Pricing Floating Rate Bonds

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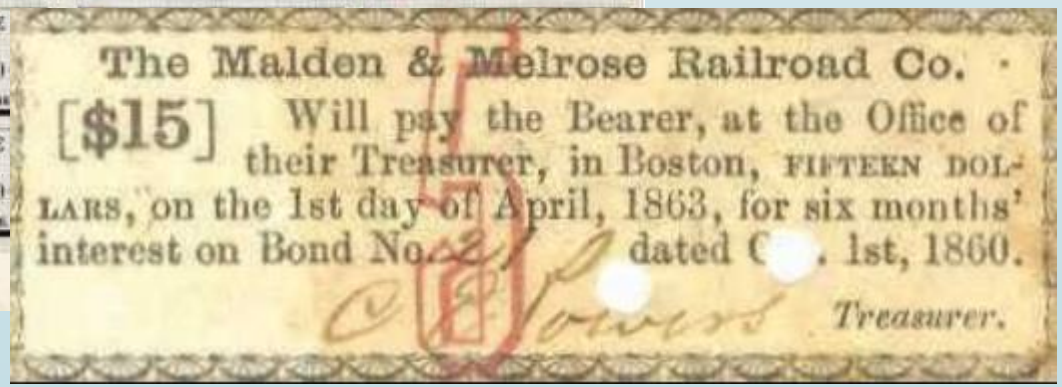
Learning Objectives

- What is a Floating Rate Bond
- Fixed vs. Floating
- How Bonds Compare
- Considering Credit Concerns
- Determining the floating rate
- Using Libor to determine floating payments
- Valuation
- Market Convexity Adjustments

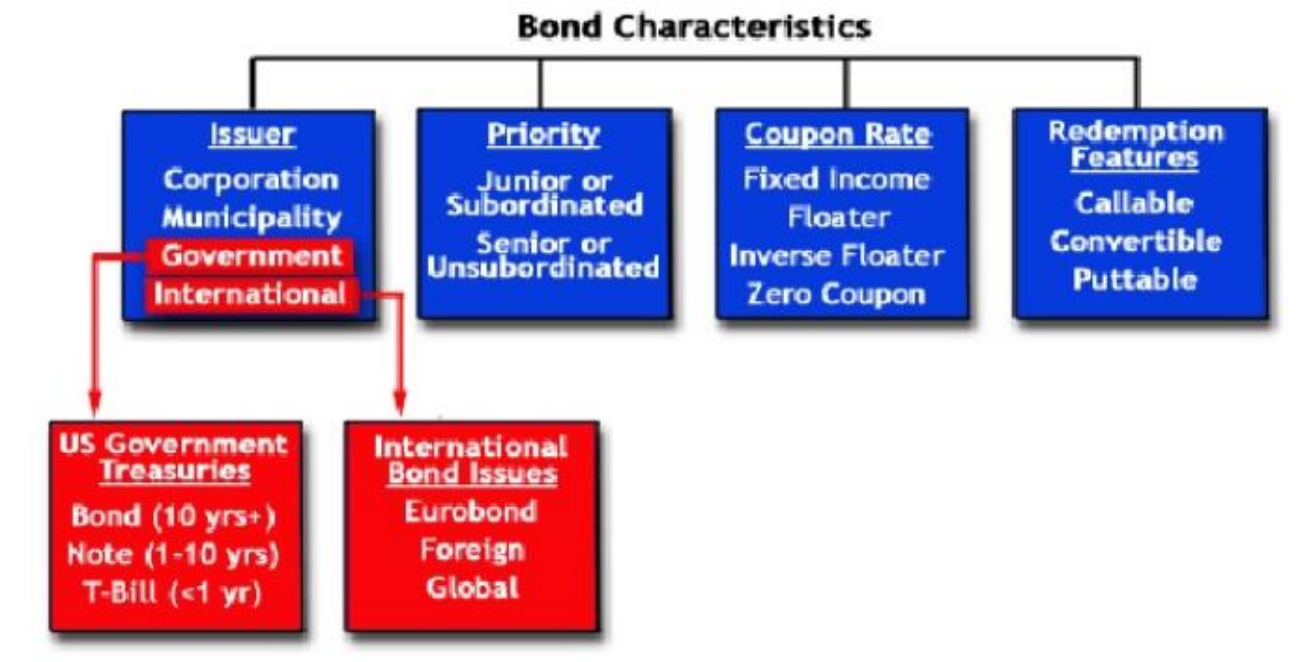
My Name is Bond…



My Name is Bond…

What is a Bond

- 'I owe you' (IOU), Debt
- Lend today for a promise of receiving in the future
- Principal constant, amortising, accreting, rollercoaster
- Coupons Variable / Floating / Adjustable / Reset (not Fixed / Locked)
- Term to Maturity can be Short, Medium, Long
- Issuers can be Corporates, Governments, Supranationals…



What is a Floating Rate Bond

Coupon 'Reset'– periodic payments (e.g. every 6M) based on some reference (e.g. LIBOR) (e.g. in advance; in arrears)

Maturity – date when principal promised to be repaid. A bond that matures in one year is more predictable (less risky?) than a bond maturing in 30 years. In general, the longer the bond, the higher the rate. All things being equal, longer term bonds fluctuate more than shorter term ones

Term to maturity – period of time until maturity

Par value (face value) – amount promised at maturity

Premium / discount – When a bond trades above face value it is said to be at a premium; If below face value, at a discount

