Who Ultimately Owns A Company?

Group 2: ONS

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Declaration of AI use

During the writing of this report, we made limited use of Artificial Intelligence (AI) tools, specifically ChatGPT developed by OpenAI. The AI was used for:

1. Improving the clarity and correctness of grammar in academic English.
2. Debugging Python code and edition error on overleaf, including identifying syntax and logic errors.

All research design, data analysis, results and findings, comparison and discussion, and conclusion was conducted independently by the authors. No AI-generated content replaced original work or interpretation.

Societal Impact Statement

This study contributes to the advancement of financial transparency and accountability by improving methods for identifying ultimate actual controllers (Ubos) in complex corporate networks. (Jafarnejad et al., 2024) Increased transparency supports anti-corruption action, financial crime prevention, and regulatory compliance, in line with international frameworks such as the United Nations Convention against Corruption. (United Nations Office on Drugs and Crime (UNODC), 2021)

From a business and industry perspective, our research provides financial institutions, regulators and multinational corporations with tools to enhance due diligence, risk assessment and compliance procedures. By being able to detect hidden control structures early, our approach helps reduce systemic financial risks that can lead to economic instability. (Li et al., 2024)

For national financial security, this study is helpful to strengthen the country's control and monitoring of capital flow, prevent the cross-border flow of illegal capital and capital outflow, and ensure the security and stability of the national financial system. (Tran, 2020)Strengthening the identification of the ultimate controller can help protect the financial system from the risk of being manipulated by foreign funds and reduce potential threats to domestic policies and markets from the external economy. In addition, transparent ownership structures help governments better deal with financial crimes such as cross-border mergers and acquisitions, tax evasion, and money laundering, thereby enhancing the country's financial regulatory capacity and ensuring national economic security. (UK Parliament, 2025)

To minimize adverse effects, we promote responsible data use and ensure compliance with privacy and data protection regulations. In addition, by working with policy makers and industry stakeholders, we aim to refine our approach, balancing transparency with confidentiality and ensuring it is implemented in an ethical manner that serves the interests of society and business. (Patel, 2024)

1 Introduction

1.1 Problem Statement

Identifying the ultimate owner of a company is crucial in today's globalized economy for addressing financial fraud, ensuring regulatory compliance, and managing wealth inequality (Polovnikov et al., 2022). The United Nations Convention against Corruption emphasizes the need for transparent and accurate records of beneficial ownership to combat illicit activities and promote financial transparency (United Nations Office on Drugs and Crime (UNODC), 2021). However, the deliberate complexity of corporate networks often obscures true ownership, making regulation difficult and masking the concentration of wealth and control (Open Ownership, 2020). This lack of clarity also presents financial and political risks, especially in cross-border ownership structures—such as the increasing foreign control of Philippine companies due to Chinese investment (Mizuno et al., 2020).

1.2 Modelling Objectives

The aim of this work is to apply multiple methodologies for identifying ultimate ownership and shareholder influence in complex corporate ownership networks. The objectives addressed in this report are:

1. Who are the ultimate owners in a corporate ownership network?
2. Which shareholders exert the most influence across the network, either directly or indirectly?

1.3 Overview of Relevant Literature

To understand ownership structures, we apply four main methods: Breadth-First Search (BFS), Depth-First Search (DFS), Cumulative Matrix, and a Stochastic matrix approach to measure influence. BFS is known for its broad node coverage and low complexity, and has been widely used in large-scale networks such as Flickr and Facebook (Cui et al., 2022; Mislove et al., 2007). However, it may introduce bias by over-sampling high-degree nodes. On the other hand, DFS focuses more exploration and performs well in sparse graphs, especially where limited hops are advantageous (Cheng et al., 2016)). The Cumulative Matrix approach computes both direct and indirect ownership shares, with mechanisms to avoid double counting (Chapelle & Szafarz, 2005).

To measure the overall influence of shareholders on the network, not just in terms of legal ownership but also their potential to impact decisions, we use a stochastic matrix approach inspired by centrality metrics such as PageRank (Brin & Page, 1998). Similar methods have been applied in studies of corporate control and systemic risk, where indirect influence is just as important as direct ownership (Battiston et al., 2012; Vitali et al., 2011). This approach emphasizes shareholders whose network position gives them disproportionate influence, even if their direct ownership percentage is relatively modest.

Together, these methods help us to analyze ultimate ownership from multiple perspectives, compare their effectiveness, and measure shareholder influence in complex ownership networks.

