Pricing Fixed Bonds

Summary: Pricing Fixed Bonds

I) General principal of bond valuation

II) Some properties of the Present Value

III) Reasons for the change in the price of a Bond

IV) Pricing a fixed coupon

V) Dirty / Clean pricing

VI) Notation, credit risk and collateral

I) General principal of bond valuation.

- The general principal of bond valuation is that its value is equal to the present value of its expected cash flows.

- 3 Steps:

A) Estimate the cash flows

B) Determine the appropriate interest rates that should be used to discount cash flows.

C) Calculate the present value of the expected cash flows found in step A) using the IR determined in step B)

A) Estimate the cash flows

- Cash flows = payment of coupons + principal
- Fixed bonds have known cash flows

B) Determine the appropriate interest rates that should be used to discount cash flows.

- The minimum interest rate than an investor should require is the yield available in the market place on a default free cash-flow.
- In the US, this is the yield on a US Treasury.
- Then the min is the Yield on the run Treasury security with the same as the security being valued.
- For a non US government security, investors will require a premium over the yield available on an on the run Treasury issue. This yield premium reflects the additional risks that the investor accepts.

C) Discounting the expected cash flows

What is the value of a single cash flow to be receive in the future?

- It is the amount of money that must be invested today to generate that future value.
- This is the present value.

$$PV = \frac{C_1}{(1+r)^n}$$

$C_1 = Cash\ Flow\ at\ period\ 1$
$r = rate\ of\ return$
$n = number\ of\ periods$

- Value = PV1+PV2 + … + PVn

Example :Discounting the expected cash flows

Bond with 4y maturity and an annual coupon of 10%

Discount rates : 8%

Cash flow will be

Y1 : 10;Y2 : 10;Y3: 10;Y4: 110

- $PV1 = 10/1{,}08 = 9{,}2593$
- $PV2 = 10/1{,}08^2 = 8{,}5734$
- $PV3 = 10/1{,}08^3 = 7{,}9383$
- $PV4 = 110/1{,}08^4 = 80{,}8533$

$PV = 9{,}2593 + 8{,}5734 + 7{,}9383 + 80{,}8533 = 106{,}6243$

II) Some properties of the PV

- For a given discount rate, the further into the future a cash flow is received, the lower its PV.

- The higher the discount rate, the lower the PV.

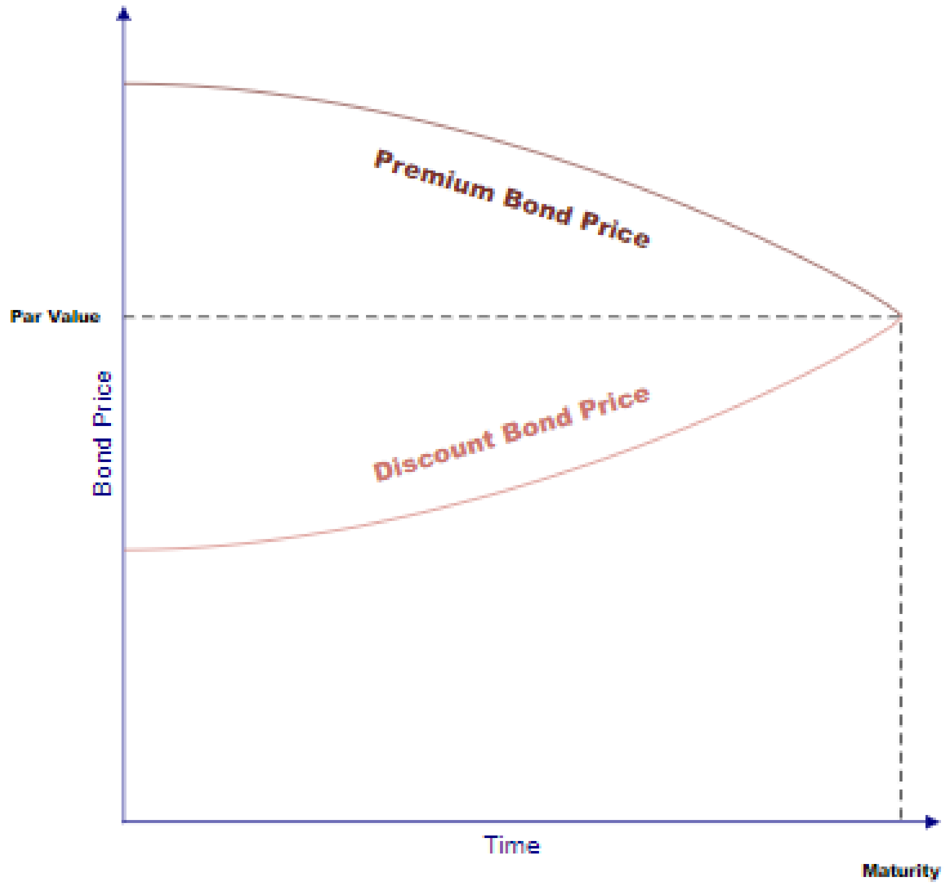
- Coupon rate = yield required by market, therefore price = par value.
- Coupon rate < yield required by market, therefore price < par value. This is discount.
- Coupon rate > yield required by market, therefore price > par value. This is premium.

