Document Title

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Contents

1 Definitions

1.1 Basic probability theory notation

Vectors are indicated with bold lower-case letters and matrices are indicated with bold capital letters, i.e. $\mathbf{v}$ is a vector and $\mathbf{M}$ is a matrix.

A multivariate, normally-distributed, random variable $\mathbf{z}$ with mean μ and covariance matrix $\mathbf{\Sigma}$ is denoted as

$$\mathcal{N}(\mathbf{z}; \boldsymbol{\mu}, \mathbf{\Sigma}) = \frac{1}{\sqrt{(2\pi)^d |\mathbf{\Sigma}|}} e^{-(\mathbf{z}-\boldsymbol{\mu})^\intercal \mathbf{\Sigma}^{-1} (\mathbf{z}-\boldsymbol{\mu})/2} \tag{1}$$

1.2 Measurements and sequences

Set of received measurements at time k.

$$\mathbf{y}_k = \left\{ \mathbf{y}^k(i) : i = 1, 2, \ldots, m_k \right\} = \left\{ \mathbf{y}^k(1), \mathbf{y}^k(2), \ldots, \mathbf{y}^k(m_k) \right\} \tag{2}$$

m_k is the number of received measurements at time k and equals the cardinality of m_k:

$$m_k = |\mathbf{y}_k| \tag{3}$$

$$\mathbf{Y}^k = \mathbf{y}_k \bigcup \mathbf{Y}^{k-1} \tag{4}$$

$$\mathbf{Y}^0 = \emptyset \tag{5}$$

For each tracked object, the event that measurement $\mathbf{y}_k(i_k)$ corresponds to that object is $\theta_k(i_k)$.

The set of possible measurement sequences is $\{\xi_k(c_k) : c_k = 1, \ldots, C_k\}$, where each measurement sequence is a set of the form

$$\xi_k(c_k) = \{i_1, \ldots, i_k\}\,; \quad i_l = 0, \ldots, m_l; \quad l = 1, \ldots, k \tag{6}$$

and C_k is the total number of possible sequences:

$$C_k = \prod_{l=1}^{k} (1 + m_l) \tag{7}$$

$$p(\mathbf{x}_k|c_k) = p(\mathbf{x}_k|\xi_k(c_k), \chi_k, \mathbf{Y}^k) \tag{8}$$

$$p(c_k) = p(\xi_k(c_k)|\chi_k, \mathbf{Y}^k) \tag{9}$$

1.3 Clusters and joint events

Set of measurements selected by track τ at time k

$$\mathbf{y}_k^{\tau} = \left\{ \mathbf{y}^k(i) : \mathbf{y}^k(i) \in \mathcal{G}(\tau, k) \right\} \tag{10}$$

where $\mathcal{G}(\tau, k)$ is the gate of track τ at time k. Clusters

$$\{\mathcal{C}\}_k = \left\{ \left(\{\tau_i\}, \{\mathbf{y}_k^{\tau_i}\} \right) : \bigcap_{\tau_i} \mathbf{y}_k^{\tau_i} \neq \emptyset \right\} \tag{11}$$

Joint event

$$\varepsilon = \bigcap_{\tau=1}^{T} \theta_k^{\tau}(i(\tau, \varepsilon)) \tag{12}$$

where $\theta_k^{\tau}(i(\tau, \varepsilon))$ is the event that $\mathbf{y}^k(i(\tau, \varepsilon))$ is the measurement originating from track τ.

$$T_0(\varepsilon) = \{\tau : \varepsilon \text{ assigns no measurement to } \tau\} \tag{13}$$

$$T_1(\varepsilon) = \{\tau : \varepsilon \text{ assigns one measurement to } \tau\} \tag{14}$$

$$\Xi(\tau, i) = \{\varepsilon : \text{measurement } i \text{ is allocated to track } \tau\} \tag{15}$$