	Coupon	Mat. date	Bid $	Yld%
Corporate				
AGT Lt	8.800	Sep 22/25	100.46	8.75
Air Ca	6.750	Feb 02/04	94.00	9.09
AssCap	5.400	Sep 04/01	100.01	5.38
Avco	5.750	Jun 02/03	100.25	5.63
Bell	6.250	Dec 01/03	101.59	5.63
Bell	6.500	May 09/05	102.01	5.95
BMO	7.000	Jan 28/10	106.55	6.04
BNS	5.400	Apr 01/03	100.31	5.24
BNS	6.250	Jul 16/07	101.56	5.95
CardTr	5.510	Jun 21/03	100.52	5.27
Cdn Pa	5.850	Mar 30/09	93.93	6.83
Clearn	0.000	May 15/08	88.50	8.61
CnCrTr	5.625	Mar 24/05	99.78	5.68
Coke	5.650	Mar 17/04	99.59	5.80

Premium – Par – Discount

Fixed vs. Floating - Exercise

How Bonds Compare – Yield and Return

- Bond prices quoted ‘dirty’ or ‘clean’
- ‘Bid’ and ‘offer’ prices
- Coupon Rate does not measure return per se
- An investor **may not pay ‘par’**, so a method of calculating the **true return** is needed
- Yield is the ‘measure of return’ or ‘rate of return’ on a bond
- ‘Required yield’ is the minimum income a bond issuer needs to offer in order to attract enough investors (e.g. bond ‘auction’)

How Bonds Compare – Yield and Return

Yields are **measures of the return** that can be earned on bonds

Flat yield

The most straightforward yield is to look at the coupon paid on a bond as a percentage of its market price – known as the **flat or running yield**

$$\text{Flat yield (\%)} = \frac{\text{Annual coupon (£)}}{\text{Bonds market price}} \text{ X } 100$$

(Price paid to purchase £100 nominal)

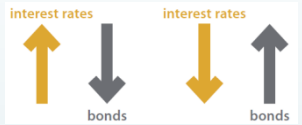
YTM / IRR

Yield to maturity is the '(internal)rate of return' an investor receives if he or she (re)invests all coupon payments at a constant rate until the bond matures. If the investor purchases the bond at a discount, its yield to maturity is higher than its coupon rate. Conversely, a bond purchased at a premium has a yield to maturity lower than its coupon rate.

How Bonds Compare – Risk

Inflation risk
If inflation rises, the 'real' value of the bond's coupon and redemption payment are eroded

Price risk/Market risk
Fluctuations in interest rates cause bond prices to change accordingly when they are traded



Liquidity Risk
The ease with which a security can be converted into cash. Some bonds are more easily sold at a fair market price than others

Risks

Default risk
There is a possibility that the issuer will not be able to pay the coupons or capital back

Exchange rate risk
Bonds denominated in a currency different from that of the investor's home currency are potentially subject to adverse exchange rate movements

Seniority Risk
Some bonds may rank behind more recently issued bonds in terms of being repaid if the issuer is unable to pay the bond back (company may go into liquidation)

Considering credit concerns

- To determine the credit-worthiness, one ought to consider:
  - Which Issuer
  - Credit Quality
  - Cash flows and payments structure
  - Legal structure
  - Country / Political

Credit – Which Issuer

- Issuer 'stability' is your main assurance of getting paid back. For example, the U.S. Government is more secure than a corporation. Their default risk -the chance that they do not pay back- is considered small, so small that U.S. government securities are known as 'risk free assets'
- A government is able to bring in future revenue through taxation
- A company must continue to make profits, which is far from guaranteed. This means corporations must offer a higher yield than governments in general in order to entice investors - this is the risk/return tradeoff in action

Credit Quality

Moody's	S&P/ Fitch	Grade	
Aaa	AAA	Investment	Highest Quality
Aa	AA	Investment	High Quality
A	A	Investment	Strong
Baa	BBB	Investment	Medium Grade
Ba,B	BB,B	Junk	Speculative
Caa,Ca,C	CCC,CC,C	Junk	Highly Spec
D	D	Junk	In Default

Credit Quality

Aaa	Bonds of best quality. Smallest degree of risk. Interest payments protected by a large or stable margin.
Aa	High quality. Margin of protection lower than Aaa.
A	Many favorable investment attributes. Adequate security of principal and interest. May be susceptible to impairment in the future.
Baa	Neither highly protected nor poorly secured. Adequate security for the present. Lacking outstanding investment characteristics. Speculative features.
Ba	Speculative elements. Future not well assured.
B	Lack characteristics of a desirable investment.
Caa	Poor standing. May be in default or danger with respect to principal or interest.
Ca	High degree of speculation. Often in default.
C	Lowest-rated class. Extremely poor chance of ever attaining any real investment standing.

