

$$\begin{aligned}
 1. \quad d &= \sqrt{[(2\sqrt{2})^2 + 2^2]} \\
 &= \sqrt{8+4} \\
 &= \sqrt{12} \\
 &= 2\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad d &= \sqrt{[(4)^2 + (2\sqrt{3})^2]} \\
 &= \sqrt{16+12} \\
 &= \sqrt{28} \\
 &= 2\sqrt{7} = r
 \end{aligned}$$

Circle equation: $(x+5)^2 + (y+\sqrt{3})^2 = 28$

3. If A(-3, 6), find the coordinate of B.

Let B(x, y)

$$(x-3)/2 = -1$$

$$x-3 = -2$$

$$x = 1$$

ans: B(1, -2)

$$(y+6)/2 = 2$$

$$y+6 = 4$$

$$y = -2$$

4. Let R(x, y)

$$(x+4)/2 = 8$$

$$x+4 = 16$$

$$x = 12$$

ans: R(12, 3)

$$(y-7)/2 = -2$$

$$y-7 = -4$$

$$y = 3$$

5. inverse point: Q(2, $2\sqrt{2}$)

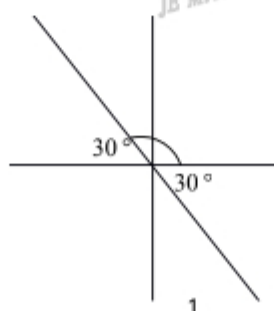
$$m = (2-2\sqrt{2})/(2\sqrt{2}-2)$$

$$= 2(1-\sqrt{2})/2(\sqrt{2}-1)$$

$$= -1$$

6. $\tan \alpha = -\sqrt{3}/3$

$$\alpha = 150^\circ$$



7.

$$\begin{aligned}\text{(a) } m_{AB} &= m_{PQ} \\ (k-4)/(1+2) &= -3 \\ k-4 &= -9 \\ k &= -5\end{aligned}$$

$$\begin{aligned}\text{(b) } m_{AB} \times m_{PQ} &= -1 \\ (k-4)/(1+2) \times -3 &= -1 \\ k-4 &= 1 \\ k &= 5\end{aligned}$$

8.

$$\begin{aligned}\text{(a) } 4/(k+2) &= (k-3)/-1 \\ -4 &= (k-3)(k+2) \\ k^2 - k - 6 + 4 &= 0 \\ k^2 - k - 2 &= 0 \\ (k-2)(k+1) &= 0 \\ k &= 2, -1\end{aligned}$$

$$\begin{aligned}\text{(b) } 4/(k+2) \times (k-3)/-1 &= -1 \\ 4(k-3)/(k+2) &= 1 \\ 4k - 12 &= k + 2 \\ 3k &= 14 \\ k &= 14/3\end{aligned}$$

9.

(a) Plot.

(b) $A(1, 0)$ and $C(7, 4)$
 $m_{AC} = 4/6 = 2/3 = \tan \theta$
 $\theta = 33.69 \dots = 34^\circ$

(c) $y - 0 = 2/3(x - 1)$
 $3y = 2x - 2$
 $2x - 3y - 2 = 0$

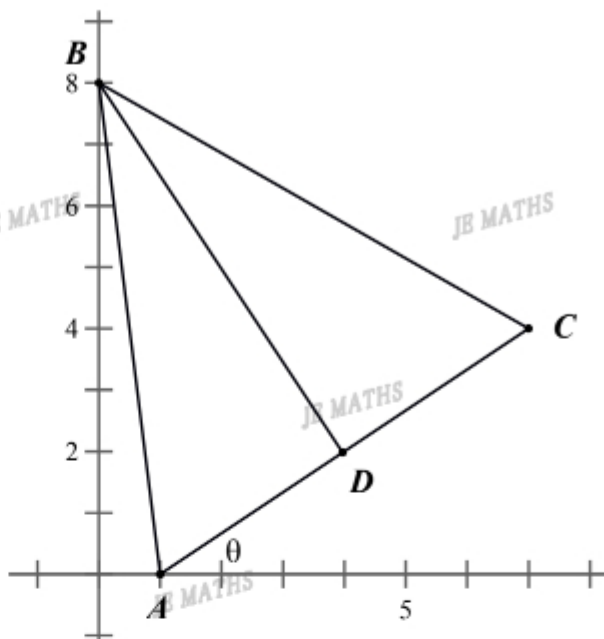
(d) $D((1+7)/2, (0+4)/2)$
 $= D(4, 2)$

(c) (i)
 $d_{AB} = \sqrt{(1^2 + 8^2)} = \sqrt{65}$
 $d_{BC} = \sqrt{(7^2 + 4^2)} = \sqrt{65}$
 $AB = BC$