2 Pseudo functions

2.1 Measurement selection MS_1

$$\mathbf{y}_k(c_k, \sigma) = \left\{ \mathbf{y} \in \mathbf{Y}_k : (\mathbf{y} - \hat{\mathbf{y}}_{k|k-1}(c_{k-1}, \sigma))^{\mathsf{T}} \mathbf{S}_k^{-1}(c_{k-1}, \sigma)(\mathbf{y} - \hat{\mathbf{y}}_{k|k-1}(c_{k-1}, \sigma)) \leq g \right\} \tag{16}$$

with $\sqrt{g}$ the gate size. The volume of the gate is

$$V_k(c_{k-1}, \sigma) = \frac{\pi^{n/2}}{\Gamma(n/2 + 1)} \sqrt{g |\mathbf{S}_k(c_{k-1}, \sigma)|} \tag{17}$$

where n is the dimensionality of the measurement-state space.

2.2 Measurement likelihoods ML_1

$$[\{p_k(i, c_{k-1}, \sigma)\}_i] = \mathrm{ML}_1 \left[\{\mathbf{y}_k(i)\}_i, \hat{\mathbf{x}}_{k|k-1}(c_{k-1}, \sigma), \mathbf{P}_{k|k-1}(c_{k-1}, \sigma), \mathbf{H}, \mathbf{R} \right] \tag{18}$$

$$p_k(i, c_{k-1}, \sigma) = \begin{cases} \frac{1}{P_G} \mathcal{N}(\mathbf{y}_k(i); \hat{\mathbf{y}}_{k|k-1}(c_{k-1}, \sigma), \mathbf{S}_k(c_{k-1}, \sigma)), & \mathbf{y}_k(i) \in V_k(c_{k-1}, \sigma) \\ 0, & \mathbf{y}_k(i) \notin V_k(c_{k-1}, \sigma) \end{cases} \tag{19}$$

On track level

$$p_k(i) \equiv p(\mathbf{y}_k(i) | \chi_k, \mathbf{Y}^{k-1}) = \sum_{c_{k-1}=1}^{C_{k-1}} p(c_k - 1) p_k(i, c_{k-1}) \tag{20}$$

2.3 Multi-object data assocation JMTDA (single track component and definite object existence)

$$\left[\left\{\cdot, \{\beta_k^\tau(i)\}_{i\geq 0}\right\}_\tau\right] = \mathrm{JMTDA}\left[\left\{1, \{p_k^\tau(i)\}_{i>0}\right\}_\tau\right] \tag{21}$$

$$\begin{aligned}\beta_k^\tau(i) &= \sum_{\varepsilon\in\Xi(\tau,i)} p(\varepsilon|\mathbf{Y}^k) \\ &= \sum_{\varepsilon\in\Xi(\tau,i)} \frac{1}{c_k}\left[\prod_{\tau_\alpha\in T_0(\varepsilon)}\left(1-P_D^{\tau_\alpha}P_G^{\tau_\alpha}\right)\prod_{\tau_\alpha\in T_1(\varepsilon)}\left(P_G^{\tau_\alpha}P_D^{\tau_\alpha}\frac{p_k^{\tau_\alpha}(i(\tau_\alpha,\epsilon))}{\rho_k(i(\tau_\alpha,\varepsilon))}\right)\right]\end{aligned} \tag{22}$$

The normalization factor c_k is found by requiring

$$\sum_\varepsilon p(\varepsilon|\mathbf{Y}^k) = 1 \tag{23}$$

2.4 Probabilistic data association $\mathbf{PDA_E}$

$$\left[\hat{\mathbf{x}}_{k|k}, \mathbf{P}_{k|k}\right] = \mathrm{PDA_E}\left[\hat{\mathbf{x}}_{k|k-1}, \mathbf{P}_{k|k-1}, \{\mathbf{y}_k(i)\}_{i=1}^{m_k}, \{\beta_k(i)\}_{i=0}^{m_k}, \mathbf{H}, \mathbf{R}\right] \tag{24}$$

