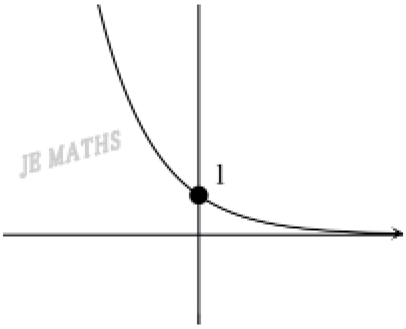
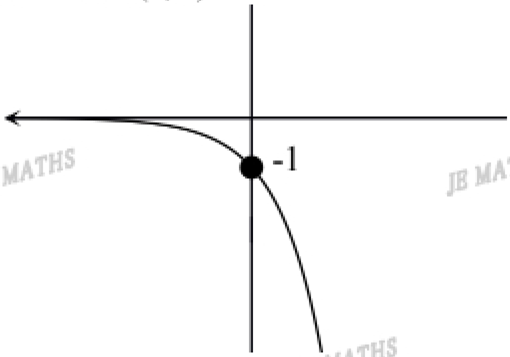
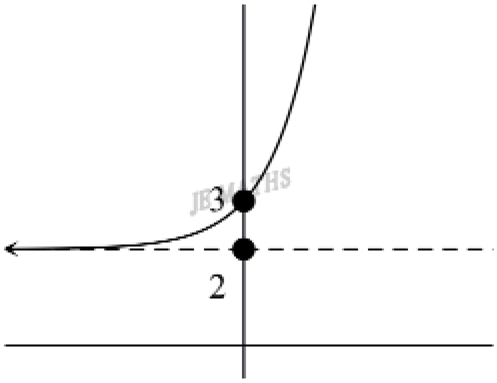
1. (a)
New centre (0, 0)



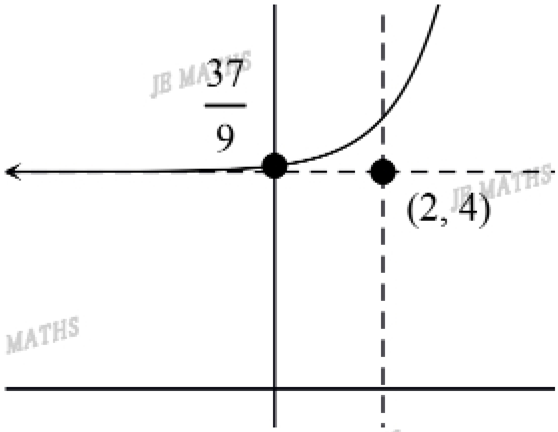
(b)
New centre (0, 0)



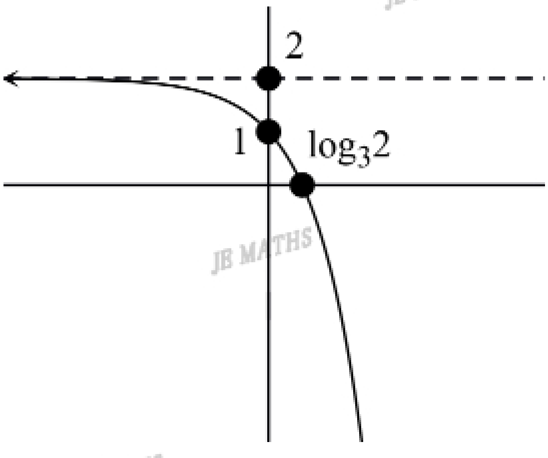
(c)
New centre (0, 2)



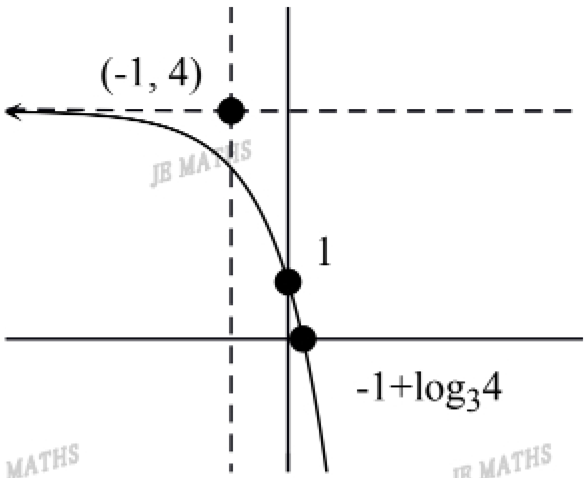
(d)
New centre (2, 4)



