

Document Title

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1 Definitions

1.1 Basic probability theory notation

Vectors are indicated with bold lower-case letters and matrices are indicated with bold capital letters, i.e. \mathbf{v} is a vector and \mathbf{M} is a matrix.

A multivariate, normally-distributed, random variable \mathbf{z} with mean μ and covariance matrix Σ is denoted as

$$\mathcal{N}(\mathbf{z}; \mu, \Sigma) = \frac{1}{\sqrt{|2\pi\Sigma|}} e^{-(\mathbf{z}-\mu)^\top \Sigma (\mathbf{z}-\mu)/2} \quad (1)$$

1.2 Measurements and sequences

Set of received measurements at time k .

$$\mathbf{y}_k = \left\{ \mathbf{y}^k(i) : i = 1, 2, \dots, m_k \right\} = \left\{ \mathbf{y}^k(1), \mathbf{y}^k(2), \dots, \mathbf{y}^k(m_k) \right\} \quad (2)$$

m_k is the number of received measurements at time k and equals the cardinality of m_k :

$$m_k = |\mathbf{y}_k| \quad (3)$$

$$\mathbf{Y}^k = \mathbf{y}_k \bigcup \mathbf{Y}^{k-1} \quad (4)$$

$$\mathbf{Y}^0 = \emptyset \quad (5)$$

For each tracked object, the event that measurement $\mathbf{y}_k(i_k)$ corresponds to that object is $\theta_k(i_k)$.

The set of possible measurement sequences is $\{\xi_k(c_k) : c_k = 1, \dots, C_k\}$, where each measurement sequence is a set of the form

$$\xi_k(c_k) = \{i_1, \dots, i_k\}; \quad i_l = 0, \dots, m_l; \quad l = 1, \dots, k \quad (6)$$

and C_k is the total number of possible sequences:

$$C_k = \prod_{l=1}^k (1 + m_l) \quad (7)$$

$$p(\mathbf{x}_k | c_k) = p(\mathbf{x}_k | \xi_k(c_k), \chi_k, \mathbf{Y}^k) \quad (8)$$

$$p(c_k) = p(\xi_k(c_k) | \chi_k, \mathbf{Y}^k) \quad (9)$$

1.3 Clusters and joint events

Set of measurements selected by track τ at time k

$$\mathbf{y}_k^\tau = \left\{ \mathbf{y}^k(i) : \mathbf{y}^k(i) \in \mathcal{G}(\tau, k) \right\} \quad (10)$$

where $\mathcal{G}(\tau, k)$ is the gate of track τ at time k . Clusters

$$\{\mathcal{C}\}_k = \left\{ (\{\tau_i\}, \{\mathbf{y}_k^{\tau_i}\}) : \bigcap_{\tau_i} \mathbf{y}_k^{\tau_i} \neq \emptyset \right\} \quad (11)$$

Joint event

$$\varepsilon = \bigcap_{\tau=1}^T \theta_k^\tau(i(\tau, \varepsilon)) \quad (12)$$

where $\theta_k^\tau(i(\tau, \varepsilon))$ is the event that $\mathbf{y}^k(i(\tau, \varepsilon))$ is the measurement originating from track τ .

$$T_0(\varepsilon) = \{\tau : \varepsilon \text{ assigns no measurement to } \tau\} \quad (13)$$

$$T_1(\varepsilon) = \{\tau : \varepsilon \text{ assigns one measurement to } \tau\} \quad (14)$$

$$\Xi(\tau, i) = \{\varepsilon : \text{measurement } i \text{ is allocated to track } \tau\} \quad (15)$$

2 Pseudo functions

2.1 Measurement selection MS₁

$$\mathbf{y}_k(c_k, \sigma) = \left\{ \mathbf{y} \in \mathbf{Y}_k : (\mathbf{y} - \hat{\mathbf{y}}_{k|k-1}(c_{k-1}, \sigma))^T \mathbf{S}_k^{-1}(c_{k-1}, \sigma) (\mathbf{y} - \hat{\mathbf{y}}_{k|k-1}(c_{k-1}, \sigma)) \leq g \right\} \quad (16)$$

with \sqrt{g} the gate size. The volume of the gate is

$$V_k(c_{k-1}, \sigma) = \frac{\pi^{n/2}}{\Gamma(n/2 + 1)} \sqrt{g |\mathbf{S}_k(c_{k-1}, \sigma)|} \quad (17)$$

where n is the dimensionality of the measurement-state space.