(ii)
 $m_{AC} = 2/3, m_{BD} = (8-2)/(0-4) = -3/2$
 $m_{AC} \times m_{BD} = 2/3 \times -3/2 = -1$
 $AC \perp BD$

(f) isosceles triangle

(g) $d_{AC} = \sqrt{(6^2 + 4^2)} = \sqrt{52}$
 $d_{BD} = \sqrt{(4^2 + 6^2)} = \sqrt{52}$
 $A = \frac{1}{2} \times AC \times BD$
 $= \frac{1}{2} \times \sqrt{52} \times \sqrt{52}$
 $= 26 \text{ u}^2$



10.

- (a) $A(-5, -1) \rightarrow B(-1, 5)$ [4, 6]
 $D(0, -3) \rightarrow C(4, 3)$ [4, 6]
 ans: $C(4, 3)$

- (b) $m_{BD} = 8/-1 = -8$
 $y+3 = -8(x-0)$
 $8x+y+3=0$

- (c) (i) $d_{BD} = \sqrt{1^2 + 8^2} = \sqrt{65}$

(ii) $d_{AB} = \sqrt{4^2 + 6^2} = \sqrt{52}$

(iii) $d_{AD} = \sqrt{5^2 + 2^2} = \sqrt{29}$

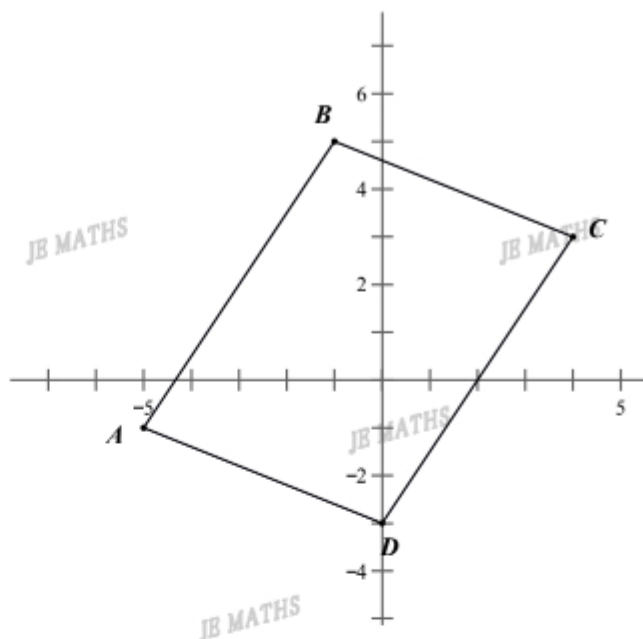
- (d) In $\triangle ABD$,

$$\cos \angle A = [(\sqrt{52})^2 + (\sqrt{29})^2 - (65)^2] / (2\sqrt{52}\sqrt{29})$$

$$\angle A = 78^\circ 6' 40.83''$$

$$= 78^\circ 7'$$

- (e) $\text{Area}(ABCD) = 2 \times \triangle ABD$
 $= 2 \times (\frac{1}{2} \times \sqrt{52} \times \sqrt{29} \times \sin 78^\circ 7')$
 $= 38.00007... = 38 \text{ u}^2$



11. Construct: make a right angle triangle ABC with C at the origin, $A(0, b)$ and $B(a, 0)$

Proof: $DA = DC = DB$

since D is the midpt of AB, $D(a/2, b/2)$

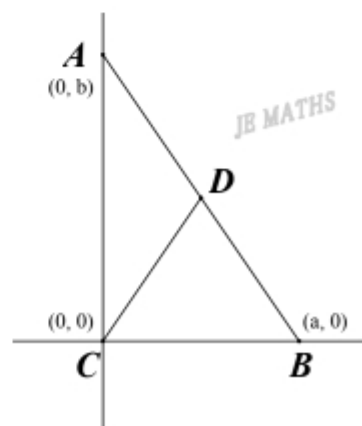
Use distance formula to find:

$$DA = \sqrt{[(a/2-0)^2 + (b/2-b)^2]} = \sqrt{(a^2/4 + b^2/4)} \quad 1)$$

$$DC = \sqrt{[(a/2-0)^2 + (b/2-0)^2]} = \sqrt{(a^2/4 + b^2/4)} \quad 2)$$

$$DB = \sqrt{[(a/2-a)^2 + (b/2-0)^2]} = \sqrt{(a^2/4 + b^2/4)} \quad 3)$$

From 1), 2) and 3): $DA = DC = DB$



12.