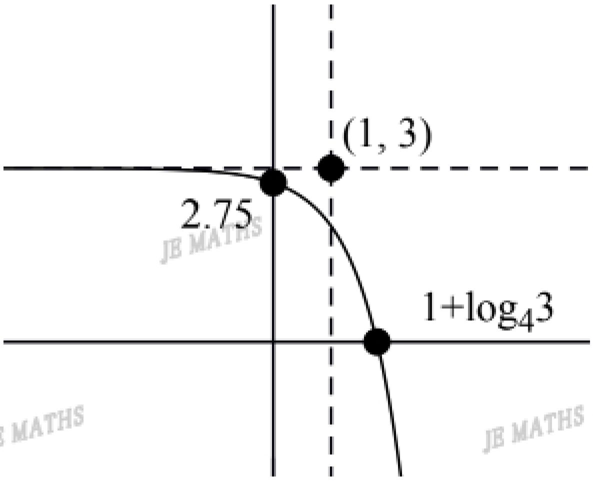
2. (a)
New centre (0, 2)



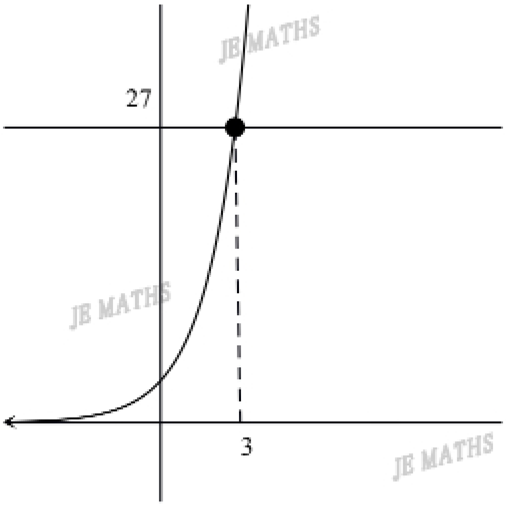
(b)
New centre (-1, 4)



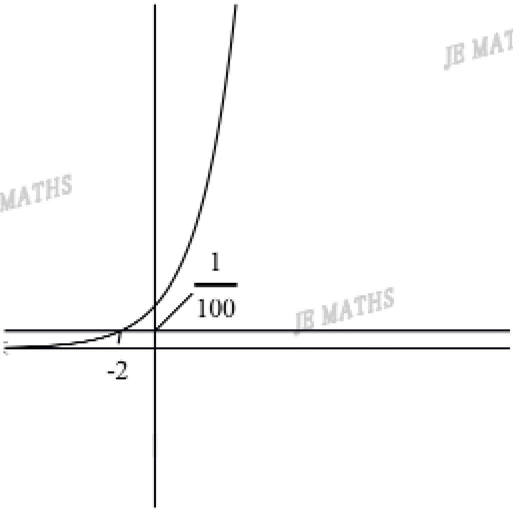
3. New centre: (1, 3)
y-int: (0, 2.75)
x-int: $(1 + log_4 3, 0)$



4. (a)
$3^x \leq 3^3$
$x \leq 3$



(b)
$10^x > 10^{-2}$
$x > -2$



5. (a)
$log_{10}3^x < log_{10}5$
$xlog_{10}3 < log_{10}5$
$x < log_{10}5/log_{10}3$
$x < log_3 5$

(b)
$log_{10}1.01^x \geq log_{10}0.1$
$xlog_{10}1.01 \geq log_{10}0.1$
$x \geq log_{10}0.1/log_{10}1.01$
$x \geq log_{1.01}0.1$

6. (a)

$3^{2x+1} > 3^2$ (take log_3)

$log_3 3^{2x+1} > log_3 3^0$

2x +1 > 2

2x > 1

x > 1/2

(b)

$\log_{10} 3^{x+3} < \log_{10} 1000$ (take log_{10})

$\log_{10} 3^{x+3} < 3\log_{10} 10$

$(x+3)\log_{10} 3 < 3$ ($\log_{10} 3 > 0$)