Change in Bond's value as it moves toward maturity

- If the discount rate doesn't change, a bond's value:

- Decrease over time if the bond is selling at a premium
- Increases over time if the bond is selling at a discount
- Is unchanged if the bond is selling at Par value.

Change in Bond's value as it moves toward maturity



III) Reasons for the Change in the Price of a Bond

The price of a bond will change because of one or more of the following reasons:

- *A change in the level of interest rates in the economy.* For example, if interest rates in the economy increase (fall) because of Fed policy, the price of a bond will decrease (increase).
- *A change in the price of the bond selling at a price other than par as it moves toward maturity without any change in the required yield :*over time a discount bond's price increases if yields do not change; a premium bond's price declines over time if yields do not change.

III) Reasons for the Change in the Price of a Bond

- *For non-Treasury bonds, a change in the required yield due to changes in the spread to Treasuries.* If the Treasury rate does not change but the spread to Treasuries changes (narrows or widens), non-Treasury bond prices will change.
- *A change in the perceived credit quality of the issuer.* Assuming that interest rates in the economy and yield spreads between non-Treasuries and Treasuries do not change, the price of a non-Treasury bond will increase (decrease) if its perceived credit quality has improved (deteriorated).

IV) Pricing a fixed coupon : Valuation using multiple discount rates

Suppose the appropriate discount rates are as follow:
Y1: 6,8%Y2: 7,2%Y3:7,6%Y4:8,0%

Then for the 4 year 10% coupon bond, the PV of each cash flow is

Y1: PV1= $10/1{,}068=9{,}3633$
Y2: PV2= $10/1{,}072^2=8{,}7018$
Y3: PV3= $10/1{,}076^3= 8{,}0272$
Y4: PV4= $110/1{,}080^4= 80{,}8533$

PV = 106,9456

IV) Valuing a zero coupon bond

$$Zero\,Coupon\;Bond\,Value = \frac{F}{(1+r)^t}$$

$$F = \;face\;value\;of\;bond$$
$$r = rate\;or\;yield$$
$$t = time\;to\;maturity$$

- A 5 year zero coupon bond is issued with a face value of $100 and a rate of 6%. Looking at the formula, $100 would be ***F***, 6% would be ***r***, and ***t*** would be 5 years.
- 100 / (1,06) ^{5}
- After solving the equation, the original price or value would be $74.73. After 5 years, the bond could then be redeemed for the $100 face value.

IV) Example of a Bond with semi-annual coupon

- The price of a bond is equal to the present value of the cash flows, and it can be determined by adding

(1) the present value of the semiannual coupon payments and

(2) the present value of the par or maturity value.

IV) Example of a Bond with semi-annual coupon

$$c\left[\frac{1-\left[\frac{1}{(1+i)^n}\right]}{i}\right]+\frac{M}{(1+i)^n}$$

where

c = semiannual coupon payment ($)
n = number of periods (number of years times 2)
i = periodic interest rate (required yield divided by 2) (in decimal)
M = maturity value

IV) Exercice

Compute the price of a 9% coupon bond with 20 years to maturity and a par value of $1,000 if the required yield is 12%.

- Price is Higher or Lower than $1,000 ?

IV) Exercice

- The cash flows for this bond are as follows:

- 40 semiannual coupon payments of \$45 and
- \$1,000 40 six-month periods from now.

The semiannual or periodic interest rate is 6%.