(a) Sketch.

(b) $(x-4)^2 + (y-6)^2 = 4$

(c) $(x-4)^2 + (y-6)^2 = 4$ 1)

$y = 8 - x$ 2)

$(x-4)^2 + (8-x-6)^2 = 4$

$(x-4)^2 + (x-2)^2 = 4$

$x^2 - 8x + 16 + x^2 - 4x + 4 = 4$

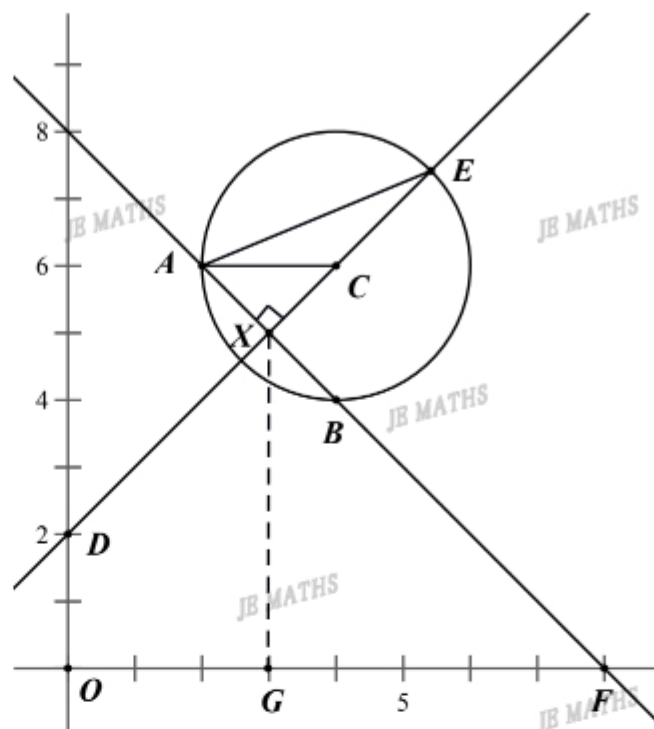
$2x^2 - 12x + 16 = 0$

$x^2 - 6x + 8 = 0$

$(x-2)(x-4) = 0$

$x=2, y=6 \rightarrow A(2, 6)$

$x=4, y=4 \rightarrow B(4, 4)$



(d) midpoint X: $((2+4)/2, (6+4)/2)$

$A(2, 6)$ and $B(4, 4)$, $m_{AB} = 2/-2 = -1$

$m_{XC} = 1$ ($m_{AB} \times m_{XC} = -1$)

mid-point X $((2+4)/2, (6+4)/2) = X(3, 5)$

$l_{XC}: y-5 = 1 \times (x-3)$

$y-5 = x-3$

$x-y+2=0$

$l_{XC}: x-y+2=0$

(c) (i)

sub $x=0$ in $x-y+2=0$

$D(0, 2)$

sub $y=0$ in $x+y=8$

$F(8, 0)$

(ii)

$A_{\text{DOGX}} = \frac{1}{2} \times 3 \times (2+5) = 21/2$

$A_{\text{XGF}} = \frac{1}{2} \times 5 \times 5 = 25/2$

$A_{\text{DEFO}} = A_{\text{DOGX}} + A_{\text{XGF}}$

$= 21/2 + 25/2$

$= 46/2$

$= 23 \text{ u}^2$

- (f) A (2, 6), X(3, 5), C(4, 6)

$$d_{AX} = \sqrt{(1^2 + 1^2)} = \sqrt{2}$$

$$d_{XC} = \sqrt{(1^2 + 1^2)} = \sqrt{2}$$

$$AX = XC = \sqrt{2}$$

$$AX \perp AB \text{ (given)}$$

ans: $\triangle AXC$ is an isosceles right angle triangle

- (g) $\angle XAC = \angle XCA$ (equal angles opposite to equal sides)

$$\begin{aligned} \angle ACX &= (180 - 90)/2 \quad (\text{angle sum of a triangle}) \\ &= 45^\circ \end{aligned}$$