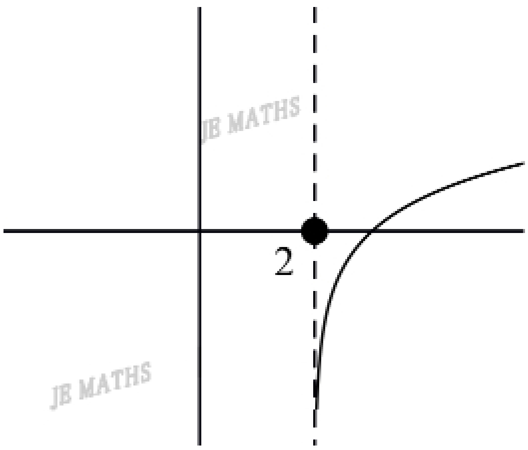
x +3 < 3/$\log_{10}$3

x < 3/$\log_{10}$3 − 3

If you take log_3, x<3log(3)10−3

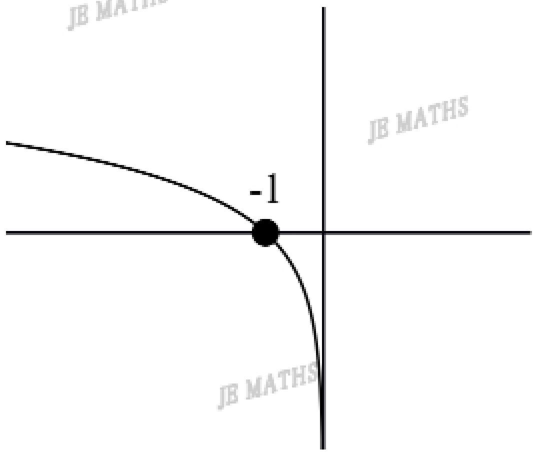
7. (a)

New centre: (2, 0)



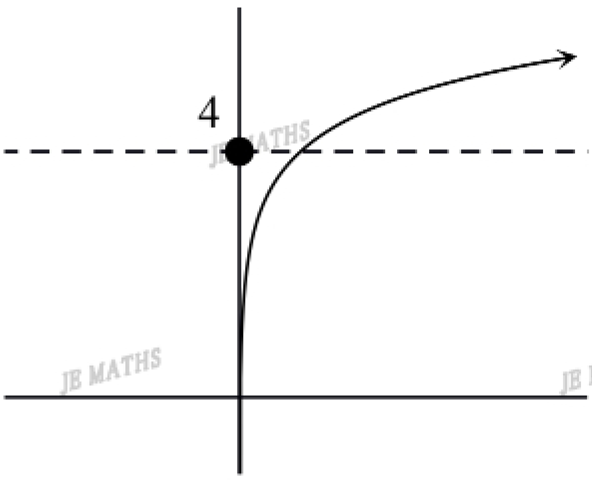
(b)

New centre: (0, 0)



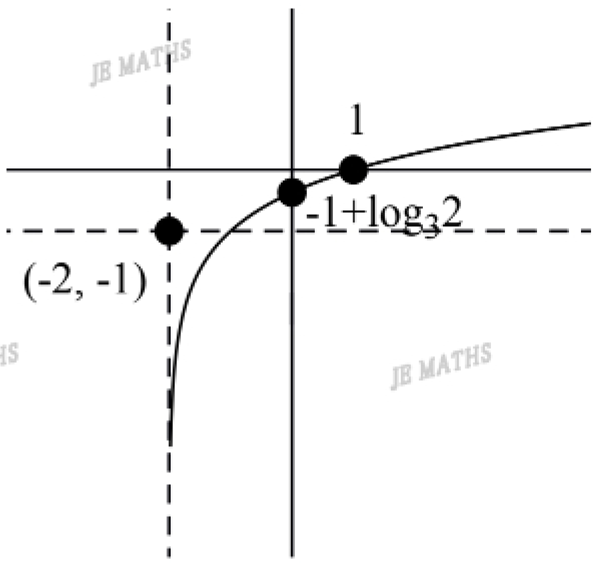
(c)

New Centre: (0, 4)



(d)

New Centre: (-2, -1)

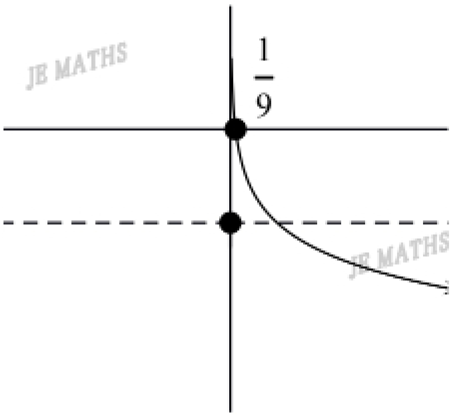


8.

