33°
52′
37″S
151°
06′
04″E

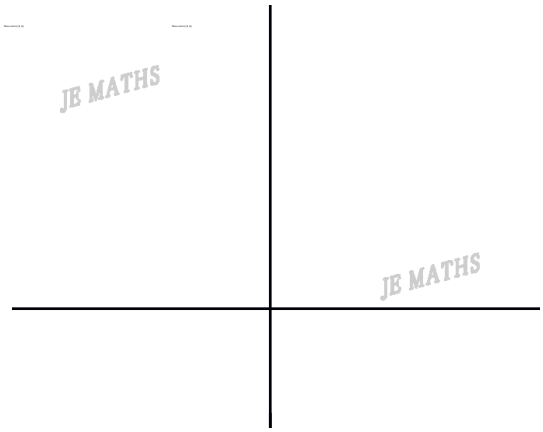
JE
MATHS

10
ADV

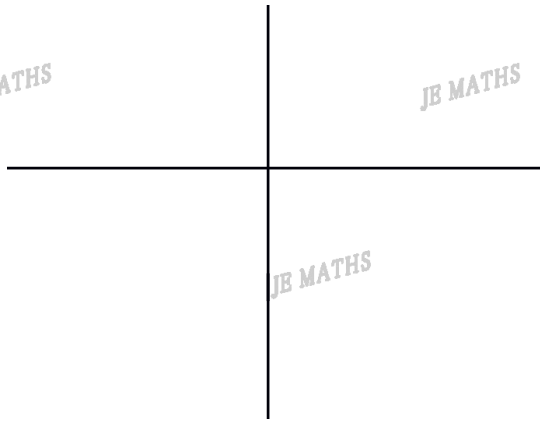
- Exponential graph sketching:

1. Sketch the following exp graphs and indicate all intercepts and the new centre.

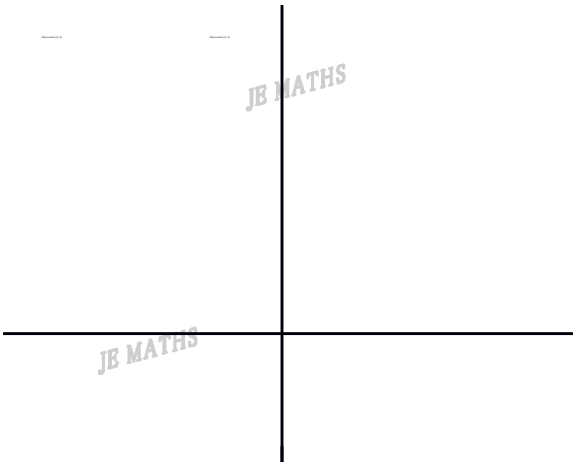
(a) $y = 2^{-x}$



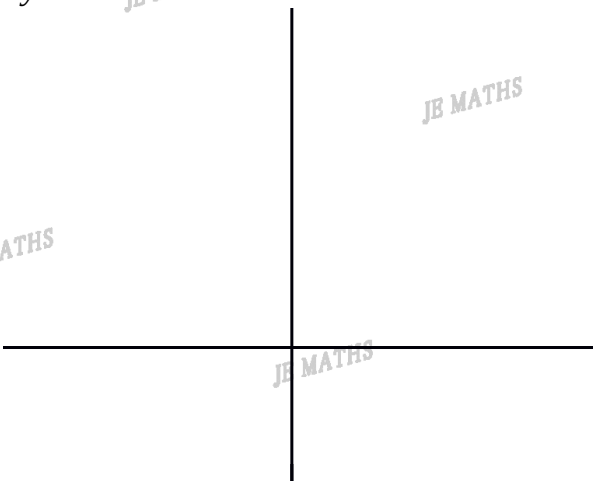
(b) $y = -3^{2x}$ (Hint: $y = -9^{x}$)