1.4 Outline of the report

- Section 2 - Methodology introduce the four methods applied to data structures and the mathematical logic behind them.
- Section 3 - Analysis and Findings describe the results obtained by each method in the analysis section.

- Section 4 - Compares the methods and discusses their implications.
- Section 5 - Summarizes the key insights and conclusions.

2 Methodology

2.1 Data Overview

The data set consists of 500 ownership links between corporate entities, where each link represents a shareholder's ownership stake in a company. Each record includes a Company ID, a Shareholder ID, and the corresponding Shareholder Percentage, captur- ing both direct and indirect ownership structures. In total, the data set contains 384 unique shareholders and 390 unique companies. In particular, the number of unique companies exceeds that of unique shareholders, indicating that some shareholders own multiple companies, resulting in ownership concentration. This structure reflects a net- work where ownership flows hierarchically, allowing for analysis of both direct control (single-step relationships) and indirect control (multilayered ownership chains).

2.2 Ultimate Owner

An *ultimate owner* is defined as a company that controls other entities but is not itself controlled by any other company within the ownership network. In other words, it sits at the top of an ownership hierarchy. This concept is critical for understanding the flow of control in corporate networks, as ultimate owners can influence multiple layers of companies below them, even if they do not directly hold large ownership shares. Identifying ultimate owners is key to mapping true control relationships in complex ownership structures (Moody's, n.d.).

2.3 Methodological Framework

2.3.1 BFS (Breadth-First Search)

Breadth-First Search (BFS) is an algorithm to traverse graph structures. The core idea is to start from a given node and visit all neighboring nodes level by level. It can be formalized as follows:

Let there be a directed graph $G = (V, E)$, where each edge $(u, v) \in E$ (Gazit & Miller, 1988)represents that shareholder u holds a certain proportion of shares in company v, denoted by $p(u, v) \in [0, 1]$. If there exists a path

$$P = (v_0 \to v_1 \to \cdots \to v_k) \tag{1}$$

in which every edge satisfies $p(v_i, v_{i+1}) \geq 0.5$, and the cumulative ownership ratio satisfies:

$$P_{\text{ownership}} = \prod_{i=0}^{k-1} p(v_i, v_{i+1}) > 0 \tag{2}$$

then we say that v_k is the Ultimate Owner of v_0.

2.3.2 Depth-First Search and UBO Analysis

Depth-First Search (DFS) explores a graph by traversing deeply along each branch before backtracking. Formally, DFS is defined recursively as:

1. Visit node v.

2. Recursively perform DFS on each unvisited neighbor of v.

In the context of the Ultimate Beneficial Owner (UBO) problem, DFS is applied specifically to a reversed ownership graph, tracing the "company $\rightarrow$ shareholder" direction until a node with no controlling shareholders (in-degree 0) is found.

The recursive mathematical definition for identifying a UBO is:

$$UBO(c) = \begin{cases} c, & \text{if } c \text{ has no controlling shareholder} \\ UBO(s), & \text{if } s \text{ controls } c \text{ and } W(s,c) \geq 0.5 \end{cases} \tag{3}$$

This recursive logic is implemented iteratively in code for efficiency, equivalent to performing DFS on the reverse graph until finding a node with no outgoing edges (no further shareholders).

The ownership percentage along a control path $c \rightarrow s$ is calculated by multiplying each shareholder percentage along that path:

$$\text{Ownership}(c,s) = \prod_{i=0}^{n-1} W(v_{i+1}, v_i) \tag{4}$$

Memorization (dynamic programming) optimizes this computation by storing previously calculated results, reducing complexity from exponential $O(b^d)$ to linear $O(|V| + |E|)$.

Finally, the control chain length (number of layers from company to UBO) corresponds to the shortest path length in the graph, computed as:

$$\text{PathLength}(c,s) = |\text{Path}(c,s)| - 1 \tag{5}$$

This path length analysis is valuable in practical applications because it reveals the complexity of the control chain—longer paths may indicate more complex company structures and potential transparency issues.

