# 0. 说明

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# 问1：证明 $\boldsymbol{z}(\boldsymbol{x}_{jk}) \approx \left(\mathbf{1}+\boldsymbol{\epsilon}_k^{\wedge}+\frac{1}{2}\boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{\epsilon}_k^{\wedge}\right)\boldsymbol{T}_{op,k}(\boldsymbol{p}_{op,j}+\boldsymbol{D}\boldsymbol{\zeta}_j)$

$$\approx \boldsymbol{T}_{op,k}\boldsymbol{p}_{op,j}+\boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{T}_{op,k}\boldsymbol{p}_{op,j}+\boldsymbol{T}_{op,k}\boldsymbol{D}\boldsymbol{\zeta}_j+\frac{1}{2}\boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{T}_{op,k}\boldsymbol{p}_{op,j}+\boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{T}_{op,k}\boldsymbol{D}\boldsymbol{\zeta}_j$$

**（三次项省略掉了）**

$$= \boldsymbol{z}(\boldsymbol{x}_{op,jk})+\boldsymbol{Z}_{jk}\delta\boldsymbol{x}_{jk}+\frac{1}{2}\sum_i \mathbf{1}_i\delta\boldsymbol{x}_{jk}^T\mathcal{Z}_{ijk}\delta\boldsymbol{x}_{jk}$$

其中

$$\boldsymbol{x}_{op,jk}=\{\boldsymbol{T}_{op,k},\boldsymbol{p}_{op,j}\}, \qquad \delta\boldsymbol{x}_{jk}=\begin{bmatrix}\boldsymbol{\epsilon}_k\\ \boldsymbol{\zeta}_j\end{bmatrix}$$

$$\boldsymbol{z}(\boldsymbol{x}_{op,jk})=\boldsymbol{T}_{op,k}\boldsymbol{p}_{op,j}$$

$$\boldsymbol{Z}_{jk}=\begin{bmatrix}(\boldsymbol{T}_{op,k}\boldsymbol{p}_{op,j})^{\odot} & \boldsymbol{T}_{op,k}\boldsymbol{D}\end{bmatrix}$$

$$\mathcal{Z}_{ijk}=\begin{bmatrix}\mathbf{1}_i^{\circledcirc}(\boldsymbol{T}_{op,k}\boldsymbol{p}_{op,j})^{\odot} & \mathbf{1}_i^{\circledcirc}\boldsymbol{T}_{op,k}\boldsymbol{D}\\ (\mathbf{1}_i^{\circledcirc}\boldsymbol{T}_{op,k}\boldsymbol{D})^T & \mathbf{0}\end{bmatrix}$$

**证：**

- 一次项

  $$\boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{T}_{op,k}\boldsymbol{p}_{op,j}+\boldsymbol{T}_{op,k}\boldsymbol{D}\boldsymbol{\zeta}_j=(\boldsymbol{T}_{op,k}\boldsymbol{p}_{op,j})^{\odot}\boldsymbol{\epsilon}_k+\boldsymbol{T}_{op,k}\boldsymbol{D}\boldsymbol{\zeta}_j=\begin{bmatrix}(\boldsymbol{T}_{op,k}\boldsymbol{p}_{op,j})^{\odot} & \boldsymbol{T}_{op,k}\boldsymbol{D}\end{bmatrix}\begin{bmatrix}\boldsymbol{\epsilon}_k\\ \boldsymbol{\zeta}_j\end{bmatrix}=\boldsymbol{Z}_{jk}\delta\boldsymbol{x}_{jk}$$

  主要用到的性质是：$\boldsymbol{x}^{\wedge}\boldsymbol{p}=\boldsymbol{p}^{\odot}\boldsymbol{x}$ (见书上表7-3)