(c) $y = 3^{x} + 2$



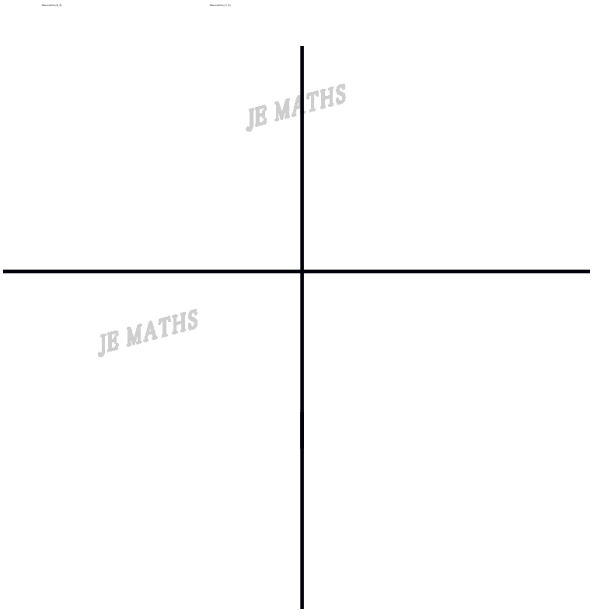
(d) $y = 3^{x-2} + 4$



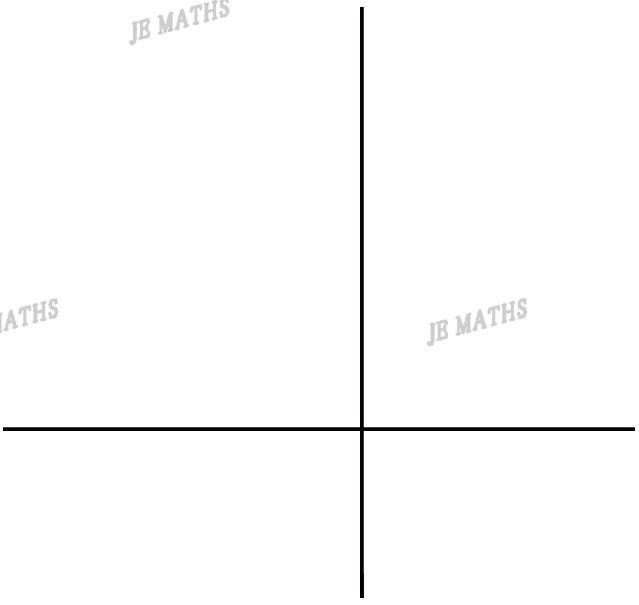
- Mix transformation (multiple):

2. Sketch the following exp graphs and indicate all intercepts and the new centre.

(a) $y = -3^{x} + 2$

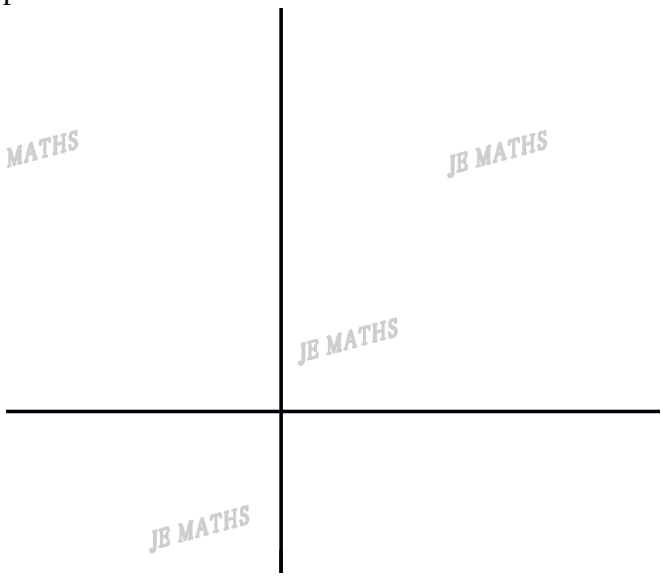


(b) $y = 4 - 3^{x+1}$



- Quick sketching technique for exponential function:

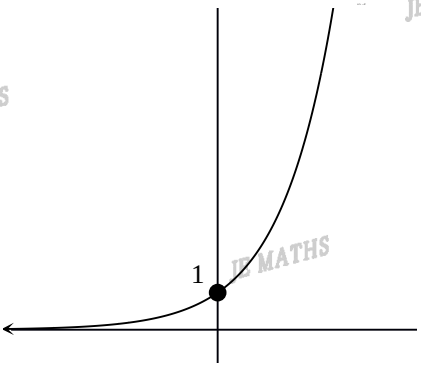
3. Sketch $y = -4^{x-1} + 3$ and indicate all intercepts and the new centre.