2.2 Measurement likelihoods ML₁

$$[\{p_k(i, c_{k-1}, \sigma)\}_i] = \text{ML}_1[\{\mathbf{y}_k(i)\}_i, \hat{\mathbf{x}}_{k|k-1}(c_{k-1}, \sigma), \mathbf{P}_{k|k-1}(c_{k-1}, \sigma), \mathbf{H}, \mathbf{R}] \quad (18)$$

$$p_k(i, c_{k-1}, \sigma) = \begin{cases} \frac{1}{P_G} \mathcal{N}(\mathbf{y}_k(i); \hat{\mathbf{y}}_{k|k-1}(c_{k-1}, \sigma), \mathbf{S}_k(c_{k-1}, \sigma)), & \mathbf{y}_k(i) \in V_k(c_{k-1}, \sigma) \\ 0, & \mathbf{y}_k(i) \notin V_k(c_{k-1}, \sigma) \end{cases} \quad (19)$$

On track level

$$p_k(i) \equiv p(\mathbf{y}_k(i) | \chi_k, \mathbf{Y}^{k-1}) = \sum_{c_{k-1}=1}^{C_{k-1}} p(c_{k-1}) p_k(i, c_{k-1}) \quad (20)$$

2.3 Multi-object data association JMTDA (single track component and definite object existence)

$$\left[\left\{ \cdot, \{\beta_k^\tau(i)\}_{i \geq 0} \right\}_\tau \right] = \text{JMTDA}[\{1, \{p_k^\tau(i)\}_{i > 0}\}_\tau] \quad (21)$$

$$\begin{aligned} \beta_k^\tau(i) &= \sum_{\varepsilon \in \Xi(\tau, i)} p(\varepsilon | \mathbf{Y}^k) \\ &= \sum_{\varepsilon \in \Xi(\tau, i)} \frac{1}{c_k} \left[\prod_{\tau_\alpha \in T_0(\varepsilon)} (1 - P_D^{\tau_\alpha} P_G^{\tau_\alpha}) \prod_{\tau_\alpha \in T_1(\varepsilon)} \left(P_G^{\tau_\alpha} P_D^{\tau_\alpha} \frac{p_k^{\tau_\alpha}(i(\tau_\alpha, \varepsilon))}{\rho_k(i(\tau_\alpha, \varepsilon))} \right) \right] \end{aligned} \quad (22)$$

The normalization factor c_k is found by requiring

$$\sum_{\varepsilon} p(\varepsilon | \mathbf{Y}^k) = 1 \quad (23)$$

2.4 Probabilistic data association PDA_E

$$[\hat{\mathbf{x}}_{k|k}, \mathbf{P}_{k|k}] = \text{PDA}_E[\hat{\mathbf{x}}_{k|k-1}, \mathbf{P}_{k|k-1}, \{\mathbf{y}_k(i)\}_{i=1}^{m_k}, \{\beta_k(i)\}_{i=0}^{m_k}, \mathbf{H}, \mathbf{R}] \quad (24)$$

The inovations $\tilde{\mathbf{y}}_k(i)$ are defined by

$$\tilde{\mathbf{y}}_k(i) = \mathbf{y}_k(i) - \hat{\mathbf{y}}_{k|k-1} \quad (25)$$

For $i = 0$, we have the special case $\tilde{\mathbf{y}}_k(0) = 0$. The estimated track mean is given by

$$\begin{aligned} \hat{\mathbf{x}}_{k|k} &= \sum_{i=0}^d \beta_k(i) \hat{\mathbf{x}}_{k|k}^i \\ &= \sum_{i=0}^d \beta_k(i) [\hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \tilde{\mathbf{y}}_k(i)] \end{aligned} \quad (26)$$

$$\begin{aligned} \mathbf{P}_{k|k} &= \sum_{i=0}^d \beta_k(i) \left[\mathbf{P}_{k|k}^i + [\hat{\mathbf{x}}_{k|k}^i - \hat{\mathbf{x}}_{k|k}] [\hat{\mathbf{x}}_{k|k}^i - \hat{\mathbf{x}}_{k|k}]^\top \right] \\ &= \sum_{i=0}^d \beta_k(i) \left\{ (\mathbf{I} - \mathbf{K}_k \mathbf{H}) \mathbf{P}_{k|k-1} \right. \\ &\quad \left. + [\hat{\mathbf{x}}_{k|k-1} - \hat{\mathbf{x}}_{k|k} + \mathbf{K}_k \tilde{\mathbf{y}}(i)] [\hat{\mathbf{x}}_{k|k-1} - \hat{\mathbf{x}}_{k|k} + \mathbf{K}_k \tilde{\mathbf{y}}(i)]^\top \right\} \end{aligned} \quad (27)$$

2.5 Clutter measurement density estimation MTT_{MK}

not yet decided

3 Algorithms

3.1 Joint probabilistic data association filter

3.2 Joint integrated probabilistic data association filter

Algorithm 1 JPDAF

1: Time k inputs:

- set \mathbf{Y}_k
- compute $\hat{\mathbf{x}}_{k-1|k-1}^\tau$ and $\mathbf{P}_{k-1|k-1}^\tau$ for each track τ

2: **for** each τ **do**

3: Track state prediction:

$$\left[\hat{\mathbf{x}}_{k|k-1}^\tau, \mathbf{P}_{k|k-1}^\tau \right] = \text{KF}_p \left[\hat{\mathbf{x}}_{k-1|k-1}^\tau, \mathbf{P}_{k-1|k-1}^\tau, \mathbf{F}, \mathbf{Q} \right]$$

