




## Stage 1:

Differentiate each function by product rule and leave your answer in the fully factored form.
Hence, find the value(s) of x where the curve has a horizontal tangent.

(a) 
$$y = x(x-10)^4$$

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(b) 
$$y = x^4 (x-1)^3$$
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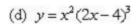
(c) 
$$y = 2x(x+5)^3$$

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(e)  $y = x^3(1-3x^2)^4$ JE MATHS

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(f)  $y = x^4 (x^2 + x)^3$   $y = x^4 (x^2 + x)^3$   $y = x^4 (x^2 + x)^3$ 

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(g)  $y = (x-2)(2x+1)^3$ 

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(h)  $y = (3x+1)^2 (4x-5)^3$ 

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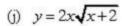
(i)  $y = -2\pi x^2 (x^2 - 3x + 1)^3$ 

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(k)  $y = -3x^2 \sqrt{3-x}$ 

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(1)  $y = x^2 \sqrt{1 - x^2}$ 

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(m) 
$$y = (3-2x^2)\sqrt{4-x^2}$$

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- 2. Consider the curve  $y = x^m (\alpha x)^n$ , where m > 1, n > 1 and  $\alpha > 0$ .
  - (i) Show that the curve has a horizontal tangent at a point T whose x-coordinate lies between 0 and  $\alpha$ . Hence, find the coordinate of point T.

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(11)	and the	coordinate	of point	7	if	m = n.	

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3. (a) Derive the product rule for a product of three:  $y = \iota \omega w$ , where u, v and w are functions of x.

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(b) Hence, differentiate  $y = x(x-1)^4 \sqrt{2x+1}$ . State value(s) of x when the derivative is zero.

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## Stage 2:

Differentiate each function by quotient rule and leave your answer in a single fraction.
Hence, find any value(s) of x that the tangent is horizontal.

(a) 
$$y = \frac{4x - 1}{2x + 1}$$

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(b) 
$$y = \frac{2x}{x^2 - 2}$$
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(c) 
$$y = \frac{x^2 - 4}{x^2 + 1}$$

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(d) 
$$y = \frac{3x^2}{x^3 - 2}$$

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(e) 
$$y = \frac{(x-1)^2}{x^2 + 2x}$$

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(f) 
$$y = \frac{(3x-1)^2}{x^2-x}$$

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(g) 
$$y = \frac{3x - 4}{\sqrt{2 - x}}$$

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(h) 
$$y = \frac{x+1}{\sqrt{x^2+1}}$$

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(i) 
$$y = \sqrt{\frac{3-x}{x-2}}$$

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- 2. Consider the hyperbola  $y = \frac{x}{x-2}$ .
  - (i) Sketch the graph of the hyperbola, showing the vertical and horizontal asymptotes, and state the domain and range.

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(ii) Show that the tangent at point P where x = p is  $2x + (p-2)^2y - p^2 = 0$ .

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(iii) Suppose the tangent at P meets x-axis at point A and the vertical line through P meets x-axis at point B. Find the value(s) of p such that OA = 2OB, where O is the origin.

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(iv) Suppose the tangent at point P passes through a fixed point C(0, c).

(a) Show that  $p = \frac{2\sqrt{c}}{\sqrt{c}-1}$  or  $p = \frac{2\sqrt{c}}{\sqrt{c}+1}$ .

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(b) State the values of c if such tangent exists.

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(c) State the values of c if only one such tangent exists.

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- (d) Suppose there are two such tangents exist.
  - (1) Show that the tangent at  $p = \frac{2\sqrt{c}}{\sqrt{c}+1}$  is always to the left branch of the hyperbola regardless the values of c.

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(2) Find values of c if the tangent at  $p = \frac{2\sqrt{c}}{\sqrt{c}-1}$  is to the right branch of the hyperbola.

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(3) Hence, state the values of c if both tangents are to the same branch of the hyperbola.

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3. Suppose that  $y = \frac{u}{x}$ , where u is a function of x. Show that  $\frac{du}{dx} = y + x \frac{dy}{dx}$ .

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4. (i) Sketch a point T on a curve y = f(x) where x, f(x) and f'(x) are all positive. Suppose the tangent, normal and vertical at T meet x – axis at point A, B and C respectively. Let the angle of inclination of the tangent be acute and denoted as  $\theta = \angle TAB$  so that  $y' = \tan \theta$ .

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- (ii) Use trigonometry to show that:
  - (a)  $AC = \frac{y}{y'}$

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(b) 
$$BC = yy'$$

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(c)  $\sec \theta = \sqrt{1 + (y')^2}$ 

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(d)  $\csc = \frac{\sqrt{1 + (y')^2}}{y'}$ 

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(e)  $AT = \frac{y\sqrt{1 + (y')^2}}{y'}$ 

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(f) 
$$BT = y\sqrt{1 + (y')^2}$$

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(iii) Hence, if  $f(x) = \frac{2x-3}{x+1}$  and x = 4, find the length of AC, BC, AT and BT.

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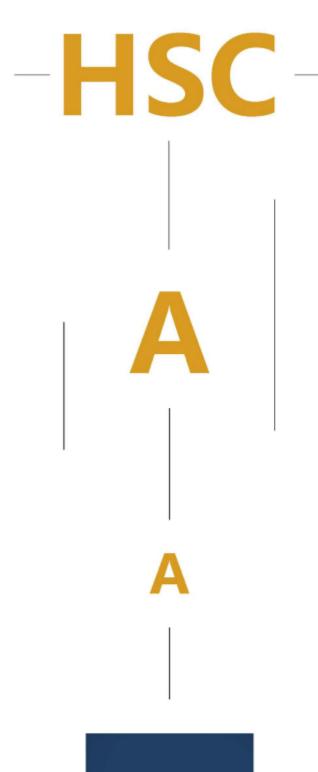
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