The inovations $\tilde{\mathbf{y}}_k(i)$ are defined by

$$\tilde{\mathbf{y}}_k(i) = \mathbf{y}_k(i) - \hat{\mathbf{y}}_{k|k-1} \tag{25}$$

For $i = 0$, we have the special case $\tilde{\mathbf{y}}_k(0) = 0$. The estimatated track mean is given by

$$\begin{aligned}\hat{\mathbf{x}}_{k|k} &= \sum_{i=0}^{d} \beta_k(i)\hat{\mathbf{x}}_{k|k}^i \\ &= \sum_{i=0}^{d} \beta_k(i)\left[\hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k\tilde{\mathbf{y}}_k(i)\right]\end{aligned} \tag{26}$$

$$\begin{aligned}\mathbf{P}_{k|k} &= \sum_{i=0}^{d} \beta_k(i)\left[\mathbf{P}_{k|k}^i + \left[\hat{\mathbf{x}}_{k|k}^i - \hat{\mathbf{x}}_{k|k}\right]\left[\hat{\mathbf{x}}_{k|k}^i - \hat{\mathbf{x}}_{k|k}\right]^\mathsf{T}\right] \\ &= \sum_{i=0}^{d} \beta_k(i)\left\{(\mathbf{I}-\mathbf{K}_k\mathbf{H})\mathbf{P}_{k|k-1}\right. \\ &\qquad \left. + \left[\hat{\mathbf{x}}_{k|k-1} - \hat{\mathbf{x}}_{k|k} + \mathbf{K}_k\tilde{\mathbf{y}}(i)\right]\left[\hat{\mathbf{x}}_{k|k-1} - \hat{\mathbf{x}}_{k|k} + \mathbf{K}_k\tilde{\mathbf{y}}(i)\right]^\mathsf{T}\right\}\end{aligned} \tag{27}$$

2.5 Clutter measurement density estimation $\mathbf{MTT_{MK}}$

not yet decided

3 Algorithms

3.1 Joint probablistic data association filter

3.2 Joint integrated probablistic data association filter

Algorithm 1 JPDAF

Time k inputs:

- set $\mathbf{Y}_k$
- compute $\hat{\mathbf{x}}^{\tau}_{k-1|k-1}$ and $\mathbf{P}^{\tau}_{k-1|k-1}$ for each track τ

for each τ **do**

  Track state prediction:

$$\left[\hat{\mathbf{x}}^{\tau}_{k|k-1}, \mathbf{P}^{\tau}_{k|k-1}\right] = \mathrm{KF_p}\left[\hat{\mathbf{x}}^{\tau}_{k-1|k-1}, \mathbf{P}^{\tau}_{k-1|k-1}, \mathbf{F}, \mathbf{Q}\right]$$

  Measurement selection (see 2.1):

$$[\mathbf{y}^{\tau}_k, V^{\tau}_k] = \mathrm{MS}_1\left[\mathbf{Y}_k, \hat{\mathbf{x}}^{\tau}_{k|k-1}, \mathbf{P}^{\tau}_{k|k-1}, \mathbf{H}, \mathbf{R}\right]$$

  Measurement likelyhood for each selected measurement i (see 2.2):

$$[\{p^{\tau}_k(i)\}_i] = \mathrm{ML}_1\left[\{\mathbf{y}_k(i)\}_i, \hat{\mathbf{x}}^{\tau}_{k|k-1}, \mathbf{P}^{\tau}_{k|k-1}, \mathbf{H}, \mathbf{R}\right]$$

end for

for each cluster **do**

$$\mathbf{y}_k = \bigcup_{\tau} \mathbf{y}^{\tau}_k$$

  if non-parametric tracking **then**

    Calculate cluster V_k

    Clutter measurement density estimation (see 2.5):

$$\rho = \mathrm{MTT_{MK}}\left(\{1\}_{\tau}, \{p^{\tau}_k(i)\}_{\tau,i}\right)/V_k$$

  end if

  Multi-object data association (see 2.3):