- (h) $AC = CE$ (radii of circle)

$$\angle EAC = \angle AEC \text{ (equal angles opposite to equal sides)}$$

$$\angle ACX = 2\angle AEC \text{ (exterior angle of a triangle)}$$

$$\angle AEC = 45^\circ/2 = 22.5^\circ = 22^\circ 30'$$

- (i) In $\triangle AXE$,

$$AX = \sqrt{2}$$

$$XE = CX + EX = 2 + \sqrt{2}$$

$$AE = \sqrt{AX^2 + XE^2}$$

$$= \sqrt{(\sqrt{2})^2 + (2 + \sqrt{2})^2}$$

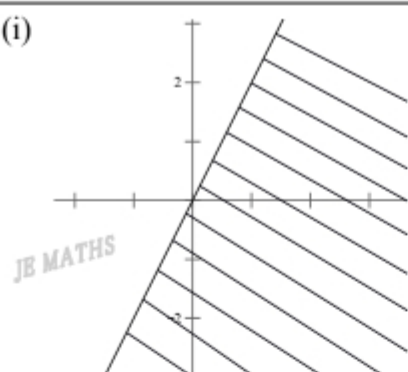
$$= \sqrt{8 + 4\sqrt{2}} \quad (AE > 0)$$

$$= 2\sqrt{2 + \sqrt{2}}$$

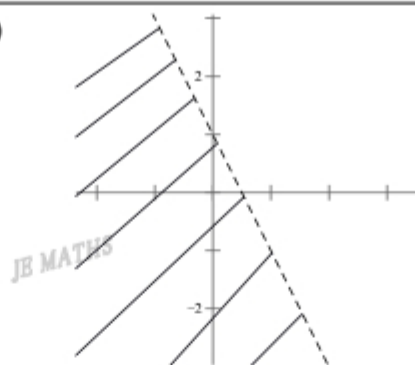
$$\cos 22^\circ 30' = EX/AE$$

$$= (2 + \sqrt{2})/2\sqrt{2 + \sqrt{2}}$$

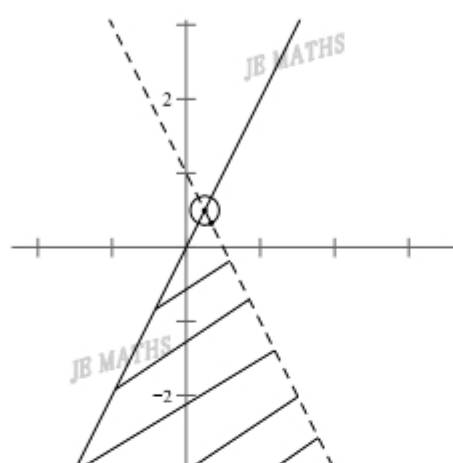
13. (a) (i)



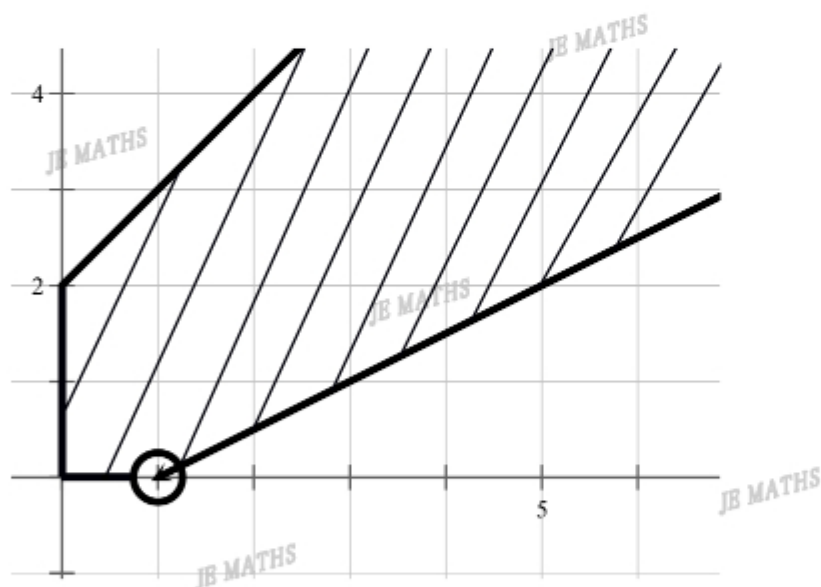
(ii)



(b)

