**Logic**

**Proposition:** a sentence declaring a fact that is either True or False. E.g. 8+2 =11 & 8+2 = 10.

**Compound Propositions:** are formed from other propositions using logical operators.

**Negation:** p is a proposition, the negation of p is ¬p. basically meaning not p. e.g. the negation of “Cardiff is the capital of France” is “Cardiff is not the capital of France”. And “8+2=10” to “8+2 ≠10”.

**Conjunction:** p and q are propositions, the conjunction of p and q, denoted p ∧ q, and is the proposition “p and q”. It is true if both p and q are True, otherwise it is False. E.g. ‘True and True = True’, ‘True and False = False’.

**Disjunction:** p and q are propositions, the disjunction of p and q, denoted p ∨ q, and is the proposition “p or q” It is True if either p or q (or both) are True, and False if both p and q are False.

**Implication example:** “p ⇒ q if you get full marks in exam you pass the module”. BUT “q ⇒ p if you pass the module you get full marks in the exam”.

**Contrapositive:** the contrapositive of the implication “p ⇒ q” is “¬q ⇒ ¬p”

**Converse:** the converse of the implication “p ⇒ q” is “q ⇒ p”

**Tautology:** a compound proposition that is always True.

**Contradiction:** a compound proposition that is always False.

**Exclusive or (XOR):** denoted as “p ⊕ q”, it is the proposition that is true if exactly one of p and q are true. Otherwise it’s False. Also written as “! =” E.g. “T T = F”, “T F = T”, “F T = T”, “F F = F”. However has a problem for interpreting in English, e.g. “Tea or Coffee XOR” v.s “Milk or Sugar OR”

**De Morgan’s Laws:**



**Logical Equivalence:** compound propositions p and q are logically equivalent if p ⇔ q (= (p ⇒ q) ∧ (q ⇒ p)) is a tautology. That is when p and q have the same truth table.

**Proposition (De Morgan’s laws):**

• ¬ (p ∨ q) is logically equivalent to ¬p ∧ ¬q

• ¬ (p ∧ q) is logically equivalent to ¬p ∨ ¬q

P ⇔ q is true only if p and q are the same

**Proof by logical equivalence:** If p ⇔ q is a tautology (always true) p and q are always the same and so are their truth tables.

**Proof by tautology:** If p and q are equivalent then they are the same, p ⇔ q are always true.

E.g. “Show p ⇒ q is logically equivalent to ¬p ∨ q”