2.3.3 Cumulative Matrix

An ownership network consisting of n entities (companies and shareholders) can be represented as A direct ownership matrix A, where:

$$A_{ij} = \begin{cases} w, & \text{if shareholder } v_i \text{ holds shares of company } v_j \text{ in proportion } w \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

This matrix representation facilitates algebraic operations to analyse direct and indirect ownership relations. To determine ownership paths between entities, we compute the powers of the ownership matrix. The matrix A to the power of k, expressed as A^k, provides ownership path information of length k. To be specific:

$$(A^k)_{ij} > 0 \Rightarrow \text{There exists an ownership path of length } k \tag{7}$$

By calculating the continuous powers of the matrix (A^2, A^3...) To track multi-step ownership paths and model how capital flows through complex networks. After multiplying and summing the matrices we can get the final ownership of the company. As k approaches infinity, the cumulative ownership matrix of this infinitely long path is equal to the Leontief inverse matrix in the limit case (Chapelle & Szafarz, 2005).

$$\text{Cumulative Matrix} = I + A + A^2 + \cdots + A^k = (I - A)^{-1} \tag{8}$$

2.3.4 Stochastic Matrix

To identify shareholders who have more influence in a synthetic company-shareholder network, we used a PageRank-inspired power iteration approach by utilizing the dominant eigenvector of the column-normalized ownership matrix (Brin & Page, 1998). Considering Perron-Frobenius theorem, this eigenvector uniquely indicates the long-term systemic influence of each shareholder, revealing hidden structural power in multilayered ownership networks. We use a square, column-stochastic ownership matrix A, where each entry a_{ij} represents the proportion of ownership that shareholder i holds in company j. The original matrix is derived from raw data and extended with dummy rows to ensure squareness.

Normalization: Each column of A is normalized so that:

$$\sum_i a_{ij} = 1 \quad \text{for all } j \tag{9}$$

This ensures A is column-stochastic, enabling a stochastic interpretation of ownership influence flows.

Power Iteration Algorithm: We apply power iteration to estimate the dominant eigenvector P^* corresponding to the largest eigenvalue $\lambda = 1$. Let $P_0 \in \mathbb{R}^{390\times 1}$ be a randomly initialized vector. This serves as the initial estimate for the dominant eigenvector:

$$P_{k+1} = A \cdot P_k \quad \Rightarrow \quad \lambda P^* = A \cdot P^* \tag{10}$$

Convergence is determined by:

$$\|P_{k+1} - P_k\| < \varepsilon, \quad (\varepsilon = 10^{-6}). \tag{11}$$

The resulting P^* is a probability vector where P_i^* represents the influence score of shareholder i.

Direct Cumulative Ownership: It is calculated as :

$$\text{Ownership Percent}_i = \left(\frac{\sum_j a_{ij}}{\sum_{i,j} a_{ij}} \right) \times 100\% \tag{12}$$

Pareto Analysis: We compute cumulative influence as:

$$C_k = \sum_{i=1}^{k} P_{(i)}^* \tag{13}$$

where $P_{(i)}^*$ are the sorted influence values. The smallest index k satisfying $C_k \geq 80\%$ identifies the top controlling shareholders.

3 Analysis and Findings

This section presents the results of applying the methods discussed in Section 2 to uncover ownership and influence patterns in the corporate network. Each subsection corresponds to a different methodological approach and highlights key findings through relevant graphs, tables, and interpretations. The aim is to identify both formal ownership structures and the hidden layers of influence that emerge from the network topology. We begin by analyzing who the ultimate owners are, then explore the complexity of ownership chains, and finally quantify systemic shareholder influence through a recursive matrix-based model.

3.1 Identifying Ultimate Owners (BFS Results)

To identify the ultimate owners, we use Breadth-First Search (BFS) algorithm to track all firms that are directly or indirectly controlled by shareholders that do not belong to any other entity. The algorithm starts at each company node and searches for its shareholders (those with an ownership percentage of $\geq 50\%$) using a hierarchically expanded queue; shareholders added to the queue are marked as visited to avoid loops. If the current node's shareholder is not in the list of company IDs, it is recognized as the ultimate controller; otherwise, the search continues along the equity chain. Afterwards, mapping the controlled companies and calculating the cumulative ownership percentages yielded the following results.

Based on the BFS statistics, we generated a result table, Table 1 is the sample table of BFS output, refer to **Appendix A** for the remaining part.

Table 1: BFS Output: Shareholder-Company Ownership Details

shareholder_id	**controlled company number**	**controlled company id**	**ownership_percent**	**cycle detected**
7	1	581	100.00%	no
8	1	725	51.00%	no
9	1	133	100.00%	no
21	1	102	100.00%	no
25	1	648	100.00%	no
42	1	846	100.00%	no
50	2	399	100.00%	no
50	2	437	99.00%	no
60	2	96	98.00%	no
60	2	474	98.00%	no

The calculated statistics below are based on the initial table. First, we found: 390 total number of companies; 192 ultimate controllers; 228 individual shareholders; no circle structure which indicates that there is no cross-holding or circularity in the ownership structure of the company. Therefore, the distribution of control is clear.

