Enrichment stage 1:

1. (a) How many month will it take \$1000 to amount to \$1239, if it is invested at 0.5% per year compound monthly? (Give your answer to the nearest month)

$1239 = 1000(1 + 0.005/12)^n$

$(2401/2400)^n = 1.239$

$\log_{10}(2401/2400)^n = \log_{10} 1.239$

$n = \log_{10} 1.239 / \log_{10} 2401/2400$

$= 514.438...$

$= 515$ months (rounded up to nearest month)

(b)

(i) How many months will it take \$A to amount to \$B, if it is invested at C% per year compound monthly?

$A = B(1 + 0.005/12)^n$

$(1+C/12)^n = A/B$

$\log_{10}(1+C/12)^n = \log_{10}(A/B)$

$n = \log_{10}(A/B) / \log_{10}(1+C/12)$

$n = \log_{1+C/12}(A/B)$

(ii) Hence, find C the subject of the relationship.

$(1+C/12)^n = A/B$

$1+C/12 = (A/B)^{1/n}$

$C/12 = (A/B)^{1/n} - 1$

$C = 12(A/B)^{1/n} - 12$

2. Using a graphical sketch, show that $\frac{3}{7}(x-1) < \log_2 x$ for $1 < x < 8$.

Therefore, for $1 < x < 8$, the curve $y = \log_2 x$ is above the line $y = 3(x-1)/7$.

Competition stage 1:

1. (a) Given that $4^x = (2^x)^2$, rewrite 25^x into a square number.

   (b) Given that $6^x = 2^x 3^x$, rewrite 10^x into the product of two index numbers.

   (c) Solve $4^x - 25^x = 10^x$ by using substitution and involving logarithm.

   (d) Hence show that the solution of $a^{2x} - b^{2x} = (ab)^x$ is $\dfrac{\log \varphi}{\log(\frac{a}{b})}$ where φ is golden ratio.

Enrichment stage 1:

1. (a)

$1239 = 1000(1 + 0.005/12)^n$

$(2401/2400)^n = 1.239$

$log_{10}\ (2401/2400)^n = log_{10}1.239$

n = $log_{10}1.239 / log_{10}2401/2400$

= 514.438...

= 515 months (rounded up to nearest month)

(b)

(i)

$A = B(1 + 0.005/12)^n$

$(1+C/12)^n = A/B$

$log_{10}\ (1+C/12)^n = log_{10}(A/B)$

n = $log_{10}(A/B)/ log_{10}(1+C/12)$

n = $log_{1+C/12}(A/B)$

(ii)

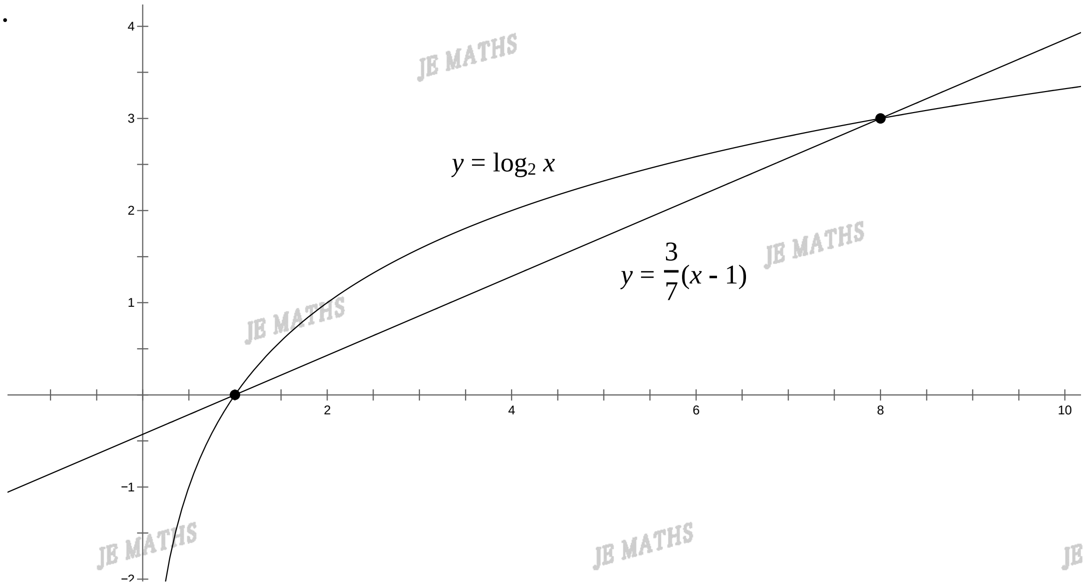
$(1+C/12)^n = A/B$

$1+C/12 = (A/B)^{1/n}$

$C/12 = (A/B)^{1/n} - 1$

$C = 12(A/B)^{1/n} - 12$

2.



Therefore, for $1 < x < 8$, the curve $y = \log_2 x$ is above the line $y = 3(x - 1)/7$.

Competition stage 1:

1. (a) $(5^x)^2$

   (b) 2^x5^x

   (c)
   $(2^x)^2-(5^x)^2=2^x5^x$
   $\div(5^x)^2$ on both side:
   $(2^x/5^x)^2-1=2^x/5^x$
   let $t=(2^x/5^x)=(2/5)^x$
   $t^2-1=t$
   $t^2-t-1=0$
   $t=(1+\sqrt{5})/2$ (since $t>0$, omit -)
   $(1+\sqrt{5})/2=(2/5)^x$
   $\log(1+\sqrt{5})/2=\log(2/5)^x$
   $\log(1+\sqrt{5})/2=x\log(2/5)$
   $x=\log(1+\sqrt{5})/2\ /\ \log(2/5)$

   (d)
   $(a^x)^2-(b^x)^2=a^xb^x$
   $\div(b^x)^2$ on both side:
   $(a^x/b^x)^2-1=a^x/b^x$
   let $t=(a^x/b^x)=(a/b)^x$
   $t^2-1=t$
   $t^2-t-1=0$
   $t=(1+\sqrt{5})/2$ (since $t>0$, omit -)
   $(1+\sqrt{5})/2=(a/b)^x$
   $\log(1+\sqrt{5})/2=\log(a/b)^x$
   $\log(1+\sqrt{5})/2=x\log(a/b)$
   $x=\log(1+\sqrt{5})/2\ /\ \log(a/b)$
   $=\log\varphi/\log(a/b)$, $\varphi=(1+\sqrt{5})/2=$golden ratio