## Problem #06 :

(a) We try to find a find a fn. $z(x)$ such that :

$$\int_0^x p(x')\,dx' = z(x)$$

We can then invert this eq$^n$ to obtain $x(z)$. We can generate random numbers $x$ with distribution $p(x)$ by generating uniform random nos. $z$ and caluclating $x(z)$.

For $w(x) = x^{-1/2}$, we have:

$$p(x) = \frac{w(x)}{\int_a^b w(x')\,dx'} = \frac{x^{-1/2}}{\int_0^1 x^{-1/2}\,dx} = \frac{x^{-1/2}}{2} = \frac{1}{2\sqrt{x}}$$

$$\Rightarrow \boxed{p(x) = \frac{1}{2\sqrt{x}}}$$

Here, $p(x) = \frac{1}{2\sqrt{x}}$

$$\Rightarrow z(x) = \int_0^x \frac{1}{2\sqrt{x'}}\,dx' = \sqrt{x}$$

$$\Rightarrow \boxed{x = z^2}$$

## Problem #10:

(a) In spherical Polar Co-ordinates, we have:

$\theta$ ranges from $0$ to $\pi$

$\phi$ ranges from $0$ to $2\pi$

Now, $$\int_0^\pi p(\theta)\,d\theta = \int_0^\pi \frac{\sin\theta}{2}\,d\theta = \frac{1}{2}\left(-\cos\theta\right)_0^\pi = \underline{\underline{1}}$$

$$\int_0^{2\pi} P(\phi)\,d\phi = \int_0^{2\pi} \frac{1}{2\pi}\,d\phi = \frac{1}{2\pi}\left(\phi\right)_0^{2\pi} = \underline{\underline{1}}$$

So, $p(\theta)$ and $p(\phi)$ are <u>Normalised</u>.

As seen before in Problem #06, we try to find fns $\Theta(\theta)$, $\Phi(\phi)$ such that

$$\Theta(\theta) = \int_0^\theta p(\theta')\, d\theta = \int_0^\theta \frac{\sin\theta'}{2}\, d\theta = \frac{1-\cos\theta}{2}$$

$$\Rightarrow \boxed{\theta = \cos^{-1}(2\Theta - 1)}$$

Similarly,

$$\Phi(\phi) = \int_0^\phi p(\phi')\, d\phi = \int_0^\phi \frac{d\phi}{2\pi} = \frac{\phi}{2\pi}$$

$$\Rightarrow \boxed{\phi = 2\pi\,\Phi}$$