Development stage 1: (logarithm equation)

- 1. (a) Given that $x^{\sqrt{x}} = (\sqrt{x})^x$, find the value of x by
 - (i) using index laws:







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(ii) taking logarithm on both sides:



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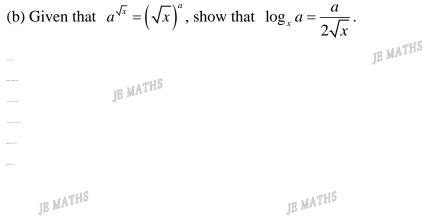


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Development stage 2: (quadratic equation involving logarithm)

1. (a) It is given that $P(x) = x^3 + x - 30$ has a integer root. Factorise P(x).

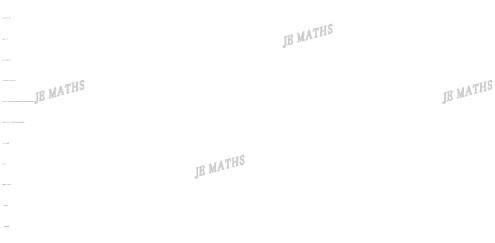
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(b) Find the value of x if $8^x + 2^x = 30$. Correct your answer to 3 decimal places. $\mathbb{B}^{\text{MATHS}}$



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Development stage 1:

1. (a)

$$x^{\sqrt{x}} = (x^{1/2})^x$$

$$\sqrt{x} = x/2$$

$$x = x 74$$

 $x^2 - 4x = 0$

$$x^2 - 4x = 0$$

$$x(x-4)=0$$

$$x = 0$$
 or 4

$$x \neq 0$$
 as 0^0 is undefined

$$x = 4$$
 only

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(ii)

$$x^{\sqrt{x}} = (\sqrt{x})^x$$

$$\log(x^{\sqrt{x}}) = \log[(\sqrt{x})^x]$$

$$\sqrt{x} \times \log x = x \times \log x^{1/2}$$

$$\sqrt{x} \times \log x = x \times 1/2 \times \log x$$

$$\sqrt{x} = x/2$$
, $\log x > 0$ for all x

$$x = x ? 4$$

$$x^2 - 4x = 0$$

$$x(x - 4) = 0$$

$$x = 0 \text{ or } 4$$

$$x \neq 0$$
 as 0^0 is undefined

$$x = 4$$
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(b)

$$a^{\sqrt{x}} = (\sqrt{x})^a$$

$$\log(a^{\sqrt{x}}) = \log[(\sqrt{x})^a]$$

$$\sqrt{x} \times \log a = a \times \log x^{1/2}$$

$$\sqrt{x} \times \log a = a \times 1/2 \times \log x$$

$$\log a/\log x = a/2\sqrt{x}$$

$$log_x a = a/2\sqrt{x}$$

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Development stage 2: (quadratic equation involving logarithm)

1. (a)
$$P(x) = x^3 + x - 30$$

Test
$$x = 3$$

$$P(3) = 3^3 + 3 - 30$$

=0

 $\therefore x = 3$ is the integer root.

$$P(x) = x^3 + x - 27 - 3$$

$$=(x-3)(x^2+3x+9)+(x-3)$$

$$=(x-3)(x^2+3x+10)$$

Alternatively, long division or sum and product of roots can be used to achieve the same result. JE MATHS IE MATHS

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(b) $8^x + 2^x = 30$

$$(2^x)^3 + 2^x = 30$$

Let
$$u = 2^x$$

$$u^3 + u - 30 = 0$$

$$(u-3)(u^2+3u+10)=0$$

 $u^2 + 3u + 10 = 0$ has a discriminant of -31 which is negative

Hence, $u^2 + 3u + 10$ has no real solutions.

 $\therefore u = 3$ only

$$2^{x} = 3$$

$$\log_2 2^x = \log_2 3$$

$$x = \log_2 3$$

$$= \log 3/\log 2$$

$$= 1.585 (3dp)$$

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$$2^{x} = 3$$

$$Z = 3$$

$$\log_2 2^x = \log_2 3$$

= 1.585 (3dp)

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