(a)

New Centre: (0, -2)

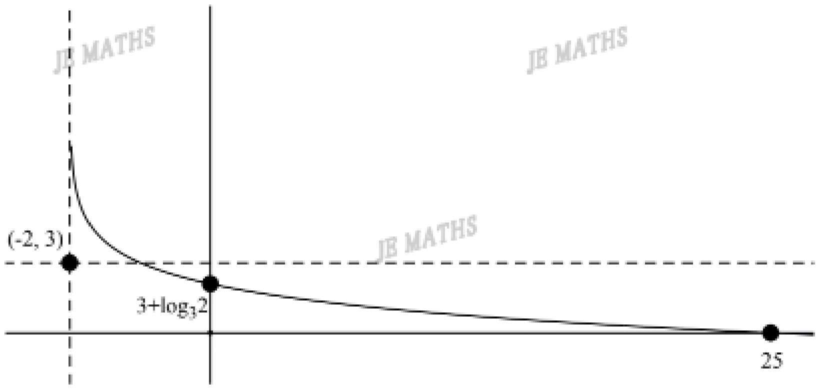
$y = log_3 x + 2 \rightarrow y = -(log_3 x + 2)$



(b)

New Centre: (-2, 3)

$y = log_3(x + 2) - 3 \rightarrow y = -(log_3(x + 2) - 3)$

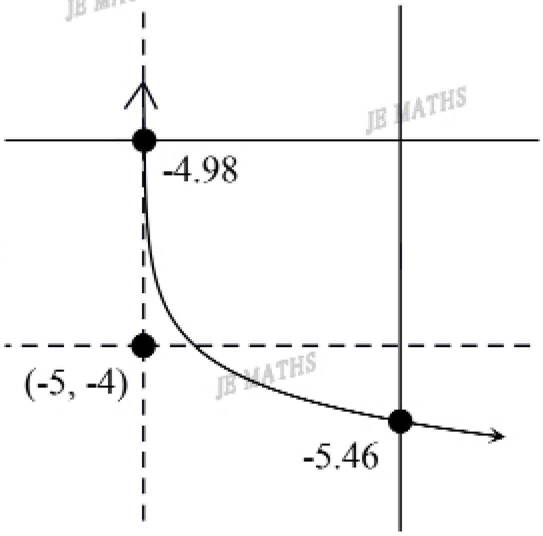


9. $y = -\log_3(x+5) - 4$

New centre: (-5, -4)

x-int: $(3^{-4} - 5$, 0)=(-4.98,0)

y-int: $(0, -4 - log_3 5)$=(0, -5.46)



- Exponential inequalities:

10.

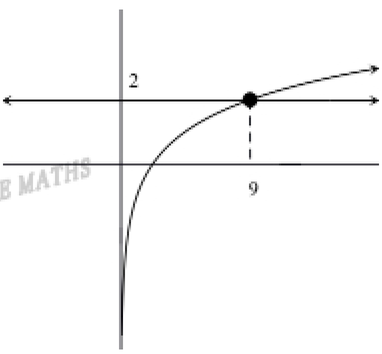
(a)

$x \leq 3^2$

$x \leq 9$

since $x > 0$

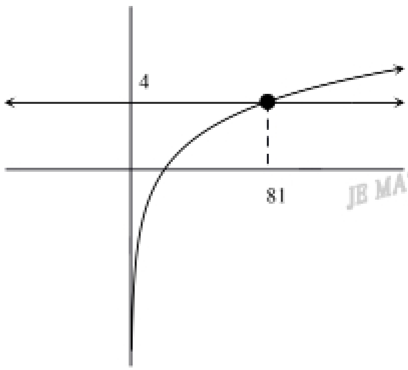
$0 < x \leq 9$



(b)

$x > 3^4$

$x > 81$



11.

(a) $\log_5 x < 9$

$5^{log_5 x} < 5^9$

$x < 5^9$

since x > 0

ans: $0 < x < 5^9$

(b) $\log_{10} x \geq \frac{1}{10}$

$log_{10} x \geq 0.1$

$10^{log_{10} x} \geq 10^{0.1}$

$x \geq 10^{0.1}$

12.

