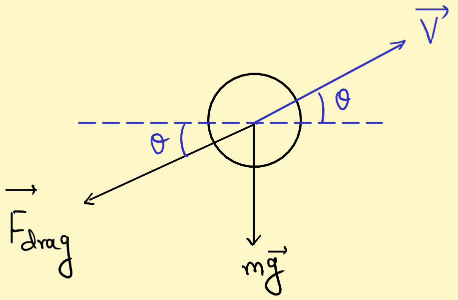
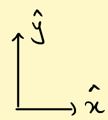
PH354 : Assignment #06

Problem #05:



◄ Free body diagram



From Newton's Second law,

$$m\vec{g} + \vec{F}_{drag} = m\vec{a}$$

$$\Rightarrow \quad \ddot{x} = \frac{(F_{drag})_x}{m} = -\frac{\frac{1}{2}\pi R^2 \rho C v^2}{m}\cos\theta \qquad \cos\theta = \frac{\dot{x}}{v}$$

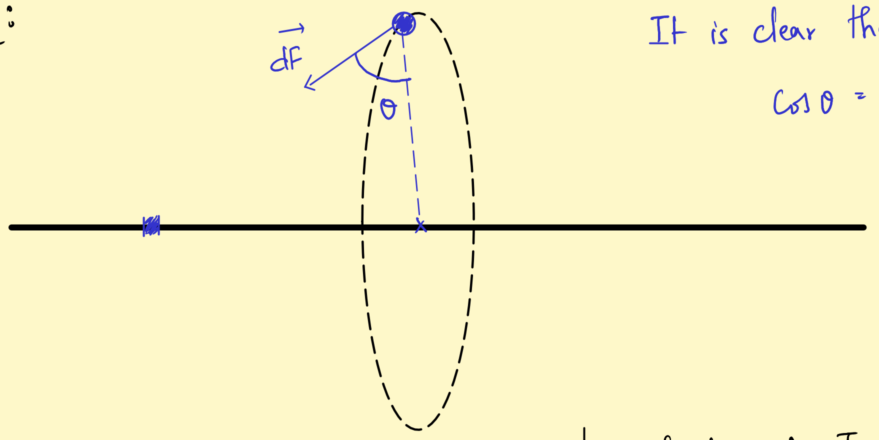
$$\Rightarrow \quad \boxed{\ddot{x} = -\frac{\pi R^2 \rho C}{2m}\,\dot{x}\,(\dot{x}^2+\dot{y}^2)^{1/2}}$$

Similarly,

$$\boxed{\ddot{y} = -g - \frac{\pi R^2 \rho C}{2m}\,\dot{y}\,(\dot{x}^2+\dot{y}^2)^{1/2}}$$

Problem #06:

(a)



It is clear that,

$$\cos\theta = \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}$$

By symmetry, the Force points toward the center of the rod. To calculate this, we have:

$$\vec{F}_{net} = \int d\vec{F} = \hat{r}\int -\frac{Gm}{(x^2+y^2+z^2)}\,dM\cdot\cos\theta$$

, where $\hat{r}$ is the unit vector pointing from the center of the rod to the ball.

$$= -\frac{GMm}{L}\,\hat{r}\int_{-L/2}^{L/2}\frac{dz}{(x^2+y^2+z^2)}\cdot\frac{(x^2+y^2)^{1/2}}{(x^2+y^2+z^2)^{1/2}}$$

$$\Rightarrow \quad \vec{F}_{net} = -\frac{GMm}{L}\,\hat{r}\int_{-L/2}^{L/2}\frac{(x^2+y^2)^{1/2}\,dz}{(x^2+y^2+z^2)^{3/2}}$$

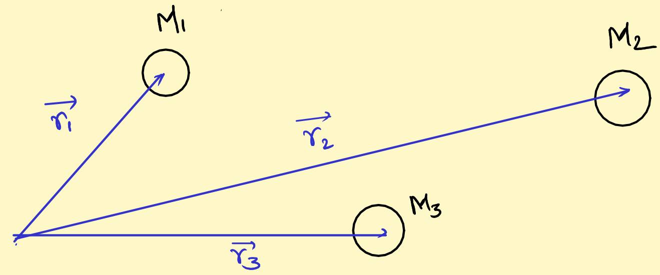
The closed form of this integral is:

$$\boxed{\vec{F}_{net} = -\frac{GMm}{(x^2+y^2)^{1/2}\,(x^2+y^2+L^2/4)^{1/2}}\,\hat{r}}$$

Writing $\hat{r} = \dfrac{x}{(x^2+y^2)^{1/2}}\,\hat{x} + \dfrac{y}{(x^2+y^2)^{1/2}}\,\hat{y}$, and $\vec{F}_{net} = m\vec{a}$, we have:

$$\boxed{\ddot{x} = -\frac{GMx}{(x^2+y^2)\,(x^2+y^2+L^2/4)^{1/2}}} \quad , \quad \boxed{\ddot{y} = -\frac{GMy}{(x^2+y^2)\,(x^2+y^2+L^2/4)^{1/2}}}$$

PROBLEM #12:



The net force on each planet is:

$$\boxed{\begin{aligned}
\vec{F}_1 &= -\frac{GM_1M_2}{|\vec{r}_1-\vec{r}_2|^3}(\vec{r}_1-\vec{r}_2) - \frac{GM_1M_3}{|\vec{r}_1-\vec{r}_3|}(\vec{r}_1-\vec{r}_3)\\
\vec{F}_2 &= -\frac{GM_1M_2}{|\vec{r}_1-\vec{r}_2|^3}(\vec{r}_2-\vec{r}_1) - \frac{GM_2M_3}{|\vec{r}_2-\vec{r}_3|}(\vec{r}_2-\vec{r}_3)\\
\vec{F}_3 &= -\frac{GM_3M_2}{|\vec{r}_3-\vec{r}_2|^3}(\vec{r}_3-\vec{r}_2) - \frac{GM_1M_3}{|\vec{r}_1-\vec{r}_3|}(\vec{r}_3-\vec{r}_1)
\end{aligned}}$$

Writing $\vec{F}_i = \dfrac{d^2\vec{r}_i}{dt^2}$, we get the required relations. Converting then into first order ODEs gives:

$$\boxed{\begin{aligned}
\frac{d\vec{r}_1}{dt} &= \vec{V}_1 , & \frac{d\vec{V}_1}{dt} &= \frac{-GM_2}{|\vec{r}_1-\vec{r}_2|^3}(\vec{r}_1-\vec{r}_2) - \frac{GM_3}{|\vec{r}_1-\vec{r}_3|}(\vec{r}_1-\vec{r}_3)\\
\frac{d\vec{r}_2}{dt} &= \vec{V}_2 , & \frac{d\vec{V}_2}{dt} &= \frac{-GM_1}{|\vec{r}_1-\vec{r}_2|^3}(\vec{r}_2-\vec{r}_1) - \frac{GM_3}{|\vec{r}_2-\vec{r}_3|}(\vec{r}_2-\vec{r}_3)\\
\frac{d\vec{r}_3}{dt} &= \vec{V}_3 , & \frac{d\vec{V}_3}{dt} &= \frac{-GM_2}{|\vec{r}_3-\vec{r}_2|^3}(\vec{r}_3-\vec{r}_2) - \frac{GM_1}{|\vec{r}_1-\vec{r}_3|}(\vec{r}_3-\vec{r}_1)
\end{aligned}}$$