Second, after using filter statistics on this table by programming, we get: **307** companies(78.7%) have **1** shareholder of the respective specific number, **70**(17.9%) companies have **2** shareholders of the respective specific number, while only **13**(3.3%) companies have **3** shareholders of the respective specific number.

Third, we began to analyze the results from the perspective of shareholders. Table 2 is the result interpretation. It presents the number of companies controlled by each ultimate owner ("Number of Companies"), the number of ultimate owners controlling that number of companies ("Number of Owners"), and top 10 owners who control most companies ("Top Owners id"). For example, "Number of Companies: 1, Number of Owners: 102" means 102 ultimate owners each control exactly one company. Results highlight that most ultimate owners control few companies, indicating a fragmented structure. However, a few owners, such as shareholder id-110 who controls 14 companies, represent significant concentrations of control.

Table 2: Ultimate Owners by Number of Companies Controlled and top 10 owners most controlled

Number of Companies	Number of Owners	Top Owners id
1	102	–
2	53	–
3	14	–
4	10	–
5	5	shareholder id 232, 644
6	2	shareholder id 793, 666
7	1	shareholder id 544
8	1	shareholder id 484
10	1	shareholder id 966
11	1	shareholder id 105
13	1	shareholder id 24
14	1	shareholder id 110

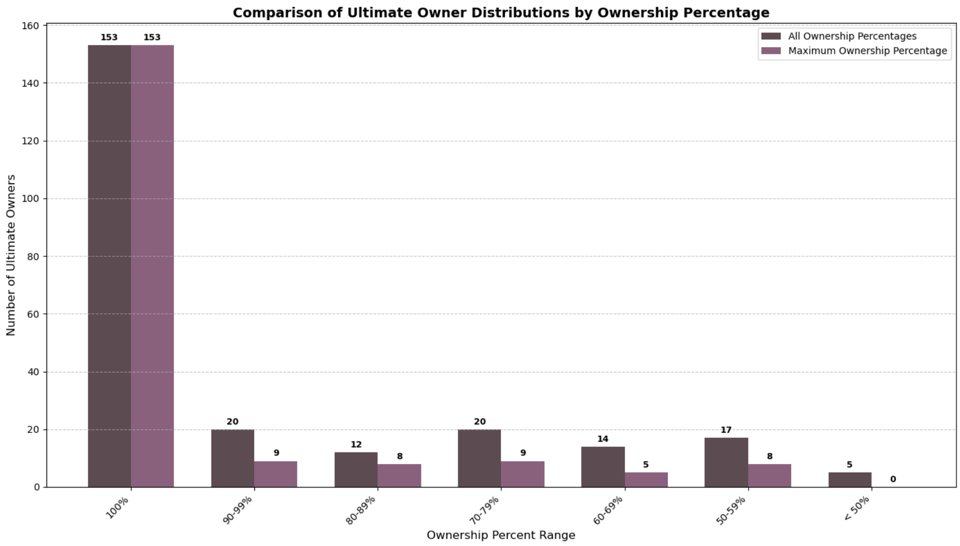

Figure 1: Comparison of Ultimate Owner Distribution by Ownership Percentages and Maximum Ownership Percentage

Finally, as Figure 1 shows, we compare two approaches to categorising ultimate owners based on their ownership percentages. The first approach, Maximum Ownership Percentage, classifies each ultimate owner into a single ownership range, based on the highest ownership percentage they hold across all companies. This ensures that each owner is counted exactly once, resulting in a total of 192 unique ultimate owners.

The second approach, All Ownership Percentages, considers every ownership stake held by an ultimate owner. In this case, a single owner can appear in multiple ownership ranges if they control different companies at different levels of ownership. This leads to a total count of 241 instances, with 35 ultimate owners appearing in more than one range. The difference (241 - 192 = 49) reflects the overlapping classifications in the second method.

This distinction highlights the importance of classification logic: while the maximum-based method offers a one-to-one mapping of owners to ownership categories, the all-inclusive method provides a more granular view of control distribution, at the cost of repeated counting.

Overall, ultimate owners mostly exhibit full control, though varied ownership levels also exist.

