}+\sqrt{x}\right)}$

$$= \lim_{x\to\infty} \frac{x+h-x}{\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$= \lim_{x\to\infty} \frac{h}{\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$= 0$$

3. (a) $\lim_{x \to 0^+} \ln x = -\infty$

(b) $\lim_{x \to 2^+} f(x) = +\infty$

$\lim_{x \to 2^-} f(x) = -\infty$

4. (a) $\lim_{x \to 5^+} \frac{6}{x-5} = \frac{6}{0^+} = +\infty$

(b) $\lim_{x \to 5^-} \frac{6}{x-5} = \frac{6}{0^-} = -\infty$

(c) $\lim_{x \to \left(\frac{\pi}{2}\right)^+} \tan x = -\infty$

(d) $\lim_{x \to \left(\frac{\pi}{2}\right)^-} \tan x = +\infty$

(e) $\lim_{x \to -1^+} \frac{2}{(x+1)(x-3)} = \frac{2}{0^+ \cdot -4} = -\infty$

(f) $\lim_{x \to -1} \frac{2}{(x+1)(x-3)} = \frac{2}{0^- \cdot -4} = +\infty$

(g) $\lim_{x \to 0} \frac{x-1}{x^2(x+2)} = \frac{0-1}{0^2(0+2)} = -\infty$

(h) $\lim_{x \to -2^+} \frac{x-1}{x^2(x+2)} = \frac{-2^+ - 1}{2^2(-2^+ + 2)} = \frac{-3}{0^+} = -\infty$