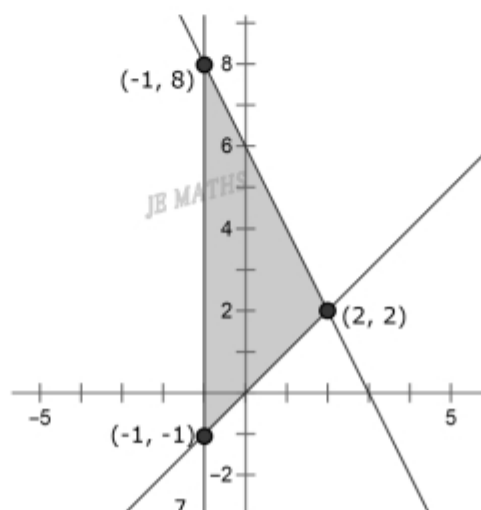


14.

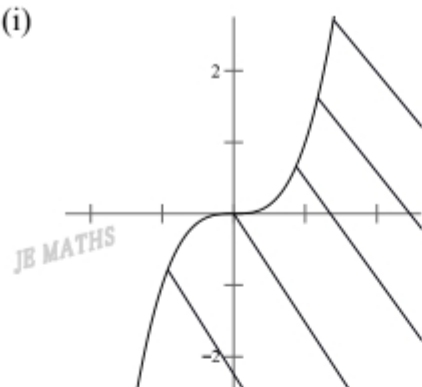


15. (a) Sketch.

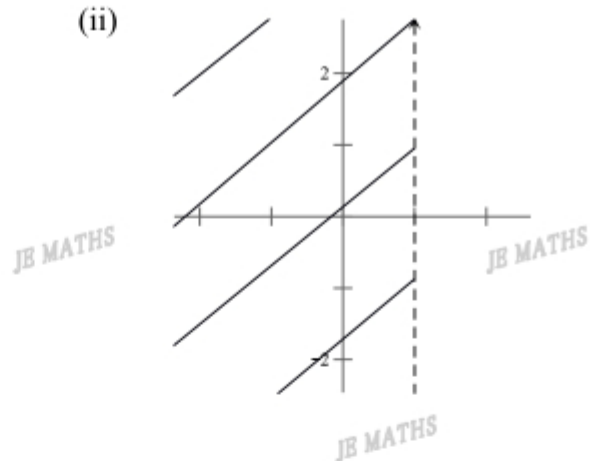
(b) $A = \frac{1}{2} \times 9 \times 3 = \frac{27}{2} \text{ u}^2$



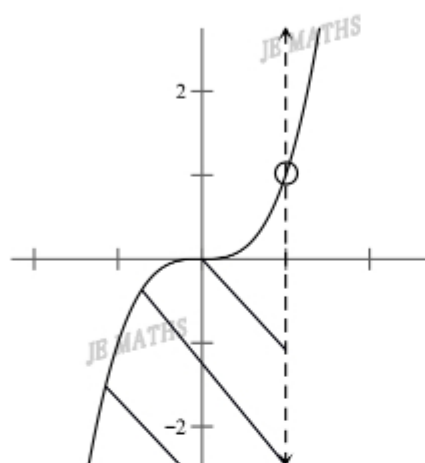
16. (a) (i)



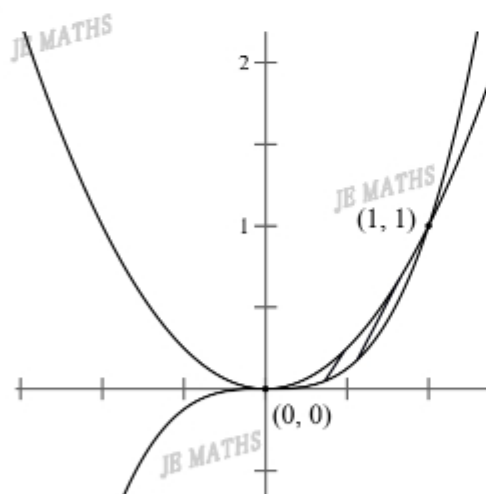
(ii)



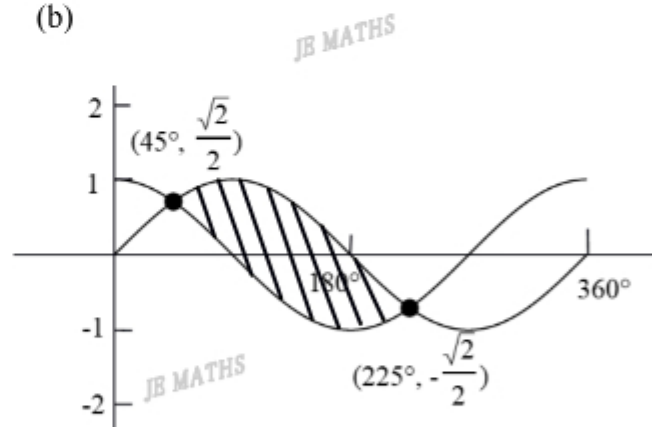
(b)