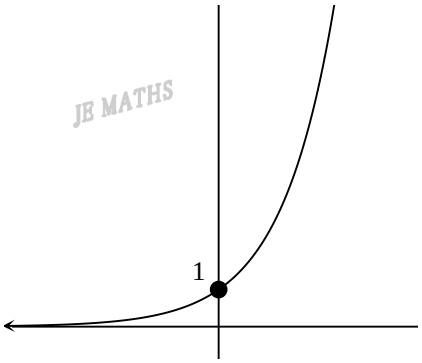


- Exponential inequalities:

4. Solve inequalities by using the given exponential graph:

(a) $3^x \le 27$ (b) $10^x > \frac{1}{100}$





5. Solve inequalities by taking $\log_{10}$ on both sides:

(a) $3^x < 5$ (b) $1.01^x \ge 0.1$

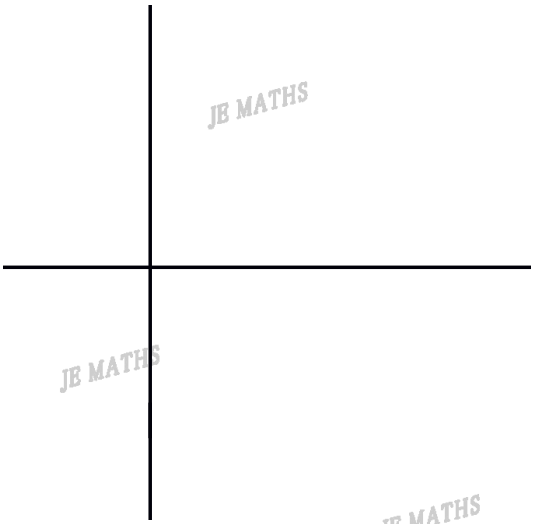
6. Solve the following **hard** inequalities:

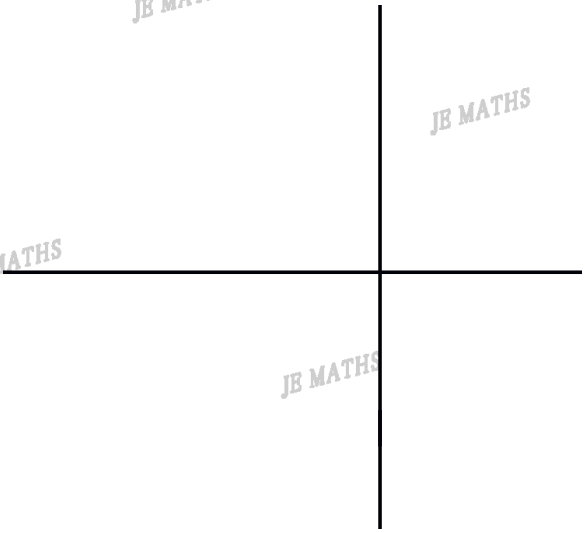
(a) $3^{2x+1} > 9$ (b) $3^{x+3} < 1000$

- Logarithmic graph sketching:

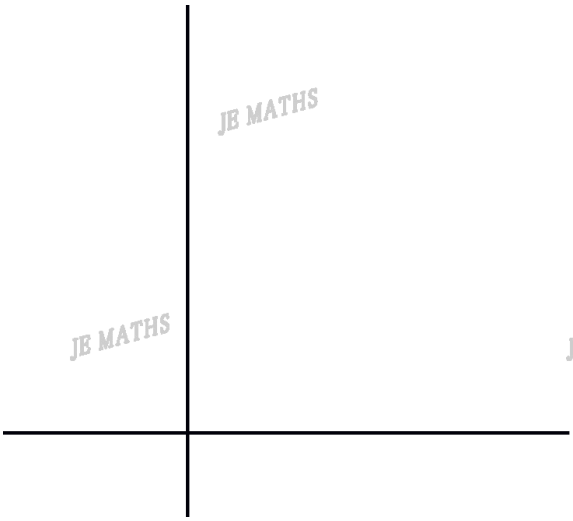
7. Sketch the following log graphs and indicate all intercepts and the new centre.