The present value of the 40 semiannual coupon payments of \$45 discounted at 6% is \$677.08, as shown below:

IV) Exercice

$$c = \$45$$

$$n = 40$$

$$i = 0.06$$

$$\$45\left\{\frac{1-\left[\frac{1}{(1.06)^{40}}\right]}{0.06}\right\}$$

$$= \$45\left[\frac{1-\left(\frac{1}{10.28572}\right)}{0.06}\right]$$

$$= \$45\left(\frac{1-0.097222}{0.06}\right)$$

$$= \$45(15.04630)$$

$$= \$677.08$$

IV) Exercice

- The present value of the par or maturity value *40 six-month periods* from now discounted at 6% is \$97.22, as shown below:

$$
\begin{aligned}
M &= \$1,000 \\
n &= 0.40 \\
i &= 0.06 \\
&\$1,000\left[\frac{1}{(1.06)^{40}}\right] \\
&= \$1,000\left(\frac{1}{10.28572}\right) \\
&= \$1,000(0.097222) \\
&= \$97.22
\end{aligned}
$$

IV) Exercice

- The price of the bond is then equal to the sum of the two present values:

$677.08 + $97.22 = $774.30

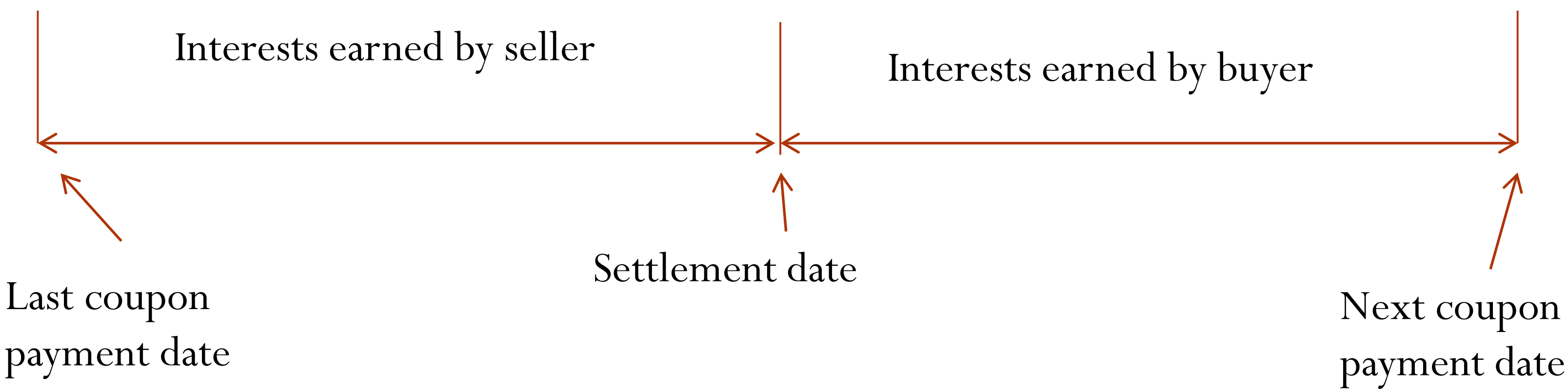
IV) Valuing a bond between a coupon payment- Remember

- *When the coupon rate equals the required yield, the price equals the par value.*
- *When the price equals the par value, the coupon rate equals the required yield.*

- *When the coupon rate is less than the required yield, the price is less than the par value.*
- *When the price is less than the par value, the coupon rate is less than the required yield.*

- *When the coupon rate is greater than the required yield, the price is greater than the par value.*
- *When the price is greater than the par value, the coupon rate is greater than the required yield.*

V) Dirty / Clean price



Dirty price = Full price

Clean Price = Full price – Accrued interest (=interest earned by buyer)

V) Computing the full price

i. Determine the number of days in the coupon period.

For a corporate bond, a municipal bond, and an agency security, the number of days in the coupon period will be 180 because a year is assumed to have 360 days.

For a coupon-bearing Treasury security, the number of days is the actual number of days. The number of days in the coupon period is called the *basis*.

Actual / Actual from July 17 to Sept 1	30/360 from July 17 to Sept 1
July 17 to July 31 : 14 Days	Remainder July : 13 Days
Aug : 31 Days	Aug : 30 Days
Sept 1: 1 Day	Sept 1: 1 Day
Total : 46 Days	Total : 44 days

ii. Compute the following ratio:

$$w = \frac{\text{number of days between settlement and next coupon payment}}{\text{number of days in the coupon period}}$$

V) Computing the full price

iii. For a bond with n coupon payments remaining to maturity, the price is

$$p = \frac{c}{(1+i)^{w}} + \frac{c}{(1+i)^{1+w}} + \frac{c}{(1+i)^{2+w}} + \cdots + \frac{c}{(1+i)^{n-1+w}} + \frac{M}{(1+i)^{n-1+w}}$$

- where
- p = price ($)
- c = semiannual coupon payment ($)
- M = maturity value
- n = number of coupon payments remaining
- i = periodic interest rate (required yield divided by 2) (in decimal)

- The period (exponent) in the formula for determining the present value can be expressed generally as $t - 1 + w$.