Credit Quality

Rating agencies evaluate many aspects including:

1.Servicing history

2.Experience

3.Originations

4.Servicing capabilities

5.Human resources

6.Financial condition

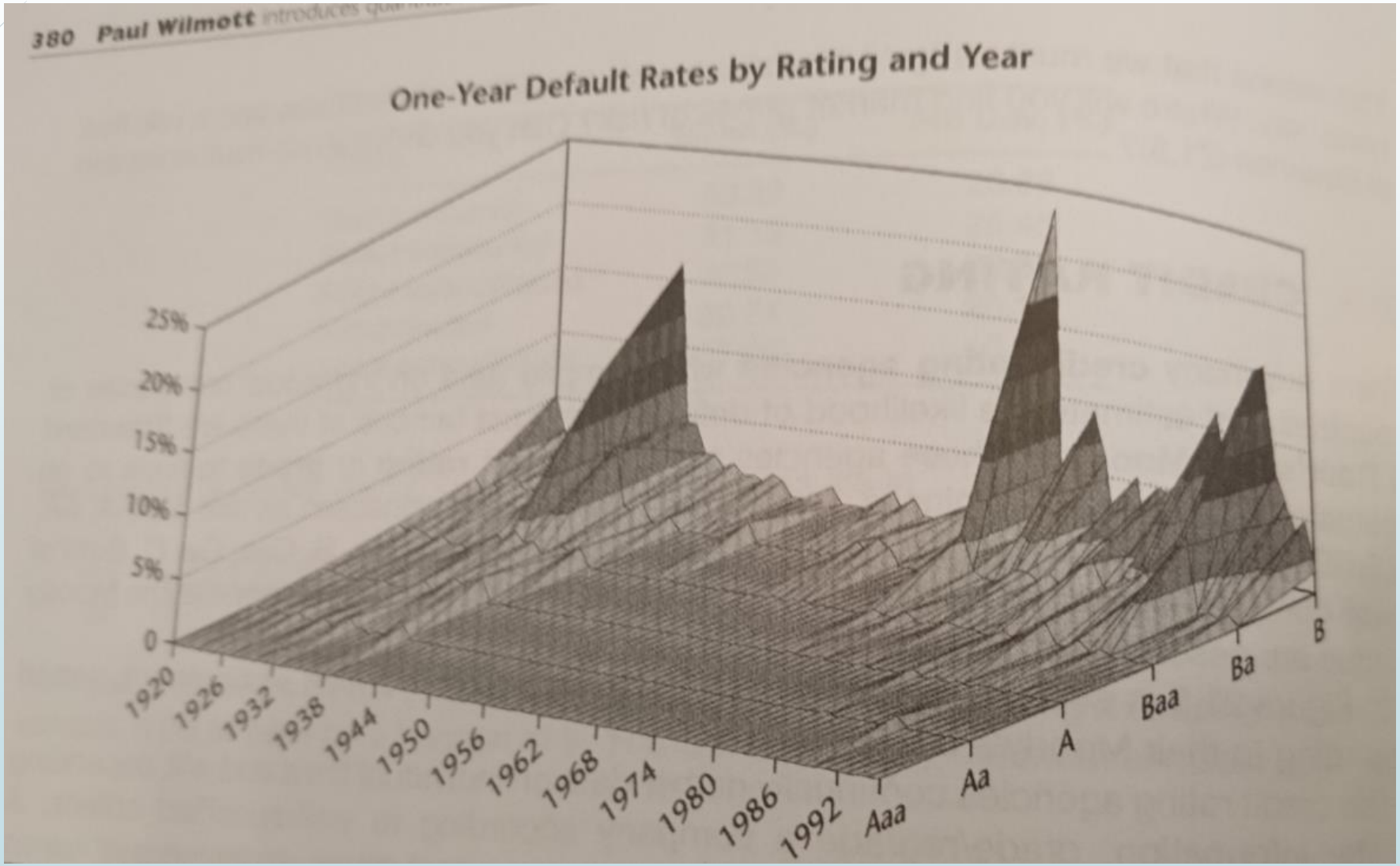
7.Growth/competition/business environment

8. Credit enhancement – provides greater protection against losses due to defaults:

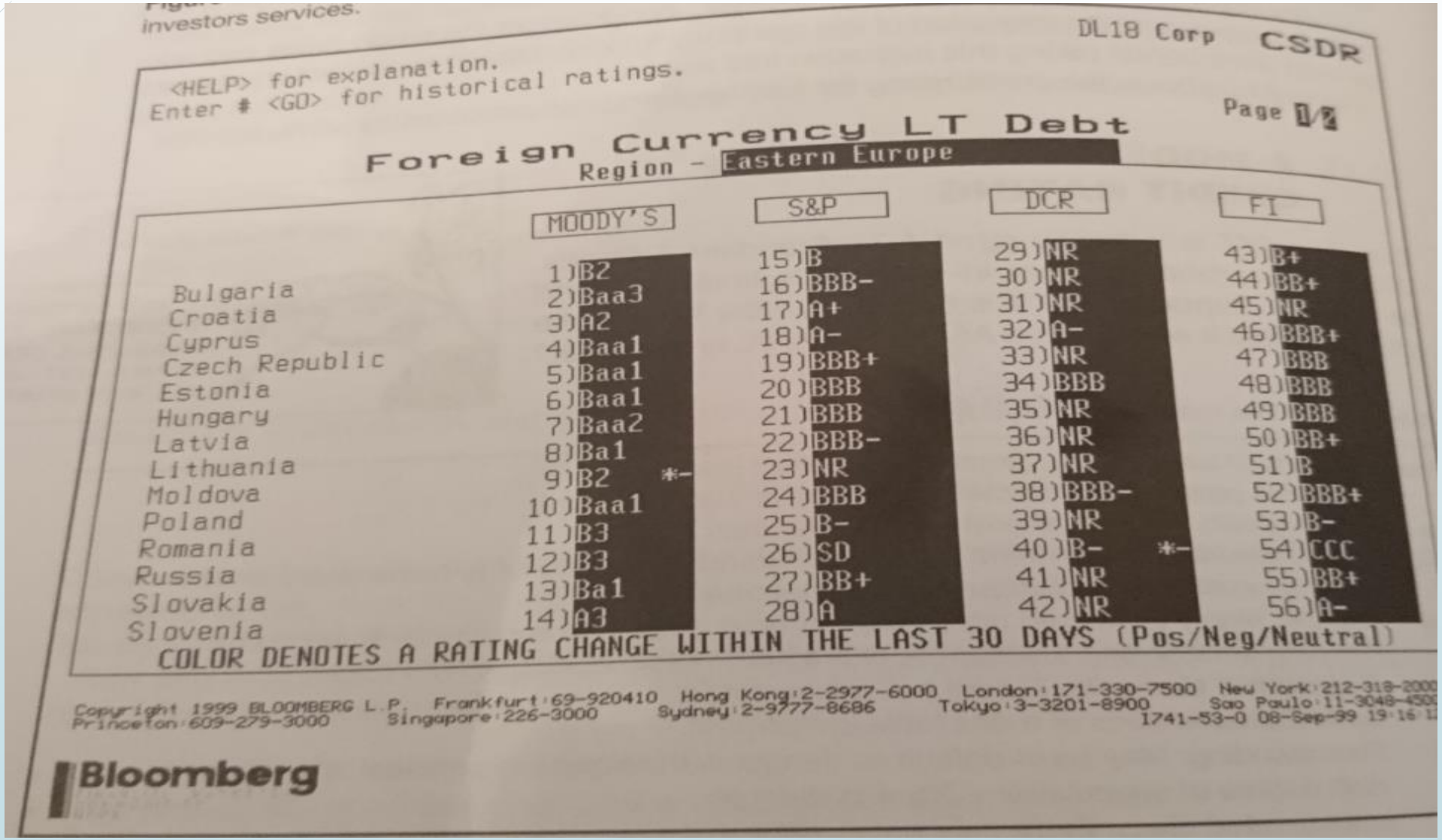
External – insurance, guarantees, letters of credit, cash collateral reserves