(a) sub t = 10 in,

$C = 2 \times 10^{10/2}$

$= 2 \times 10^5$

$= 200{,}000$

ans: C = 200,000

(b) $C = 2 \times 10^{t/2}$

$C/2 = 10^{t/2}$

$log_{10}(C/2) = t/2$

$2log_{10}(C/2) = t$

ans: $t = 2log_{10}(C/2)$

(c) sub C = 1000 in $t = 2log_{10}(C/2)$

$t = 2log_{10}(1000/2)$

$= 2log_{10}500$

$= 5.3979 \ldots$

ans: 5.4

13.

(a) sub Q = 8 in,

$t = 5log_3(2 \times 8)$

$= 5log_3 16$

$= 12.6185 \ldots$

ans: 12.6

(b) $t = 5log_3 2Q$

$t/5 = log_3 2Q$

$3^{t/5} = 2Q$

$1/2 \times 3^{t/5} = Q$

ans: $Q = 1/2 \times 3^{t/5}$

(c) sub t = 15 in $Q = 1/2 \times 3^{t/5}$

$Q = 1/2 \times 3^{15/5}$

$= 1/2 \times 3^3$

$= 27/2$

ans: 13.5

14. $3^x < 5^{100}$

$log_3 3^x < log_3 5^{100}$

$x < 100 log_3 5$

$x <146.4973...$

ans: 146

15.

(a) sub n =10 in

$C = 1{,}000{,}000 \times 1.05^{10}$

$= 1{,}000{,}000 \times 1.05^{10}$

$= 1{,}628{,}894.627$

ans: $1,628,894.6

(b) $1{,}000{,}000 \times 1.05^{10} > 2{,}000{,}000$

$1.05^{10} > 2$

$log_{1.05} 1.05^x > log_{1.05} 2$

$x > log_{1.05} 2$

$x > 14.2066 \ldots$

ans: more than 14.2 yr

16.

(a) n = 0, P = 2

n = 10, P = 4

ie, the population is doubling every 10 years

(b) (i)

sub n = 50 in

$P = 2 \times 2^{50/10}$

$= 2 \times 2^5$

$= 64$

ans: 64 mil

(64mil)

(ii)

sub n = 99 in

$P = 2 \times 2^{99/10}$

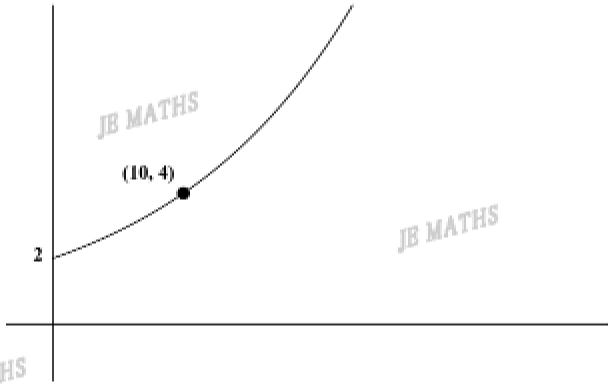
$= 2 \times 2^{9.9}$

$= 1910.851 \ldots$

ans: 1911 mil

(c)

n	0	10	20	30	40
P (mil)	2	4	8	16	32



(d) $P = 2 \times 2^{n/10}$

$P/2 = 2^{n/10}$

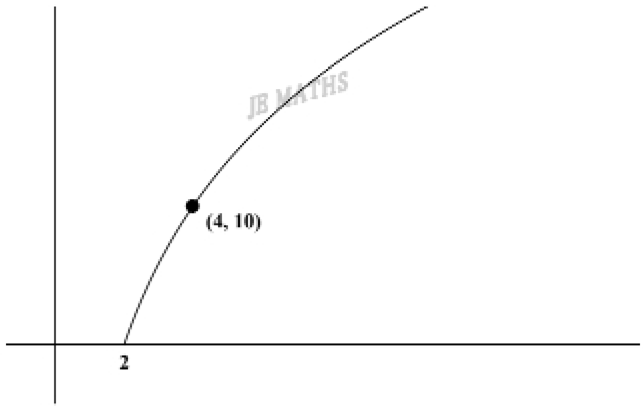
$log_2(P/2) = n/10$

$10log_2(P/2) = n$

ans: $n = 10log_2(P/2)$

(c)

P (mil)	2	4	8	16	32
n	0	10	20	30	40



(f) Initial population is 2 mil, tripled will be 2×3 = 6 mil.

sub P = 6 in $n = 10log_2(P/2)$

$n = 10log_2(6/2)$

$= 10log_2 3$

$= 15.8496 \ldots$

ans: 15.8 years

17.

