

1. (a)  $(1/2)^{-2} = 1/4$

(b)  $2^{-2} = 1/4$

(c)  $(5/2)^{-2} = (2/5)^2 = 4/25$

(d)  $(1/5)^{-2} = 5^2 = 25$

2.  $= 1/(1-2) + 1/(1+2/3)$

$= -1 + 3/5$

$= -2/5$

3. (a)  $x^{-2}$

(b)  $-1/2 \times x^{-2}$

4. (a)  $2^{2x} \times 2^{-4x} = 2^{-2x} = 1/(2^{2x})$

(b)  $2^{-2x} \div 2^{-x} = 2^{-x}$

(c)  $8m^{-6} \times n^6$

(d)  $6m^4 \times n^{-6} \div (4m^{-4} \times n^6) = 3/2 \times m^8 \times n^{-12} = 3m^8/2n^{12}$

5. (a)  $2 \times 2^n = 2^{n+1}$

(b)  $2^{-1} \times 2^{2n} = 2^{2n-1}$

(c)  $(4 \times 2^n - 2 \times 2^n) / 2^n = 2 \times 2^n / 2^n = 2$

(d)  $2^{-n} \times 2^{4n} \div 2^{-n} = 2^{4n}$

6. (a)  $x^{-2} + 1/x^2$

(c)  $= 1/(x + 1/y)^2$   
 $= 1/(x^2 + 2xy/y + 1/y^2)$

(b)  $x^4 + 2 + 1/x^4$

(d)  $= 1/(1/x + 1/y)^2$   
 $= 1/(1/x^2 + 2/xy + 1/y^2)$   
 $= x^2 y^2 (y^2 + 2xy + x^2)$   
 $= x^2 y^2 (x+y)^2$  (better leave like that)

7. (a)  $((3^n - 1)/(3^n - 1)) = (3^n + 1)(3^n - 1)/(3^n - 1) = 3^n + 1$

(b)  $(2^n - 1)(2^{2n} + 2^n + 1)$

8. (a)  $(2/3)^x = (2/3)^{-2}$   
 $x = -2$

(b)  $(3/5)^{-3x} = (3/5)^{-2}$   
 $-3x = -2$   
 $x = 2/3$

(c)  $2^{4-2n} = 2^3$   
 $4 - 2x = 3$   
 $1 = 2x$   
 $x = 1/2$

(d)  $(1/2)^{2x+4} = (1/2)^{3x-6}$   
 $2x+4 = 3x-6$   
 $10 = x$

9. (a)  $x - y = 0$   
 $x + y = 2$   
 $2x = 2$   
 $x = 1$   
 $y = 1$

(b)  $x + y = 3$   
 $2(x - y) = 3 \rightarrow x - y = 1.5$   
 $2x = 4.5$   
 $x = 9/4$   
 $y = 3 - 9/4 = 3/4$

10.  $2^{2n+2} - 2^{2n-1} = 1792$   
 $2^{2n-1}(2^3 - 1) = 1792$   
 $2^{2n-1} \times 8 = 1792$   
 $2^{2n-1} = 256$   
 $2^{2n-1} = 2^8$   
 $2n - 1 = 8$   
 $n = 9/2$

11. (a)  $\sqrt{4} = 2$

(b)  $\sqrt{1/4} = 1/2$

(c)  $(3/2)^2 = 9/4$

(d)  $(27/8)^{4/3} = (3/2)^4 = 81/16$

12. (a)  $15a^{1/2-1/3} b^{1/3+1/2} = 15a^{1/6}b^{5/6}$

(b)  $64x^3 \times y^{-4} \div (x^{-2} \times y^{-1}) = 64x^5 \times y^{-3}$

13. (a)  $-x^{3/2}$

(b)  $3x^{-2/3}$

(c)  $xy^{-3/2}$

(d)  $x^{3/2}y^{-2/3}$

14. (a)  $x^{-2} + 1/x$

(b)  $x^3 + 2 + 1/x^3$

15. (a)  
 $x = 5^{1/5}$

(c)  
 $x = \pm 81^{1/4}$   
 $x = \pm 3$

(b)  
 $x = 243^{3/5} = 3^3 = 27$

(d)  
 $x = \pm 9^{3/2}$   
 $x = \pm 27$

16. (a)  
 $2^{3x-3} = 2^2$   
 $3x-3=2$   
 $3x=5$   
 $x=5/3$

(c)  
 $2^{2x} = 2^{3/2}$   
 $2x = 3/2$   
 $x=3/4$

(b)  
 $(2/5)^{3x+6} = (2/5)^{-x}$   
 $3x+6 = -x$   
 $4x = -6$   
 $x = -3/2$

(d)  
 $3\sqrt{2} \times 3^{2x+2} = 9\sqrt{6} \quad (\div 3\sqrt{2})$   
 $3^{2x+2} = 3\sqrt{3}$   
 $3^{2x+2} = 3 \times 3^{1/2}$   
 $3^{2x+2} = 3^{3/2}$   
 $2x + 2 = 3/2$   
 $2x = -1/2$   
 $x = -1/4$

17. (a)  $5\log_x x = 5$

(b)  $\log_x x^{-2} = -2\log_x x = -2$

(c)  $\log_x x^{1/2} = 1/2 \times \log_x x = 1/2$

(d)  $\log_x x^{-1/2} = -1/2 \times \log_x x = -1/2$

(e)  $\log_x x^{5/2} = 5/2 \times \log_x x = 5/2$

(f)  $\log_x x^{-3/2} = -3/2 \times \log_x x = -3/2$

18. (a)  $-\log_6 2 + \log_6 2 = 0$

(b)  $\log_2 (96 \div 6) = \log_2 16 = \log_2 2^4 = 4\log_2 2 = 4$

(c)  $\log_3 (4 \times 18 \div 8) = \log_3 9 = 2\log_3 3 = 2$

(d)  $\log_2 (4 \div 8 \div 16) = \log_2 (1/32) = \log_2 2^{-5} = -5\log_2 2 = -5$