Internal – reserve funds, senior/subordinated debentures

Considering credit concerns – Default



Credit – Country / Political



Credit – Other

- Cash flows and payments structure
  - 1. Stable (Paper Industry), Seasonal (Farming Industry), Cyclical (Housing)
  - 2. Concentrated (a few large customers)
  - 3. Payment priorities: Amortization/Accreting; how excess cash flow is used, collateral structures and types
- Legal Provisions
  - 1. Subordination
  - 2. Call Provision
  - 3. Sinking Fund Provision
  - 4. Serial Bonds
  - 5. Restrictions on Future Issues
  - 6. Collateral
  - 7. Convertibility (Sweetener)

Determining the floating rate

Floating rate notes (**FRNs**) are bonds that have a variable coupon, equal to a money market reference rate, like LIBOR or federal funds rate, plus a **quoted spread** (also known as **quoted margin**). The spread is a rate that remains constant. Almost all FRNs have quarterly coupons, i.e. they pay out interest every three months. At the beginning of each coupon period, the coupon is calculated by taking the fixing of the reference rate for that day and adding the spread.[1][2][3] A typical coupon would look like 3 months USD LIBOR +0.20%.

FRNs issued trade in the secondary market. Reasons for price movements include:

1.Accrued interest - As a note gets closer to the interest payment date (ex-coupon) it builds up more accrued interest. When the interest is paid the price falls by the amount of the payment and will again start to accrue interest until the next payment date.

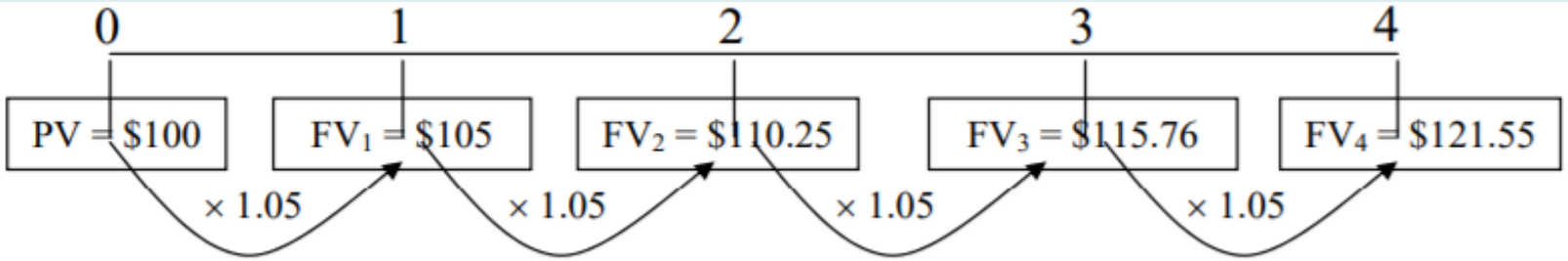
2.Short term rate movements - Due to the reset mechanism on the payment dates, FRNs will pay a fixed rate until the next coupon reset date. Therefore an investor is locked in at the current rate until it resets at the next reset date.

3.Traded margin - The most important factor that will cause fundamental changes to the price of an FRN is the movement in the traded margin or spread movement. That is the extent to which the traded margin diverges from the margin set at first issue which is reflection of perceived relative creditworthiness.