- 二次项
  - 方法1：直接带入各个符号的表达式计算，得到 $\frac{1}{2}\boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{T}_{op,k}\boldsymbol{p}_{op,j}+\boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{T}_{op,k}\boldsymbol{D}\boldsymbol{\zeta}_j=\frac{1}{2}\sum_i \mathbf{1}_i\delta\boldsymbol{x}_{jk}^T\mathcal{Z}_{ijk}\delta\boldsymbol{x}_{jk}$
  - 方法2：

  $$\frac{1}{2}\boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{T}_{op,k}\boldsymbol{p}_{op,j}+\boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{T}_{op,k}\boldsymbol{D}\boldsymbol{\zeta}_j=\frac{1}{2}\boldsymbol{\epsilon}_k^{\wedge}(\boldsymbol{T}_{op,k}\boldsymbol{p}_{op,j})^{\odot}\boldsymbol{\epsilon}_k+\boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{T}_{op,k}\boldsymbol{D}\boldsymbol{\zeta}_j$$

  取其第 $i$ 项

  $$\mathbf{1}_i^T\left(\frac{1}{2}\boldsymbol{\epsilon}_k^{\wedge}(\boldsymbol{T}_{op,k}\boldsymbol{p}_{op,j})^{\odot}\boldsymbol{\epsilon}_k+\boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{T}_{op,k}\boldsymbol{D}\boldsymbol{\zeta}_j\right)=\frac{1}{2}\mathbf{1}_i^T\boldsymbol{\epsilon}_k^{\wedge}(\boldsymbol{T}_{op,k}\boldsymbol{p}_{op,j})^{\odot}\boldsymbol{\epsilon}_k+\mathbf{1}_i^T\boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{T}_{op,k}\boldsymbol{D}\boldsymbol{\zeta}_j$$

  $$=\frac{1}{2}\boldsymbol{\epsilon}_k^T\mathbf{1}_i^{\circledcirc}(\boldsymbol{T}_{op,k}\boldsymbol{p}_{op,j})^{\odot}\boldsymbol{\epsilon}_k+\boldsymbol{\epsilon}_k^T\mathbf{1}_i^{\circledcirc}\boldsymbol{T}_{op,k}\boldsymbol{D}\boldsymbol{\zeta}_j$$

$$= \frac{1}{2}\left(\begin{bmatrix}\boldsymbol{\epsilon}_k^T & \boldsymbol{\zeta}_j^T\end{bmatrix}\begin{bmatrix}\mathbf{1}_i^{\circledcirc}(\boldsymbol{T}_{op,k}\boldsymbol{p}_{op,j})^{\odot} & \mathbf{1}_i^{\circledcirc}\boldsymbol{T}_{op,k}\boldsymbol{D} \\ (\mathbf{1}_i^{\circledcirc}\boldsymbol{T}_{op,k}\boldsymbol{D})^T & \mathbf{0}\end{bmatrix}\begin{bmatrix}\boldsymbol{\epsilon}_k \\ \boldsymbol{\zeta}_j\end{bmatrix}\right)$$

$$= \tfrac{1}{2}\delta\boldsymbol{x}_{jk}^T\mathcal{Z}_{ijk}\delta\boldsymbol{x}_{jk}$$

$$\therefore \mathbf{1}_i\mathbf{1}_i^T\left(\tfrac{1}{2}\boldsymbol{\epsilon}_k^{\wedge}(\boldsymbol{T}_{op,k}\boldsymbol{p}_{op,j})^{\odot}\boldsymbol{\epsilon}_k + \boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{T}_{op,k}\boldsymbol{D}\boldsymbol{\zeta}_j\right) = \tfrac{1}{2}\sum_i \mathbf{1}_i\delta\boldsymbol{x}_{jk}^T\mathcal{Z}_{ijk}\delta\boldsymbol{x}_{jk}$$

主要用到的性质是：$\boldsymbol{x}^{\wedge}\boldsymbol{p} = \boldsymbol{p}^{\odot}\boldsymbol{x}$ 和 $\boldsymbol{p}^T\boldsymbol{x}^{\wedge} = \boldsymbol{x}^T\boldsymbol{p}^{\circledcirc}$(见书上表7-3)