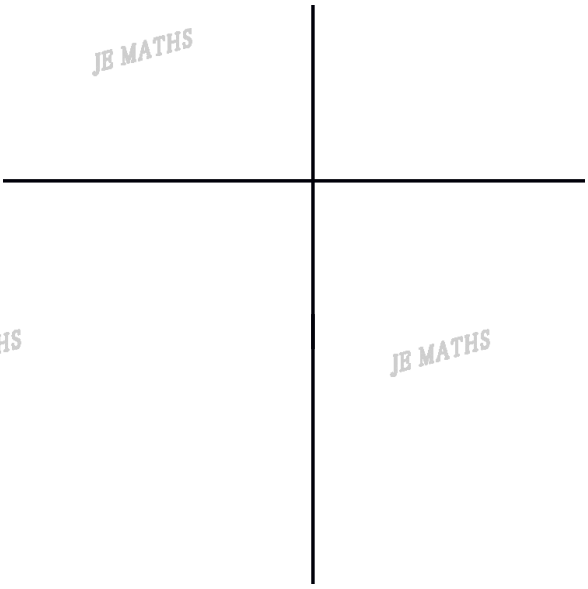
(a) $y = \log_3(x-2)$ (b) $y = \log_3(-x)$





(c) $y = \log_3 x + 4$ (d) $y = \log_3(x+2) - 1$

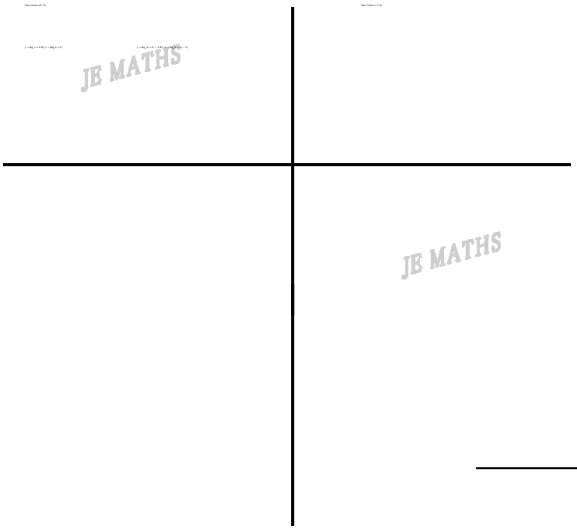




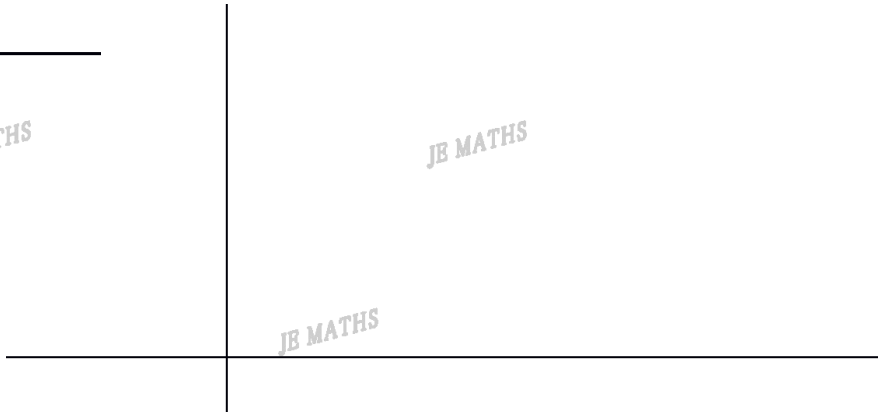
- Mix transformation (multiple):

8. Sketch the following log graphs and indicate all intercepts and the new centre.

(a) $y = -\log_3 x - 2$



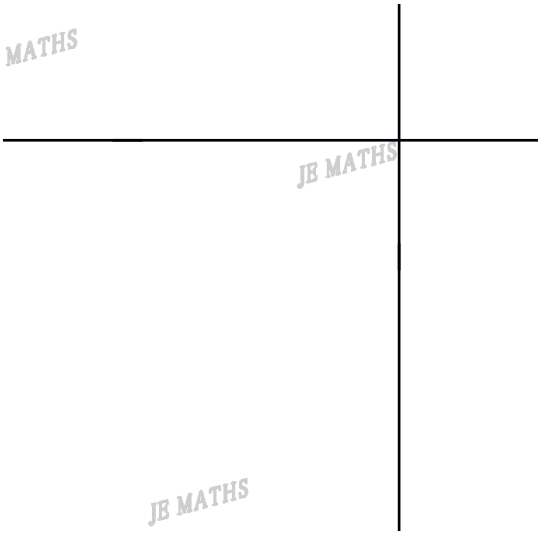
(b) $y = 3 - \log_3(x+2)$



- Quick sketching technique for exponential function:

9. Sketch the following log graphs, intercepts are in one decimal place if necessary.

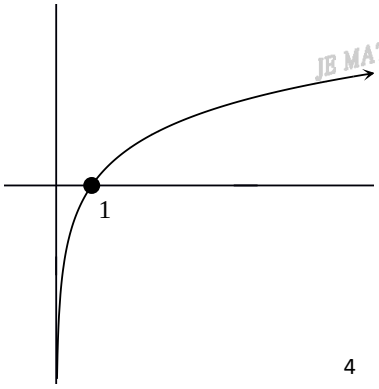
$y = -\log_3(x+5) - 4$



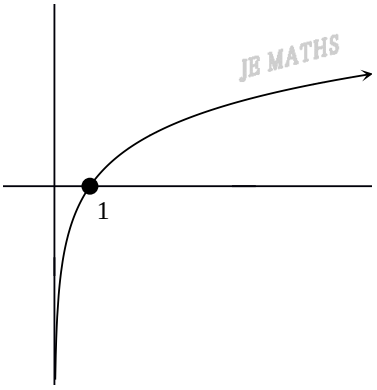
- Exponential inequalities:

10. Solve inequalities by using the given logarithmic graph:

(a) $\log_3 x \le 2$



(b) $\log_3 x > 4$



11. Solve **hard** inequalities by taking the proper base number on both sides:

(a) $\log_5 x < 9$

(b) $\log_{10} x \geq \frac{1}{10}$

- Problems involving exp and log equation:

12. The cost C is varying over time t according to the formula: $C = 2 \times 10^{\frac{t}{2}}$.

(a) Find C when $t = 10$.

(b) Show that $t = 2\log_{10} \frac{C}{2}$.

by making t the subject of the formula.

(c) Hence, find t when $C = 1000$. (1dp)

13. The quantity Q is varying over time t according to the formula: $t = 5\log_3 2Q$.

(a) Find t when $Q = 8$. (1dp)

(b) Show that $Q = \frac{1}{2} \times 3^{\frac{t}{5}}$

by making Q the subject of the formula.

(c) Find Q when $t = 15$. (1dp)

- Problems involving exp and log inequalities:

14. How many positive integer powers of 3 are less than 5^{100}?

Hint: taking $\log_3$ on both sides of the inequality.

15. Aaron bought a rising property at \$1,000,000 with the inflation rate at 5% p.a.

If the cost C after n years is given as the formula: $C = 1,000,000 \times 1.05^n$.

(a) Find the total cost after 10 years. (1dp)

(b) How many years will the property cost more than \$2,000,000? (1dp)

- Application involving making an equation of log and exp:

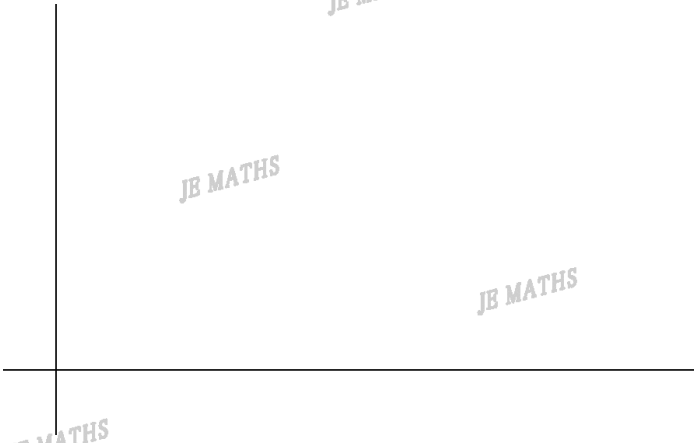
16. **The population of rabbits** on an island doubles every 10 years and is 2 million at the start.

(a) Check if the population P after another n years is $P = 2 \times 2^{\frac{n}{10}}$, where P is in million.

(b) Find the population, in million, after
(i) 50 years (ii) 99 years (0dp)

(c) Fill in the following table and plot it on the given number plane below.

n	0	10	20	30	40
P (mil)					



(d) Show that $n = 10\log_2 \frac{P}{2}$.

(e) Fill in the following table and plot it on the given number plane below:

P (mil)	2	4	8	16	32
n					

(f) Find n when the population will be tripled. (1dp)

17. Biologists use **Petri dishes**, which are shallow cylindrical glasses, to culture bacteria. In an experiment conducted to culture bacteria from a hand, it is observed that the number of bacteria is doubling every five hours. If initially, there are 10,000 bacteria inside the dish,

(a) make the formula for the population P after n hours.