Valuation

- $FV = PV\,(1+r)^n$
- $PV = FV\,/\,(1+r)^n$
- Discount Factor (DF) = PV of \$1 = $1\,/\,(1+r)^n$
- $PV = FV * DF$

$$PV = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \ldots$$



$FV_4 = \$100 \times (1.05) \times (1.05) \times (1.05) \times (1.05)$

$= PV \times (1+i) \times (1+i) \times (1+i) \times (1+i)$

$= PV \times (1+i)^4$

In general, the future value of an initial lump sum is: $FV_n = PV \times (1+i)^n$

Valuation


Future value
Timeline
0
1
2
3
4
Duration
Maturity date


Last cashflow
Cashflows
Timeline
0
1
2
3
4
5
Duration
Maturity date

Valuation

$$P = \left(\frac{C}{1+i} + \frac{C}{(1+i)^2} + \cdots + \frac{C}{(1+i)^N}\right) + \frac{M}{(1+i)^N}$$
$$= \left(\sum_{n=1}^{N} \frac{C}{(1+i)^n}\right) + \frac{M}{(1+i)^N}$$
$$= C\left(\frac{1-(1+i)^{-N}}{i}\right) + M(1+i)^{-N}$$

$$S_n = P(1+r)^n$$

$$S_n = P\left(1+\frac{r}{m}\right)^{\mathrm{mt}}$$

$$S = P\,e^{r\,t}$$

Semi Annual

$$B_0 = \frac{C}{2}\left[\frac{1-\left[1+\frac{r}{2}\right]^{-2t}}{\frac{r}{2}}\right] + \frac{F}{\left[1+\frac{r}{2}\right]^{-2t}}$$

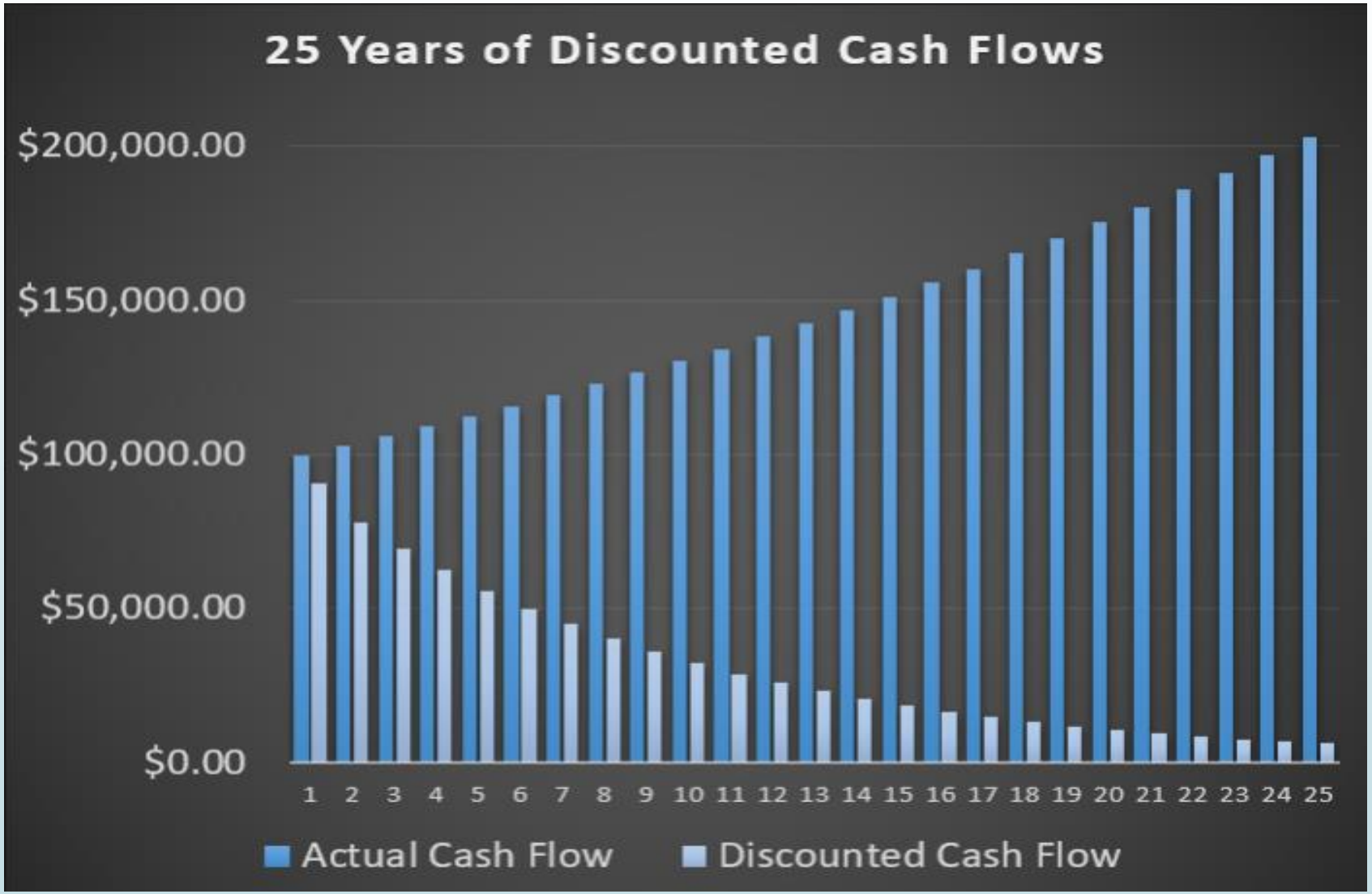
Annual

$$B_0 = C\left[\frac{1-\left[1+r\right]^{-t}}{r}\right] + \frac{F}{\left[1+r\right]^{t}}$$