For example, for the first cash flow, the period is $1 - 1 + w$, or simply w.

For the second cash flow, it is $2 - 1 + w$, or simply $1 + w$.

If the bond has 20 coupon payments remaining, the last period is $20 - 1 + w$, or simply $19 + w$

V) Example

- A corporate bond with a coupon rate of 10% maturing March 1, 2012 is purchased with a settlement date of July 17, 2006. What would the price of this bond be if it is priced to yield 6.5%?

- Higher or Lower than 100?

V) Example

- The next coupon payment will be made on September 1, 2006. Because the bond is a corporate bond, based on a 30/360 day-count convention, there are 44 days between the settlement date and the next coupon date. The number of days in the coupon period is 180. Therefore, w = 44/180 = 0,24444

- The number of coupon payments remaining, *n*, is 12. The semiannual interest rate is 3.25% (6.5%/2).

- The full price is $120.0281

V) Example

Period	Cash Flow per $100 of Par	Present Value of $1 at 3.25%	Present Value of Cash Flow
0.24444	$ 5.000	$0.992212	$4.961060
1.24444	5.000	0.960980	4.804902
2.24444	5.000	0.930731	4.653658
3.24444	5.000	0.901435	4.507175
4.24444	5.000	0.873060	4.365303
5.24444	5.000	0.845579	4.227896
6.24444	5.000	0.818963	4.094815
7.24444	5.000	0.793184	3.965922
8.24444	5.000	0.768217	3.841087
9.24444	5.000	0.744036	3.720181
10.24444	5.000	0.720616	3.603081
11.24444	105.000	0.697933	73.283000
		Total	$120.028100

V) Example

- The clean price is is the full price of the bond minus the accrued interest.
- Accrued Interest

$$\text{AI} = c\left(\frac{\begin{array}{c}\text{number of days from last coupon}\\ \text{payment to settlement date}\end{array}}{\text{number of days in coupon period}}\right)$$

c = semiannual coupon payment ($)

- Because the number of days between settlement (July 17, 2006) and the next coupon payment (September 1, 2006) is 44 days and the number of days in the coupon period is 180, the number of days from the last coupon payment date (March 1, 2006) to the settlement date is 136 (180 − 44)

$$\text{AI} = \$5\left(\frac{136}{180}\right) = \$3.777778$$

VI) Credit Spreads and valuation of Non-Treasury Securities

- The Treasury spot rate can be used to value any default free security.
- The value of a non-Treasury security is found by discounting the cash flow by the Treasury spot rates plus a yield spread to reflect the additional risk.
- It is expected that credit spreads increase with the maturity of the bond

VI) Credit Rating

[illegible]	Moody's		S&P		Fitch	
	Long Term	Short Term	Long Term	Short Term	Long Term	Short Term
Investment Grade: Highest (Triple A)	Aaa		AAA		AAA	
	Aa1		AA+		AA+	
Investment Grade: Very high	Aa2	P-1	AA	A-1+	AA	F1+
	Aa3	(Prime-1)	AA-		AA-	
	A1		A+	A-1	A+	F1/F1+
Investment Grade: High	A2	P-2/P-1	A		A	F1
	A3	P-2/P-1	A-		A-	F2/F1
	Baa1	P-2 (Prime-2)	BBB+	A-2	BBB+	F2
Investment Grade: Good	Baa2	P-3/P-2	BBB		BBB	F3/F2
	Baa3	P-3 (Prime-3)	BBB-	A-3	BBB-	F3
	Ba1		BB+		BB+	
Speculative Grade:	Ba2		BB		BB	
Speculative	Ba3		BB-	B	BB-	B
	B1		B+		B+	
Speculative Grade:	B2		B		B	
Highly speculative	B3	Not Prime	B-		B-	
	Caa1		CCC+		CCC	
Speculative Grade:	Caa2		CCC			
Very high risks	Caa3		CCC-	C		C
Speculative Grade:	Ca		CC		CC	
Very near to default			C		C	
In default	C		SD/D	D	RD/D	RD/D

VI) Valuation models

- A valuation model provides the fair value of a security.
- This assumes that the security doesn't have an embedded option.
- If a security has an embedded option, the binomial and the Monte-Carlo simulation model will be used.
- The binomial model is used to value callable bonds, puttable bonds, floating rates notes and structured notes in which the coupon formula is based on an interest rate.
- The MC simulation model is used to value MBS and ABS.
- Both models will make assumption of the volatility of interest rates.

Questions?

- My Contacts:

david.saab@apertureinvestors.com

+33 7 87 06 94 26