(b) Hence, find out how many bacteria are there after

(i) 10 hours? (ii) 1 day? (0dp)

(c) Show that $n = 5\log_2 \dfrac{P}{10000}$.

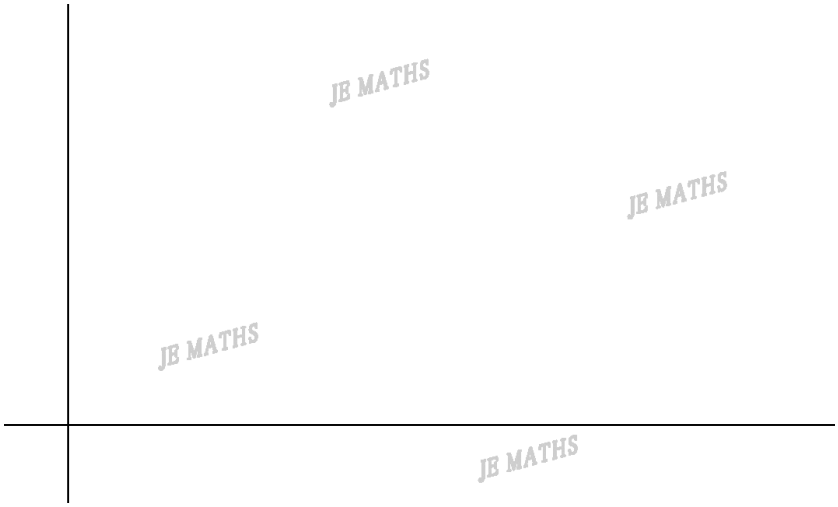
(d) After how many hours, to the nearest hour, will the bacteria reaches

(i) 888888?

(ii) 1 million times its initial population?

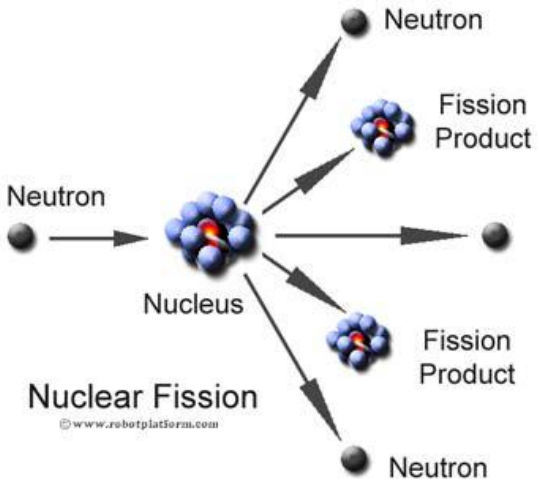
(e) Use the log law to simplify $n = 5\log_2 \frac{P}{10000}$ into $n = 5\log_2 P - 20\log_2 10$.

(f) Hence, sketch $n = 5\log_2 P - 20\log_2 10$ and indicate one point when the population is tripled. (1dps if necessary)



18. **Uranium-235** is a critical material in nuclear fission, capable of sustaining a fission chain reaction and releasing a tremendous amount of energy in a nuclear bomb. Its half-life is 703.8 million years, implying that after 703,800,000 years, half of its original amount will decay into other elements.

(a) Check if the mass of uranium-235 in the Earth is $M = M_0\left(\frac{1}{2}\right)^{\frac{n}{703,800,000}}$ after n years from the present, where M_0 is the mass of total uranium-235 in the Earth now.



(b) When the Milky Way and Andromeda galaxies collide four billion years from now, what percentage of the current uranium-235 on Earth will still remain?

(c) Clair Cameron Patterson, an American geochemist, pioneered the method of uranium-lead radiometric age dating for finding the age of rocks and minerals. In 1956, he used this method to obtain one of the earliest estimates of the Earth's age, which was 4,600,000,000 years old. Assuming that the amount of lead-207 (the decay product of uranium-235) was negligible at the time of the Earth's formation, how many times more uranium-235 was present then compared to now? (Round your answer to the nearest whole number.)

(d) Show that $n = -703800000 \log_2 \frac{M}{M_0}$.

Clair Cameron Patterson
(1922-1995)
obtained one of the earliest estimates of the age of the Earth

(e) After how many years, in billion, will the percentage of the present uranium-235 remaining on the Earth drop to 1%? (1dp)

1
2
3
Avg:

Week___

www.jeeducation.com.au

HSC

A

A

JE MATHS

Want to learn? We will help u.

Don't want to learn? We will change u.

ph: 0422 777 073