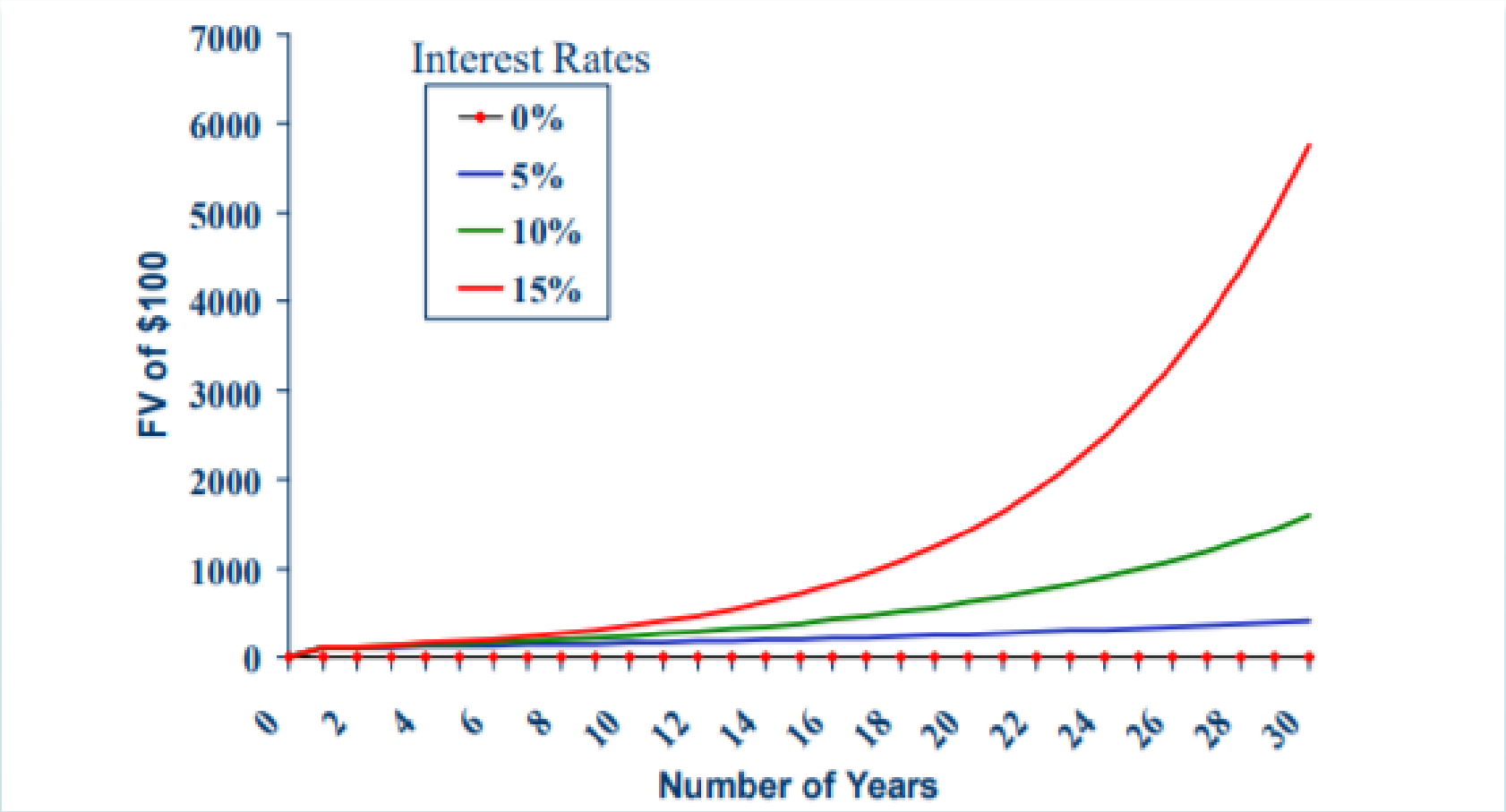
Where

Bo - Bond Price
C - Coupon Payment
r - Interest Rate
F - Face Value
t - Years/Periods

Valuation


25 Years of Discounted Cash Flows
$200,000.00
$150,000.00
$100,000.00
$50,000.00
$0.00
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
Actual Cash Flow
Discounted Cash Flow

