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JE
MATHS

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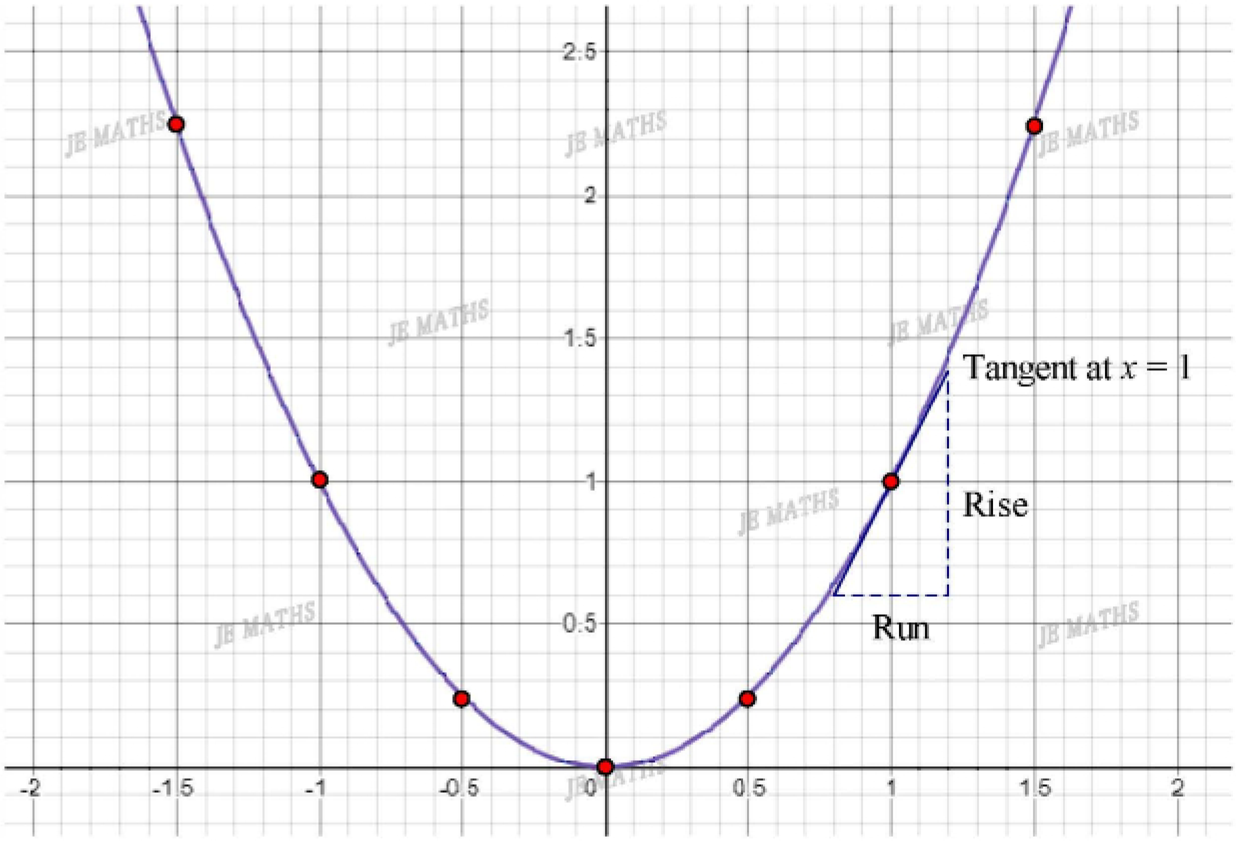
Adv

Stage 1:

1. Consider the parabola $f(x) = x^2$.



(i) The tangent to $f(x)$ at $x = 1$ has been shown on the diagram. Measure the gradient of this tangent by using $gradient = \frac{rise}{run}$. Hence, estimate the value of $f'(1)$.

