A. Question one: Could you please calculate your best estimate of the TOSMHT for the $M_J = 1$ state?

TABLE I: The convergence test for the tune-out wavelengths of TOSMHT polarizability $[\alpha^S(\omega) - \frac{1}{2}\alpha^T(\omega)]$ for $M_J = \pm 1$ states of ^{4}He.

	$\alpha(\omega) = \alpha^S(\omega) - \frac{1}{2}\alpha^T(\omega)$		
(ℓ_{max}, N)	$\lambda_{TO}(nm)$	$\omega_{TO}(a.u.)$	$\frac{d\alpha(\omega)}{d\omega}\|_{\omega=\omega_{TO}}$
(10, 30)	413.083 0819	0.110 300 6986	7134.656
(10, 40)	413.082 9492	0.110 300 7341	7134.621
(10, 50)	413.083 0025	0.110 300 7198	7134.600
Extrap.	413.082 98(2)	0.110 300 73(1)	7134.59(1)
QED Corr.	0.004 26(1)		
Total in nm	413.087 24(3)		
Total in MHz	725 736 428(53)		

Answer: The detailed results are listed in the Table I. The columns 2 and 3 are the tune-out wavelength in nm and in a.u. units for the TOSMHT polarizability. The first-order derivative of polarizability $\frac{d\alpha(\omega)}{d\omega}|_{\omega=\omega_{TO}}$ at the tune-out wavelength is also given in the column 4. The QED correction (QED Corr.) includes the α^3-order with $\partial^2_\varepsilon \ln k_0$ term and α^4-order QED corrections. The final value of the TOSMHT for the $M_J = \pm 1$ state is 725 736 428(53) MHz.

B. Question two: How bad is the approximation of the "method part 2"?

Answer: In order to estimate the effectiveness of the approximate method, Table II lists our ab-initio calculations of the tune-out wavelength for the scalar polarizability, the first-order derivative of the scalar polarizability $\frac{d\alpha^S(\omega)}{d\omega}|_{\omega=\omega^S_{TO}}$ and the tensor polarizability $\alpha^T(\omega^S_{TO})$ at the tune-out wavelength of ω^S_{TO}. Using the values of ω^S_{TO}, $\frac{d\alpha^S(\omega)}{d\omega}|_{\omega=\omega^S_{TO}}$, and $\alpha^T(\omega^S_{TO})$ in Table I, we can get the tune-out wavelength by using the approximate formula of $\omega_{TO} = \omega^S_{TO} + \frac{1}{2}\alpha^T(\omega^S_t)/\frac{d\alpha(\omega)}{d\omega}$, please see the last column of Table II. You will see the values from the approximated formula in the last column of Table II are the same with the ab-initio values in the third-column of Table I. So we can draw that the approximation of the "method part 2" is reasonable.

TABLE II: Convergence test for the effectiveness of the approximate method of $\omega_{TO} = \omega^S_{TO} + \frac{1}{2}\beta^T$, where $\beta^T = \alpha^T(\omega^S_t)/\frac{d\alpha(\omega)}{d\omega}$ for $M_J = \pm 1$ states of ^{4}He. Where ω^S_{TO} represents the tune-out wavelength for the scalar polarizability $\alpha^S(\omega)$.

	$\alpha^S(\omega)$				
(ℓ_{max}, N)	$\lambda^S_{TO}(nm)$	$\omega^S_{TO}(a.u.)$	$\frac{d\alpha^S(\omega)}{d\omega}\|_{\omega=\omega^S_{TO}}$	$\alpha^T(\omega^S_{TO})$	$\omega_{TO} = \omega^S_{TO} + \frac{1}{2}\frac{\alpha^T(\omega^S_t)}{\frac{d\alpha(\omega)}{d\omega}}\|_{\omega=\omega^S_{TO}}$
(10, 30)	413.084 0595	0.110 300 4376	7134.442	3.724 543[-3]	0.110 300 698 6255294
(10, 40)	413.083 9279	0.110 300 4727	7134.406	3.728 913[-3]	0.110 300 734 0331089
(10, 50)	413.083 9775	0.110 300 4595	7134.386	3.714 754[-3]	0.110 300 719 8415346
Extrap.	413.083 96(2)	0.110 300 46(2)	7134.37(1)	3.71(1)[-3]	0.110 300 73(1)