$$\left[\left\{\cdot, \{\beta^{\tau}_k(i)\}_{i\geq 0}\right\}_{\tau}\right] = \mathrm{JMTDA}\left[\left\{1, \{p^{\tau}_k(i)\}_{i>0}\right\}_{\tau}\right]$$

end for

for each track τ **do**

  Estimation/merging (see 2.4):

$$\left[\hat{\mathbf{x}}^{\tau}_{k|k}, \mathbf{P}^{\tau}_{k|k}\right] = \mathrm{PDA_E}\left[\hat{\mathbf{x}}^{\tau}_{k|k-1}, \mathbf{P}^{\tau}_{k|k-1}, \{\mathbf{y}_k(i)\}_{i=1}^{m_k}, \{\beta_k(i)\}_{i=0}^{m_k}, \mathbf{H}, \mathbf{R}\right]$$

  Output trajectory

end for

Algorithm 2 JIPDAF

Time k inputs:

- set $\mathbf{Y}_k$
- object existance probability $p(\chi^\tau_{k-1|k-1})$ for each track τ
- compute $\hat{\mathbf{x}}^\tau_{k-1|k-1}$ and $\mathbf{P}^\tau_{k-1|k-1}$ for each track τ

for each τ **do**

  Track state prediction:

$$p(\chi^\tau_k|\bar{\mathbf{Y}}^{k-1}) = \gamma_{11} p(\chi^\tau_{\bar{\mathbf{Y}}^{k-1}})$$

$$\left[\hat{\mathbf{x}}^\tau_{k|k-1}, \mathbf{P}^\tau_{k|k-1}\right] = \mathrm{KF_p}\left[\hat{\mathbf{x}}^\tau_{k-1|k-1}, \mathbf{P}^\tau_{k-1|k-1}, \mathbf{F}, \mathbf{Q}\right]$$

  Measurement selection (see 2.1):

$$[\mathbf{y}^\tau_k, V^\tau_k] = \mathrm{MS}_1\left[\mathbf{Y}_k, \hat{\mathbf{x}}^\tau_{k|k-1}, \mathbf{P}^\tau_{k|k-1}, \mathbf{H}, \mathbf{R}\right]$$

  Measurement likelyhood for each selected measurement i (see 2.2):

$$[\{p^\tau_k(i)\}_i] = \mathrm{ML}_1\left[\{\mathbf{y}_k(i)\}_i, \hat{\mathbf{x}}^\tau_{k|k-1}, \mathbf{P}^\tau_{k|k-1}, \mathbf{H}, \mathbf{R}\right]$$

end for

for each cluster **do**

$$\mathbf{y}_k = \bigcup_\tau \mathbf{y}^\tau_k$$

  if non-parametric tracking **then**

    Calculate cluster V_k

    Clutter measurement density estimation (see 2.5):

$$\rho = \mathrm{MTT_{MK}}\left(\{1\}_\tau, \{p^\tau_k(i)\}_{\tau,i}\right)/V_k$$

  end if

  Multi-object data association (see 2.3):

$$\left[\left\{p(\chi^\tau_k|\bar{\mathbf{Y}}), \{\beta^\tau_k(i)\}_{i\geq 0}\right\}_\tau\right] = \mathrm{JMTDA}\left[\left\{p(\chi^\tau_k|\bar{\mathbf{Y}}^{k-1}), \{p^\tau_k(i)\}_{i>0}\right\}_\tau\right]$$

end for

for each track τ **do**

  Estimation/merging (see 2.4):

$$\left[\hat{\mathbf{x}}^\tau_{k|k}, \mathbf{P}^\tau_{k|k}\right] = \mathrm{PDA_E}\left[\hat{\mathbf{x}}^\tau_{k|k-1}, \mathbf{P}^\tau_{k|k-1}, \{\mathbf{y}_k(i)\}_{i=1}^{m_k}, \{\beta_k(i)\}_{i=0}^{m_k}, \mathbf{H}, \mathbf{R}\right]$$

  Output trajectory

end for