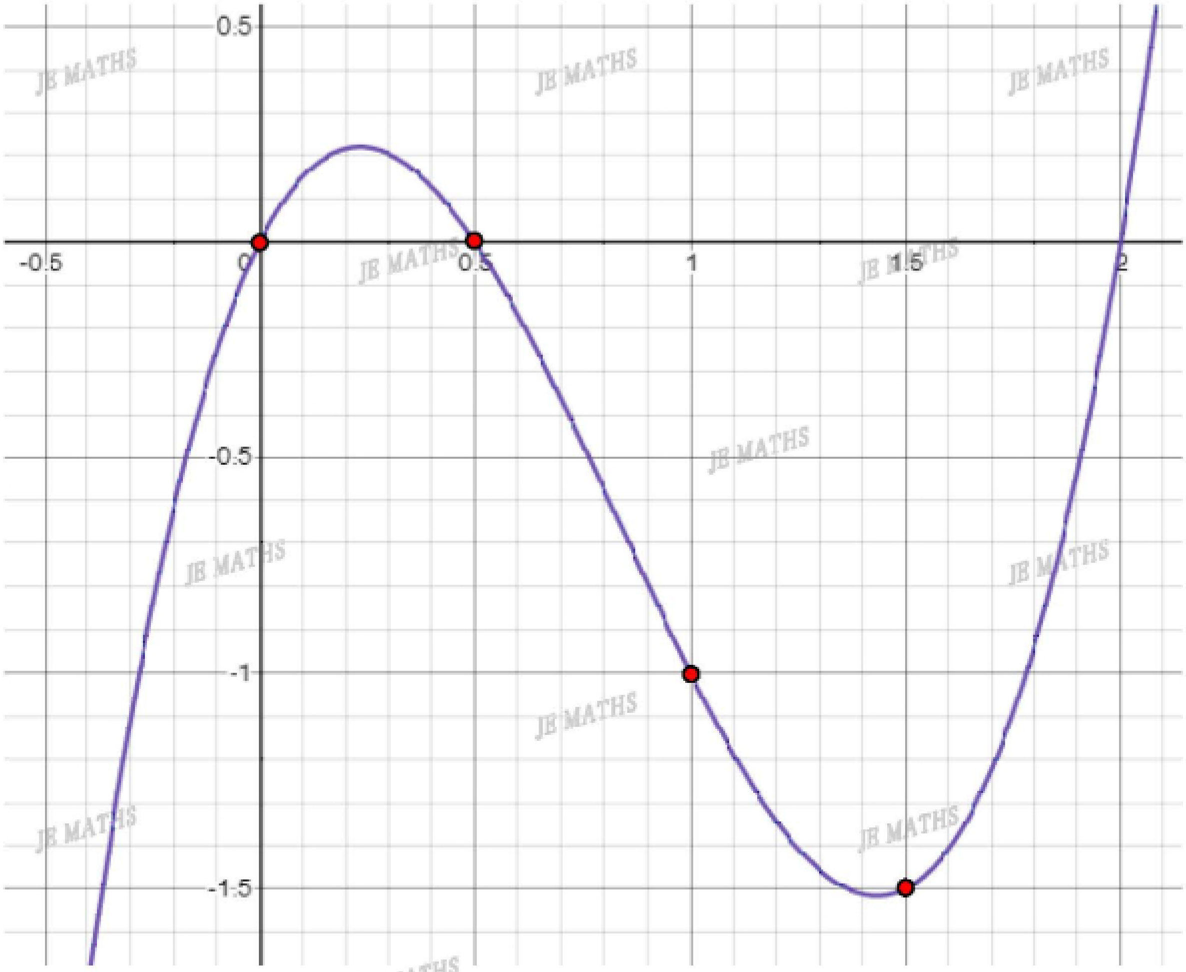
(ii) Construct the tangents to $f(x)$ at the given points. Measure the gradient of each tangent by using $gradient = \frac{rise}{run}$. Hence, estimate the derivatives. (1dp if necessary)

$f'(0) =$ $\qquad\qquad$ $f'(-0.5) =$

$f'(0.5) =$ $\qquad\qquad$ $f'(-1) =$

$f'(1.5) =$ $\qquad\qquad$ $f'(-1.5) =$

2. Given the graph of $f(x)$, construct the tangents at the given points and measure their gradient by $gradient = \dfrac{rise}{run}$.



Estimate the derivatives and correct to one decimal place if necessary.

$f'(0) =$ $f'(0.5) =$

$f'(1) =$ $f'(1.5) =$

3. Find the derivative $f'(x)$ of the function $f(x)$:

(a) $f(x) = 4$

(b) $f(x) = -\dfrac{5}{2}$

(c) $f(x)=5x-3$

(d) $f(x)=-3-7x$

(e) $f(x)=\dfrac{5(x-3)}{2}$

(f) $f(x)=\dfrac{7}{4}(1-3x)$

4. Find the derivative $f'(x)$ of the function $f(x)$:

(a) $f(x)=3(1-2x)-2(1-3x)$

(b) $f(x)=\dfrac{1}{2}(3-11x)-\dfrac{1}{2}(5-9x)$

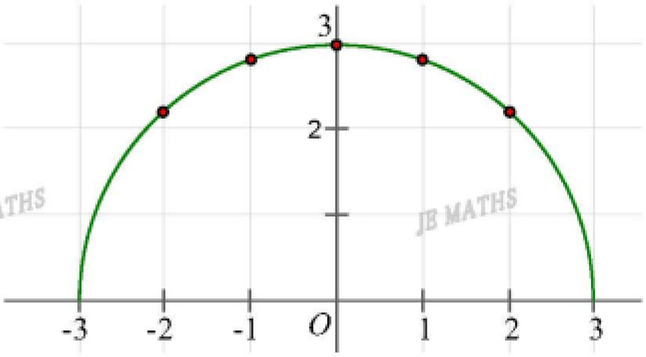
(c) $f(x)=(2x-7)^2-(2x+7)^2$

(d) $f(x)=\dfrac{x-c}{a}-\dfrac{b-x}{a}$, where a, b, c are constants.