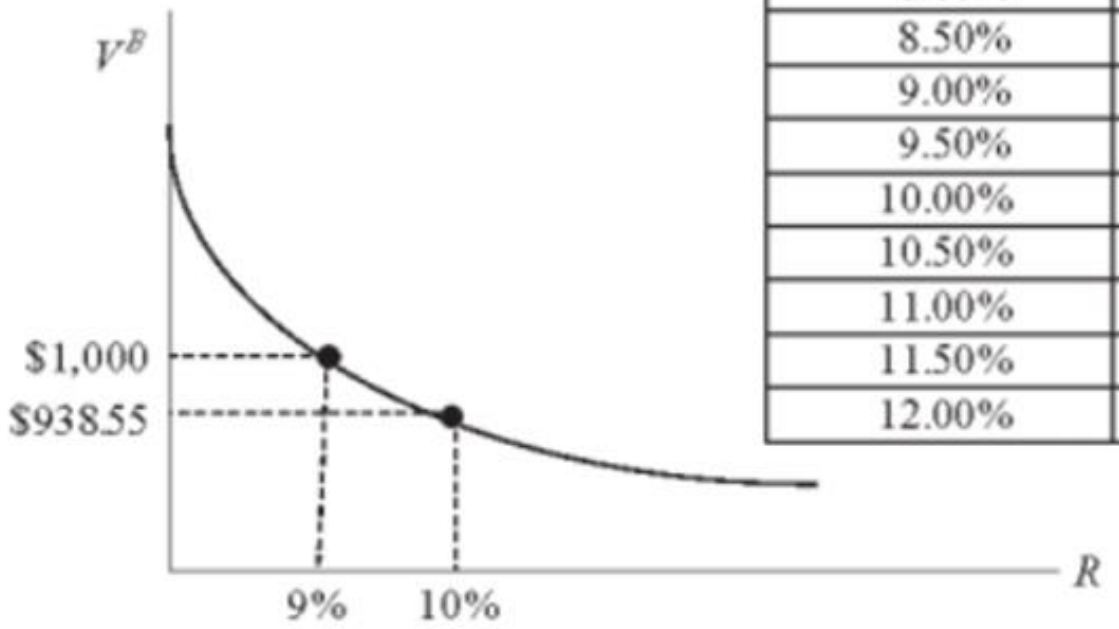
Valuation


Interest Rates
0%
5%
10%
15%
FV of $100
7000
6000
5000
4000
3000
2000
1000
0
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
Number of Years

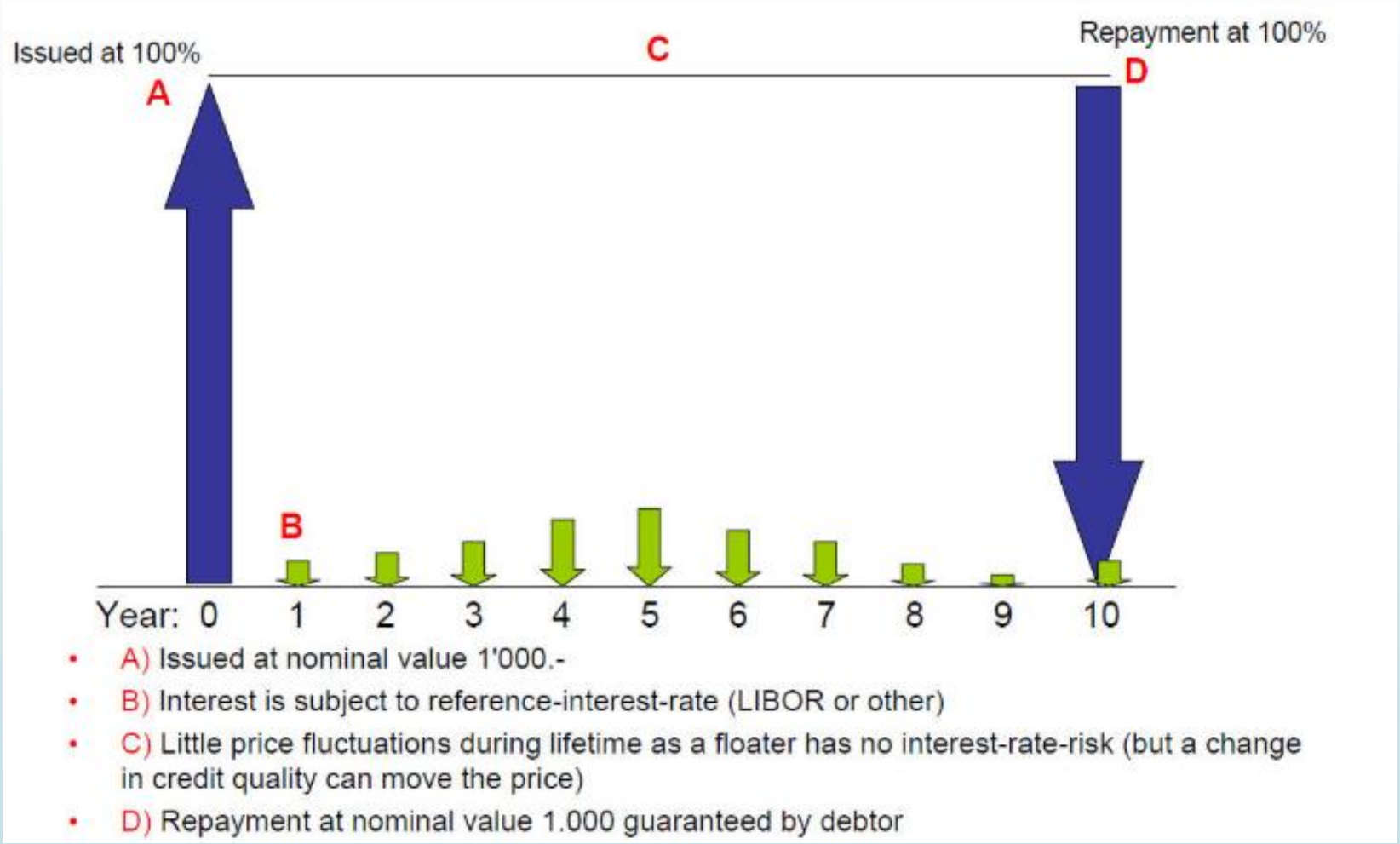
Valuation

Bond Relation : Price-yield curve depicts the inverse relation between V and R. The price-yield curve for the 10-year, 9% coupon bond:

Rate	Bond Value
7.00%	$1,140.47
7.50%	$1,102.96
8.00%	$1,067.10
8.50%	$1,032.81
9.00%	$1,000.00
9.50%	$968.61
10.00%	$938.55
10.50%	$909.78
11.00%	$882.22
11.50%	$855.81
12.00%	$830.49



