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JE MATHS

10 Adv

Stage 1:

1. Use Parametric Differentiation to find $\frac{dy}{dx}$ in terms of the parameter t.

(a) $x = 2t + 1$, $y = t^2 - 4t$

(b) $x = t^2 - 3$, $y = \frac{1}{t}$

(c) $x = t + 4$, $y = \frac{1}{t^3}$

(d) $x = \sqrt{t}$, $y = 2t^3 + 5$

(e) $x = \frac{1}{t^2}$, $y = \frac{1}{\sqrt{t}}$

2. A curve is defined by $x = \frac{(t+1)^2}{2}$, $y = \frac{(t+1)^3}{3}$.

(i) Find the derivative $\frac{dy}{dx}$ of the curve in terms of t.

(ii) Find the gradient of the tangent and hence the point of contact if:

(a) $t = -3$

(b) $t = 0$

(c) $t = \frac{1}{2}$

(iii) Find where on the curve the tangent has:

(a) gradient 5

(b) an angle of inclination of 135°.

(iv) Find the equation of tangent at the point where $x = \frac{(t+1)^2}{2}$.

(v) Find the area of the triangle formed by the tangent in (iv) and xy axis.

3. (i) Show that the rectangular hyperbola $xy = c^2$ can be described by the two parametric equation $x = ct$ and $y = \frac{c}{t}$.

(ii) Use parametric differentiation to find the equation of tangent at $T\left(ct, \frac{c}{t}\right)$.

(iii) Let tangent at T meet axis at point A and B.
Show that the area of triangle OAB is constant as point T varies.

(iv) Show that point T bisects AB. Hence, Show that $AB = 2OT$.

Stage 2:

1. Use $\frac{dy}{dx} = \frac{1}{dx/dy}$ to differentiate:

   (a) $y = \sqrt{4 - x}$

   (b) $y = \frac{1}{\sqrt[5]{x}}$

   (c) $y = \frac{1}{\sqrt[3]{x+1}}$

2. (i) Use $\frac{dy}{dx} = \frac{1}{dx/dy}$ to differentiate $y = x^{\frac{1}{k}}$ for non-zero integer k.

(ii) Use chain rule $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ to prove the Power Rule for rational indices:

$$\frac{d}{dx}\left(x^{\frac{n}{k}}\right) = \frac{n}{k} \times x^{\frac{n}{k}-1}, \text{ where } n,\ k \text{ are integers.}$$

Stage 3:

1. Find where on the curve the tangent is horizontal.

   (a) $y = \dfrac{-3}{x^4 - 2x^2 - 1}$

   (b) $y = \sqrt{x^2 - 4x}$

2. Find the equations of the tangent and normal to the curve:

   (a) $y = \dfrac{1}{1+x^2}$ at $x = 1$

(b) $y=\sqrt{8-x}$ at $x=4$

3. Find the value of k if:

(a) the tangent to $y=\dfrac{1}{x^2+k}$ at $x=1$ has a gradient $-\dfrac{1}{2}$

(b) the normal to $y=\sqrt{x+k}$ at $x=4$ has gradient -6

4. Let the function $y = \dfrac{1}{8-x}$.

(i) Find the equation of tangent at the point where $x = p$.

(ii) Hence, find the equation of tangents passing through:

(a) the origin

(b) $(10, 0)$

Stage 4:

1. Consider the semi circle $y=\sqrt{16-x^2}$ and semi ellipse $y=k\sqrt{16-x^2}$ ($k>0$ and $k\neq 1$).

(i) Find the equation of tangent to the semi circle and semi ellipse at $x=t$, where $0<t<4$, respectively.

(ii) Find the $x-$intercept of these two tangents in part (i).
Hence, show that these two tangents in part (i) meet each other at $x-$axis.

(iii) Suppose the tangent to the semi ellipse at point T where $x = t$ $(0 < t < 4)$ meet $x-$axis at point A, the semi ellipse meet $x-$axis at $B(4, 0)$ and the vertical line through T meet $x-$axis at $C(t, 0)$. Show that $OB \times OB = OA \times OC$.

2. Consider the parabola $y = a(x-h)^2 + k$.

(i) Find point P and Q on the curve where the tangent has gradient m and $-m$.

(ii) Show that the area of the quadrilateral formed by the tangents and normals at P and Q is $\frac{m(1+m^2)}{4a^2}$.

3. Consider the parabola $y = a(x-h)^2 + k$.

(i) Show that the equation of tangent at point T where $x = t$ is $y = 2a(t-h)x - at^2 + ah^2 + k$.

(ii) Find the $x-$coordinate of the point(s) on the curve where the tangent passes through the origin.

(iii) Let the vertical line through the vertex $V(h,k)$ meet the tangent at T at point P.

Find the length of VP. Hence, show that VP is proportional to the square of the horizontal distance from point T to the axis of symmetry of the parabola.

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