4: Measurement selection (see 2.1):

$$[\mathbf{y}_k^\tau, V_k^\tau] = \text{MS}_1 \left[\mathbf{Y}_k, \hat{\mathbf{x}}_{k|k-1}^\tau, \mathbf{P}_{k|k-1}^\tau, \mathbf{H}, \mathbf{R} \right]$$

5: Measurement likelihood for each selected measurement i (see 2.2):

$$[\{p_k^\tau(i)\}_i] = \text{ML}_1 \left[\{\mathbf{y}_k(i)\}_i, \hat{\mathbf{x}}_{k|k-1}^\tau, \mathbf{P}_{k|k-1}^\tau, \mathbf{H}, \mathbf{R} \right]$$

6: **end for**

7: **for** each cluster **do**

$$\mathbf{y}_k = \bigcup_{\tau} \mathbf{y}_k^\tau$$

8: **if** non-parametric tracking **then**

9: Calculate cluster V_k

10: Clutter measurement density estimation (see 2.5):

$$\rho = \text{MTT}_{\text{MK}} \left(\{1\}_\tau, \{p_k^\tau(i)\}_{\tau,i} \right) / V_k$$

11: **end if**

12: Multi-object data association (see 2.3):

$$\left[\left\{ \cdot, \{\beta_k^\tau(i)\}_{i \geq 0} \right\}_\tau \right] = \text{JMTDA} \left[\{1, \{p_k^\tau(i)\}_{i > 0}\}_\tau \right]$$

13: **end for**

14: **for** each track τ **do**

15: Estimation/merging (see 2.4):

$$\left[\hat{\mathbf{x}}_{k|k}^\tau, \mathbf{P}_{k|k}^\tau \right] = \text{PDA}_E \left[\hat{\mathbf{x}}_{k|k-1}^\tau, \mathbf{P}_{k|k-1}^\tau, \{\mathbf{y}_k(i)\}_{i=1}^{m_k}, \{\beta_k(i)\}_{i=0}^{m_k}, \mathbf{H}, \mathbf{R} \right]$$

16: Output trajectory

17: **end for**

Algorithm 2 JIPDAF

1: Time k inputs:

- set \mathbf{Y}_k
- object existance probability $p(\chi_{k-1|k-1}^\tau)$ for each track τ
- compute $\hat{\mathbf{x}}_{k-1|k-1}^\tau$ and $\mathbf{P}_{k-1|k-1}^\tau$ for each track τ

2: **for** each τ **do**

3: Track state prediction:

$$p(\chi_k^\tau | \bar{\mathbf{Y}}^{k-1}) = \gamma_{11} p(\chi_{\bar{\mathbf{Y}}^{k-1}}^\tau)$$

$$\left[\hat{\mathbf{x}}_{k|k-1}^\tau, \mathbf{P}_{k|k-1}^\tau \right] = \text{KF}_p \left[\hat{\mathbf{x}}_{k-1|k-1}^\tau, \mathbf{P}_{k-1|k-1}^\tau, \mathbf{F}, \mathbf{Q} \right]$$

4: Measurement selection (see 2.1):

$$[\mathbf{y}_k^\tau, V_k^\tau] = \text{MS}_1 \left[\mathbf{Y}_k, \hat{\mathbf{x}}_{k|k-1}^\tau, \mathbf{P}_{k|k-1}^\tau, \mathbf{H}, \mathbf{R} \right]$$

5: Measurement likelihood for each selected measurement i (see 2.2):

$$\{p_k^\tau(i)\}_i = \text{ML}_1 \left[\{\mathbf{y}_k(i)\}_i, \hat{\mathbf{x}}_{k|k-1}^\tau, \mathbf{P}_{k|k-1}^\tau, \mathbf{H}, \mathbf{R} \right]$$

6: **end for**

7: **for** each cluster **do**

$$\mathbf{y}_k = \bigcup_{\tau} \mathbf{y}_k^\tau$$

8: **if** non-parametric tracking **then**

9: Calculate cluster V_k

10: Clutter measurement density estimation (see 2.5):

$$\rho = \text{MTT}_{\text{MK}} \left(\{1\}_\tau, \{p_k^\tau(i)\}_{\tau,i} \right) / V_k$$

11: **end if**

12: Multi-object data association (see 2.3):

$$\left[\left\{ p(\chi_k^\tau | \bar{\mathbf{Y}}), \{\beta_k^\tau(i)\}_{i \geq 0} \right\}_\tau \right] = \text{JMTDA} \left[\left\{ p(\chi_k^\tau | \bar{\mathbf{Y}}^{k-1}), \{p_k^\tau(i)\}_{i > 0} \right\}_\tau \right]$$

13: **end for**

14: **for** each track τ **do**

15: Estimation/merging (see 2.4):

$$\left[\hat{\mathbf{x}}_{k|k}^\tau, \mathbf{P}_{k|k}^\tau \right] = \text{PDA}_E \left[\hat{\mathbf{x}}_{k|k-1}^\tau, \mathbf{P}_{k|k-1}^\tau, \{\mathbf{y}_k(i)\}_{i=1}^{m_k}, \{\beta_k(i)\}_{i=0}^{m_k}, \mathbf{H}, \mathbf{R} \right]$$

16: Output trajectory

17: **end for**