Valuation



- A) Issued at nominal value 1'000.-
- B) Interest is subject to reference-interest-rate (LIBOR or other)
- C) Little price fluctuations during lifetime as a floater has no interest-rate-risk (but a change in credit quality can move the price)
- D) Repayment at nominal value 1.000 guaranteed by debtor

Valuation

Consider a floating rate note that pays according to 6 month LIBOR every 6 months for n years. LIBOR is determined at the beginning of each period and paid at the end of the period. Let $\ell[t_i, t_{i+1}]$ represent the Libor rate at date t_i for the time period $[t_i, t_{i+1}]$. The cash flow of $N\ell[t_i, t_{i+1}]\Delta t_i$ occurs at date t_{i+1}. We shall assume the face value N is \$100. To convert 6 month LIBOR into a semiannually compounded rate, we have

$$\frac{y}{2} = \ell[t_i, t_{i+1}] \times \frac{\text{Days in period}}{360}$$

We have seen that the price of a floating rate note at date 0, is given by:

$$V_{FLOAT} = 100P(0, t_0) = \frac{100}{(1 + y/2)^p}$$

where p is the fraction of a six month period remaining to the next reset date.

Valuation - Exercise

A bank buys a 3×6 FRA and read as "three by six", with a notional of \$100m. This quote convention identifies the point in time when the contract begins (t_0 is 3 months) and ends (t_1 is 6 months). Assume the agreed FRA rate is 4%. ($FRA_0[t_0, t_1] = 0.04$) The buyer has committed to pay 4% on a hypothetical deposit that starts in 3 months and ends in 6 months. Assume the exact deposit period is 92 days and payment is based on "actual/360" day basis. In this case $\Delta t = 92/360$.

Assume three months later, three month LIBOR is at 6%. ($\ell[t_0, t_1] = 0.06$.)

Market Convexity Adjustments

Similar to a FRA, an interest rate futures contract allows to manage the exposure to the simple spot rate $L(T_0, T_1)$ prevailing over a future period $[T_0, T_1]$ with length $\delta = T_1 - T_0$.

In contrast to FRAs, interest rate futures are daily marked to market (resettled).

Marking to market works as follows:

- At $t \leq T_0$: the futures price is quoted as

$$P_{futures}(t, T_0, T_1) = 100 \times (1 - R_{futures}(t, T_0, T_1))$$

  where $R_{futures}(t, T_0, T_1)$ is the futures rate prevailing at t

- At $t + \Delta t$: cash flow to holder of futures contract

$$\Delta P_{futures}(t + \Delta t) := P_{futures}(t + \Delta t, T_0, T_1) - P_{futures}(t, T_0, T_1)$$

Market Convexity Adjustments

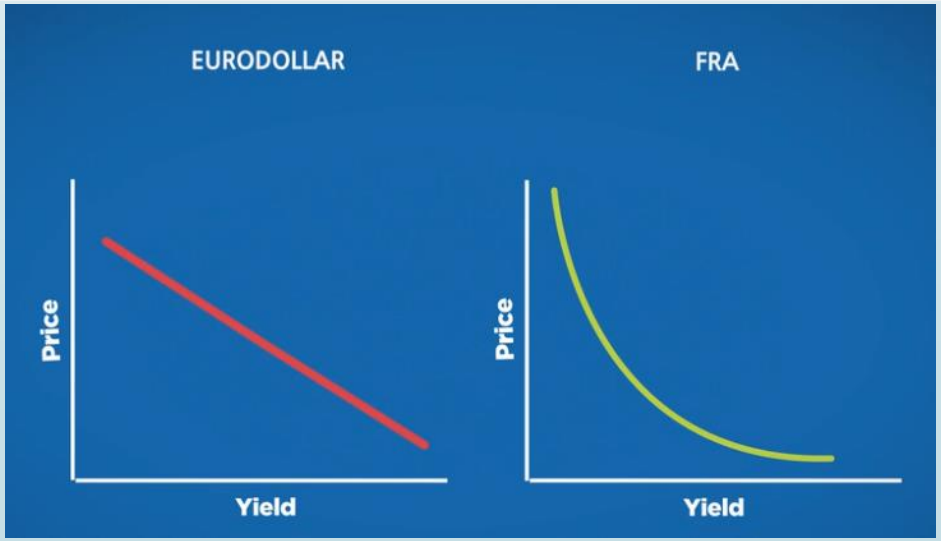
- What is a '(Market) Convexity Adjustment'
- A convexity adjustment is (in this context) the change required to be made to a forward interest rate or yield to get the expected future interest rate or yield. Convexity adjustment refers to the difference between the forward interest rate and the future interest rate; this difference has to be added to the former to arrive at the latter. The need for this adjustment arises because of the non-liner relationship between bond prices and yields.

$$\text{Forward rate} = \text{Futures Rate} - \tfrac{1}{2}\sigma^2 T_1 T_2$$

Convexity - Exercise

The table shows the convexity bias between a position of short 1000 Eurodollar (ED) futures and an offsetting short $1005m 3-month FRA (slightly more than $1000m to compensate for discounting methodology), both instigated at a rate of 2%.

Rates	1.70%	1.80%	1.90%	2.00%	2.10%	2.20%	2.30%
Short 1000 ED STIR payoff							
Short $1005m 3-month FRA payoff							
NET	560	249	62	0	62	249	559



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