(a) Make the formula for the population P after n hours.

n = 0, P = 10000

n = 5, P = 20000

ie, $P = 10000 \times 2^{n/5}$

(b) (i)

sub n = 10 in

$P = 10000 \times 2^{10/5}$

$= 10000 \times 2^2$

$= 40000$

ans: 40000

(ii)

sub n = 24 in

$P = 10000 \times 2^{24/5}$

$= 10000 \times 2^{4.8}$

$= 278576.1803$

ans: 278576

(c) $P = 10000 \times 2^{n/5}$

$P/10000 = 2^{n/5}$

$log_2 P/10000 = n/5$

$5log_2 P/10000 = n$

ans: $n = 5log_2 P/10000$

(d) (i)

sub P=888888 in $n = 5log_2 P/10000$

$n = 5log_2 888888/10000$

$= 5log_2 88.8888$

$= 32.3696 ...$

ans: 33 hours

(ii)

sub P/10000=1,000,000 in $n = 5log_2 P/10000$

$n = 5log_2 1{,}000{,}000$

$= 30log_2 10$

$= 99.6578 ...$

ans: 100 hours

(e) $n = 5log_2 P/10000$

$= 5(log_2 P - log_2 10000)$

$= 5(log_2 P - log_2 10^4)$

$= 5(log_2 P - 4log_2 10)$

$= 5log_2 P - 20log_2 10$

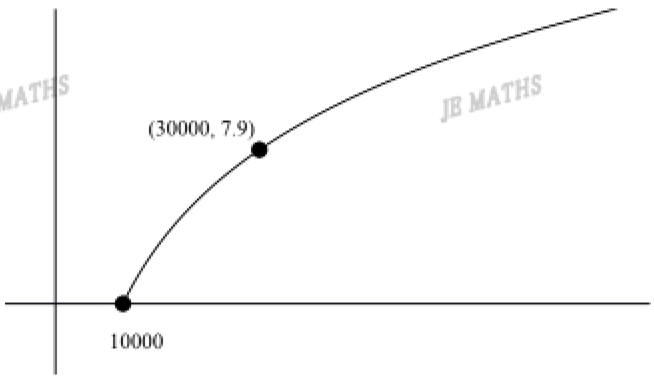
(f) sub P=30000 in $n = 5log_2 P/10000$

$n = 5log_2 30000/10000$

$= 5log_2 3$

$= 7.924 ...$

ans: (30000, 7.9)



18.

(a) n = 0, $M = M_0 \times (1/2)^0 = M_0$
n = 703,800,000, $M = M_0 \times 1/2$
ie, the mass is halved every 703,800,000 years

(b) sub n= 4,000,000,000 in
$M = M_0 \times (1/2)^{4,000,000,000/703,800,000}$
$M/M_0 = (1/2)^{4,000,000,000/703,800,000}$
$M/M_0 = 0.019$ ...
$M/M_0 = 1.9\%$
ans: 1.9% of the present uranium-235 will still be there on the Earth when they collide

(c) sub n= -4,600,000,000 in
$M = M_0 \times (1/2)^{-4,600,000/703,800,000}$
$M/M_0 = (1/2)^{-4,600,000/703,800,000}$
$M/M_0 = 92.7932$ ...
ans: there are 93 times more uranium 235 was there when the Earth was born

(d) $M = M_0 \times (1/2)^{n/703,800,000}$
$M/M_0 = (1/2)^{n/703,800,000}$
$log_{0.5}(M/M_0) = n/703,800,000$
$703,800,000 log_{0.5}(M/M_0) = n$
since $log_{0.5}(M/M_0) = log_{1/2}(M/M_0) = log_{2^{-1}}(M/M_0) = -log_2(M/M_0)$
$n = -703,800,000 log_2(M/M_0)$

(e) sub $M/M_0 = 0.01$ in $n = -703,800,000 log_2(M/M_0)$
$n = -703,800,000 log_2 0.01$
$= 4,675,945,986$
ans: after 4.7 billion years, the percentage of the present uranium-235 remaining on the Earth will drop to 1%.