17. (a)



(b)



18. $A(0, a), B(b, 0), C(b/2, a/2)$

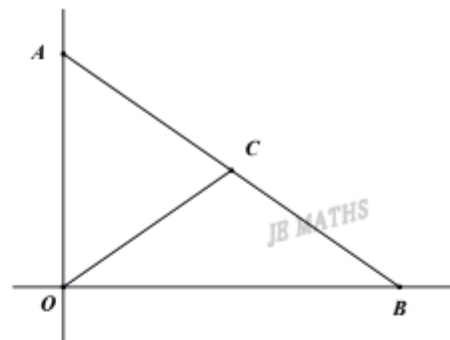
$$d_{AC} = \sqrt{[(b/2)^2 + (a/2 - a)^2]} = \sqrt{(b^2/4 + a^2/4)} = \sqrt{(a^2 + b^2)/2}$$

$$d_{CO} = \sqrt{[(b/2)^2 + (a/2)^2]} = \sqrt{(b^2/4 + a^2/4)} = \sqrt{(a^2 + b^2)/2}$$

$$d_{BC} = \sqrt{[(b - b/2)^2 + (a/2)^2]} = \sqrt{(b^2/4 + a^2/4)} = \sqrt{(a^2 + b^2)/2}$$

$$d_{AC} = d_{CO} = d_{BC}$$

ans: $AC=CO=BC$



19.

(a) (i)

$$C(r, r), m_{OC} = r/r = 1$$

$$l_{OC}: y = x$$

(ii)

$$\text{Circle: } (x-r)^2 + (y-r)^2 = r^2$$

(b) sub $y = x$ in to the circle

$$(x-r)^2 + (x-r)^2 = r^2$$

$$2(x-r)^2 = r^2$$

$$(x-r)^2 = r^2/2$$

$$x-r = r/\sqrt{2} \quad (\text{omit } -)$$

$$x = r/\sqrt{2} + r = r(1/\sqrt{2} + 1)$$

$$D(r(1/\sqrt{2} + 1), r(1/\sqrt{2} + 1))$$

(c) $l_{OC}: y = x,$

$$\tan \angle DOB = 1$$

$$\angle DOB = 45^\circ$$

$$\angle AOD = 90^\circ - \angle DOB$$

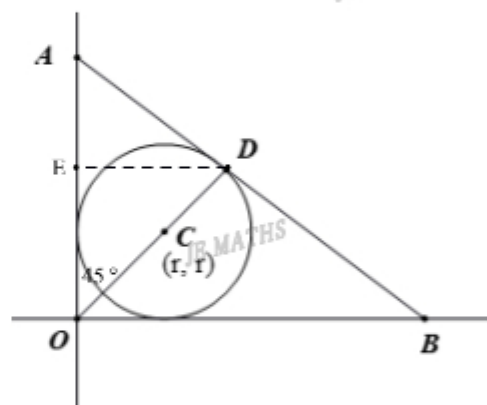
$$= 90^\circ - 45^\circ$$

$$= 45^\circ$$

$AD = OD$ (equal angles opposite to equal sides)

$OD \perp AB$ (tangent is perpendicular to the radius)

ans: $\triangle ADO$ is an isosceles right angle triangle



(d) Make a height on OA from D, named E.

$$D(r(1/\sqrt{2} + 1), r(1/\sqrt{2} + 1)) \text{ (proved)}$$

$$DE = r(1/\sqrt{2} + 1)$$

$$\angle AOD = \angle OAD = 45^\circ \text{ (equal angles opposite to equal sides)}$$

$$\angle ADE = 90 - 45 = 45^\circ, \angle EDO = 90 - 45 = 45^\circ \text{ (angle sum of a triangle)}$$

$$AE = ED = EO \text{ (equal angles opposite to equal sides)}$$

$$AO = 2ED$$

$$= 2 \times r(1/\sqrt{2} + 1)$$

$$= r(\sqrt{2} + 2)$$

$$\text{ans: } A(r(\sqrt{2} + 2), 0)$$