19. (a)  $3/2\log_2 2 = 3/2$

(b)  $3/2\log_3 3 = 3/2$

(c)  $-1/(1/2) \times \log_3 3 = -2$

(d)  $3n/2n \times \log_2 2 = 3/2$

20. (a) (5)

(b) (3)

(c) (a)

(d) (b)

21. (a)  $\log_2 3 + \log_2 5$

(b)  $-\log_2 3 \times 5$   
 $= -\log_2 3 - \log_2 5$

(c)  $\log_2 8 - \log_2 5$   
 $= 3\log_2 2 - \log_2 5$   
 $= 3 - \log_2 5$

(d)  $\log_2 10 - \log_2 3$   
 $= \log_2 2 + \log_2 5 - \log_2 3$   
 $= 1 + \log_2 5 - \log_2 3$

(e)  $\log_2 2 + \log_2 3 + \log_2 5$   
 $= 1 + \log_2 3 + \log_2 5$

(f)  $\log_2 27 - \log_2 501$   
 $= 3\log_2 3 - \log_2 25 \times 2$   
 $= 3\log_2 3 - 2\log_2 5 - \log_2 2$   
 $= 3\log_2 3 - 2\log_2 5 - 1$   
 $= 3\log_2 3 - 2\log_2 5 - 1$

22. (a)  $\log_{10} 10 + \log_{10} x^2$   
 $= 1 + 2\log_{10} x$

(b)  $\log_3 x - \log_3 3$   
 $= \log_3 x - 1$

(c)  $3\log_2 2 + 3\log_2 x$   
 $= 3 + 3\log_2 x$

(d)  $\log_3 (1/3x)$  (9/27=1/3)  
 $= -(\log_3 3 - \log_3 x)$   
 $= -1 + \log_3 x$

23. (a)  $4^{\log_4 (5/6)} = 5/6$

(b)  $t^{\log_t (b^a)} = b^a$

(c)  $a^{\log_a (x^x)} = x^x$

(d)  $a^{1/x \times \log_a x} = a^{\log_a (x^{1/x})} = x^{1/x}$

24. (a)

$$\begin{aligned}
 \text{LHS} &= \log_2 5 + \log_2 2 + \log_2 3 + \log_2 \sqrt{3} \\
 &= \log_2 5 + \log_2 2 + \log_2 3 + 1/2 \times \log_2 3 \\
 &= \log_2 5 + 1 + 3/2 \times \log_2 3 = \text{RHS}
 \end{aligned}$$

(b)

$$\begin{aligned}
 \text{LHS} &= \log_2 8 - \log_2 25 - \log_2 \sqrt{30} \\
 &= 3\log_2 2 - 2\log_2 5 - 1/2 \log_2 3 \times 5 \\
 &= 3\log_2 2 - 2\log_2 5 - 1/2 \times (\log_2 3 + \log_2 5) \\
 &= 3 - 2\log_2 5 - 1/2 \times (\log_2 3 + \log_2 5) \\
 &= 3 - 2\log_2 5 - 1/2 \times \log_2 3 - 1/2 \times \log_2 5 \\
 &= 3 - 5/2 \times \log_2 5 - 1/2 \times \log_2 3 = \text{RHS}
 \end{aligned}$$

25. (a)

$$\begin{aligned}
 \text{LHS} &= \log_x 8 - \log_x 5 - \log_x x^3 \quad (\text{simplify } 24/15=8/5) \\
 &= 3\log_x 2 - \log_x 5 - 3 \\
 &= 3p - r - 3 = \text{RHS}
 \end{aligned}$$

(b)

$$\begin{aligned}
 \text{LHS} &= \log_x 36 - \log_x 25 - \log_x x \\
 &= 2\log_x 6 - 2\log_x 5 - 1 \\
 &= 2\log_x 2 \times 3 - 2\log_x 5 - 1 \\
 &= 2\log_x 2 + 2\log_x 3 - 2\log_x 5 - 1 \\
 &= 2p + 2q - 2r - 1 = \text{RHS}
 \end{aligned}$$

26. LHS =  $\log_{an} x$ 

$$\begin{aligned}
 &= \log_a x / \log_a an \\
 &= \log_a x / (\log_a a + \log_a n) \\
 &= \log_a x / (1 + \log_a n) \\
 &= \text{RHS}
 \end{aligned}$$

27. (a)  $3^{\log_3 5}$ (b)  $3^{\log_3 m}$ (c)  $2^{\log_2 0.4}$ (d)  $x^{\log_x y}$ 

28. (a)

$$\begin{aligned}
 \log_2 \sqrt{x} + \log_2 2 &= \log_2 y^3 \\
 \log_2 2\sqrt{x} &= \log_2 y^3 \\
 2\sqrt{x} &= y^3
 \end{aligned}$$

(b)

$$\begin{aligned}
 \log_5 (2x - 1)^3 &= \log_5 (2x + 1)^2 + \log_5 5 \\
 \log_5 (2x - 1)^3 &= \log_5 5(2x + 1)^2 \\
 (2x - 1)^3 &= 5(2x + 1)^2
 \end{aligned}$$