(e) $f(x)=\dfrac{P+Qx}{R}+\dfrac{P-Qx}{S}$

Stage 2:

1. Consider the upper semicircle $f(x)=\sqrt{9-x^2}$.



(a) (i) Find the coordinates of the points shown in the diagram.

(ii) Construct the tangents to the semicircle at the given points. Draw the radius of the semicircle at the given points.

(iii) Use the fact that the tangent to the circle is perpendicular to the radius at the point of contact to find:

$f'(0)=$ $f'(1)=$

$f'(-1)=$ $f'(2)=$

$f'(-2)=$

(b) (i) Let point $P\left(x,\sqrt{9-x^2}\right)$ on the semicircle, find the gradient of the radius OP.

(ii) Use the fact in (a)(iii) to find $f'(x)$, the gradient of the tangent at P.

(iii) Hence, find the $x-$coordinate of the point on the semicircle where the gradient of tangent is equal to 1.

2. Consider the lower semicircle $f(x) = -\sqrt{16 - x^2} - 2$.

(i) Find the centre C and radius r of the semicircle and hence sketch the graph.

(ii) Find the gradient of radius CP at $P\left(x, -\sqrt{16-x^2} - 2\right)$.

(iii) Use the fact that the tangent to a circle is perpendicular to the radius at the point of contact to find $f'(x)$, the gradient of tangent at $P\left(x, -\sqrt{16-x^2} - 2\right)$.

(iv) Hence, find:

(a) $f'(-1) =$

(b) $f'(2) =$

(c) $f'(-3) =$

(v) Find where the gradient of tangent to the semicircle is $f'(x)$ = -1.

3. Consider the semicircle $f(x)=\sqrt{25-(x-1)^2}$.

(i) Find the centre C and radius r of the semicircle and hence sketch the graph.

(ii) Find the gradient of radius CP at $P\left(x,\sqrt{25-(x-1)^2}\right)$.

(iii) Use the radius and tangent theorem to find $f'(x)$.

(iv) Find the equation of tangent at $x=4$.

(v) Find $x-$coordinate of the point on the semicircle where the gradient of tangent is $\frac{1}{2}$.

4. Consider the semicircle $f(x) = -\sqrt{16-(x-5)^2}$.

(i) Find the centre and radius of the semicircle and sketch the graph.

(ii) Use the radius and tangent theorem to find the derivative of $f(x)$.

5. Consider the semicircle $f(x) = 7+\sqrt{9-(4-x)^2}$.

(i) Find the centre and radius of the semicircle and sketch the graph.

(ii) Use the radius and tangent theorem to find the derivative $f'(x)$.

6. Consider the semicircle $f(x) = 2 - \sqrt{3-(x+1)^2}$.

(i) Find the centre and radius of the semicircle and sketch the graph.

(ii) Use the radius and tangent theorem to find the derivative of $f(x)$.

7. Consider the semicircle $f(x) = 9 - \sqrt{4x - x^2}$.

(i) Find the centre and radius of the semicircle and sketch the graph.

(ii) Use the radius and tangent theorem to find the derivative $f'(x)$.

Stage 3:

1. Consider two ways to deduce the derivative of $f(x)=3x^2$.

(a) Let the point $P(a,3a^2)$ on the curve $y=3x^2$.

Let the line passing through P with gradient m be $y-3a^2=m(x-a)$.

(i) Find the $x-$coordinates of the two intersection points of the line and the curve in terms of a and m.

(ii) When the two intersection points overlap, the line $y-3a^2=m(x-a)$ is a tangent to the curve $y=3x^2$ at $P(a,3a^2)$. Find the gradient of tangent m in terms of a.

(iii) Hence, find the derivative $f'(x)$ of $f(x)=3x^2$.

(b) Let $y = mx + b$ be a tangent to the curve $y = 3x^2$.

(i) Find the $x-$coordinates of the point of contact.

(ii) Hence, deduce that the derivative $f'(x)$ of $f(x) = 3x^2$.

2. Consider the function $f(x) = 3x^2 - x + 2$.

(i) Let $y = mx + b$ be a tangent to $f(x)$, find the $x-$coordinates of the point of contact.

(ii) Hence, find the derivative of $f(x) = 3x^2 - x + 2$.

3. Consider the general function $f(x) = Kx^2$, where K is a constant.

(i) Let $y = mx + b$ be a tangent to $f(x)$, find the $x-$coordinates of the point of contact.

(iii) Hence, find the derivative of $f(x) = Kx^2$ in terms of K.

4. Consider the general parabola $f(x) = Ax^2 + Bx + C$, where A, B, C are constants.

(i) Let $y = mx + b$ be a tangent to $f(x)$, find the $x-$coordinates of the point of contact.

(ii) Hence, find the derivative of $f(x) = Ax^2 + Bx + C$ in terms of A, B and C.

5. Consider the curve $f(x)=\dfrac{1}{x}$. Let $y=mx+b$ be a tangent.

(i) Find the $x-$coordinate of the point of contact in terms of m and b.

(ii) Express b in terms of m.

(iii) Hence, deduce the derivative of $f(x)=\dfrac{1}{x}$.

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