3.2 Ownership Chain Complexity (DFS Results)

While the previous section identified which shareholders control the most companies, it does not capture the complexity of ownership chains — in particular, how deeply layered control structures may be. To explore this dimension, we apply Depth-First Search (DFS) to calculate the longest ownership path from each ultimate owner to the furthest terminal company they control. For each ultimate owner (identified via BFS), we apply DFS to traverse the ownership graph and find the maximum path length from that owner to any company they control. Each path represents a chain of ownership, and the longest one shows the depth of control hierarchy. Along this longest path, we also calculate the maximum cumulative ownership percentage, which we use to categorize each ultimate owner as 100% (Full Ownership), 70–99% (Strong Majority), 50–69% (Simple Majority), and <50% (Minority Control)

This approach captures not just how many companies an owner controls, but how complex and indirect their control relationships are.

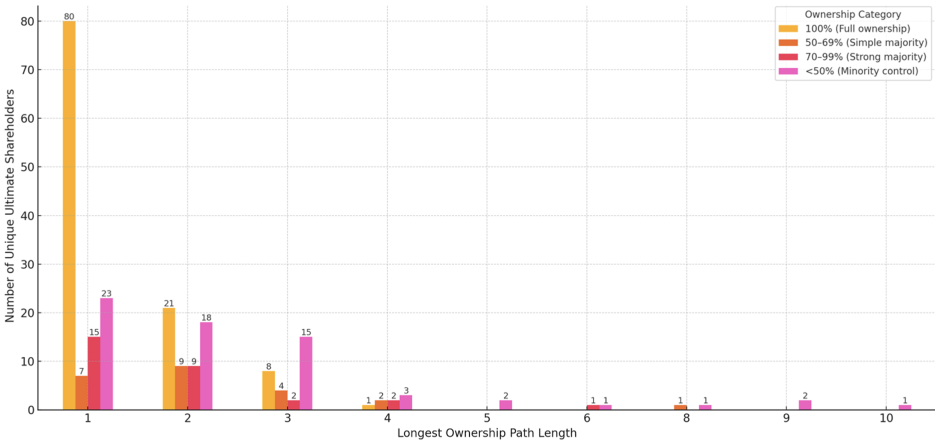

Figure 2: Distribution of Unique Ultimate Shareholders by Longest Ownership Path and Ownership Category

The results are visualized in Figure 2, which shows the distribution of the ultimate unique owners by the length of their longest control path and their associated ownership category. This analysis clarifies that while structural control is mostly direct and centralized, a small number of owners has complex and indirect control through long, multilayered chains — particularly in cases of minority ownership. Understanding this layered control is important for revealing hidden influence and tracing real-world corporate entanglements.

3.3 Identifying Ultimate Owners (Cumulative Matrix Results)

The ultimate control relationship of the enterprise is revealed through the matrix calculation method. First, the basic ownership matrix $A_{618\times618}$ is constructed, in which each element represents the direct shareholding ratio of shareholders in the company. If only the equity is greater than 50% is not set as the effective controlling shareholder, the final equity relationship can be directly calculated using the Leontief matrix, and a total of 1052 ownership relationships can be obtained.

However, the pure matrix approach has limitations in identifying specific control paths, because the cumulative matrix only preserves the total ownership value of the company by the ultimate shareholder. To overcome this limitation, the algorithm creates a specialized dictionary to record shareholder ids that meet ownership requirements greater than 50%, and dynamically builds and updates the control chain as new control paths are discovered, enabling accurate tracking of each control path and calculating its length.

The sum (in degree) of each column of the matrix represents the total number of times each company is controlled by other companies, and the sum of the rows of the matrix represents how many companies are controlled by shareholders (out degree). A company entry of 0 indicates that it is not controlled by any other entity, while an exit of 0 indicates that company controls at least one other company, which is the starting point of the control chain. For an effective controlling shareholder, if it controls a company (UBO matrix[i, j] ¿ 0), record the control relationship in the control chains dictionary, including: terminal shareholder, chain of control (path from terminal shareholder to controlled company), chain length, effective ownership percentage.

Figure 3 shows the ultimate actual controller (UBO) distribution analysis shows that control relationships are concentrated within short path lengths (1-3), where direct ownership (path length 1) dominates, especially in the 100% full ownership category. With the increase of path length, the number of UBO decreased significantly, and the control type gradually shifted from full ownership to majority control and minority control. In the longer path (6-10), a small number of control relationships are prominent, which may indicate that some entities use complex structures to downplay the apparent degree of control, which is particularly important for regulators and risk assessment, as complex ownership structures may hide potential risks that require closer scrutiny and regulatory attention.

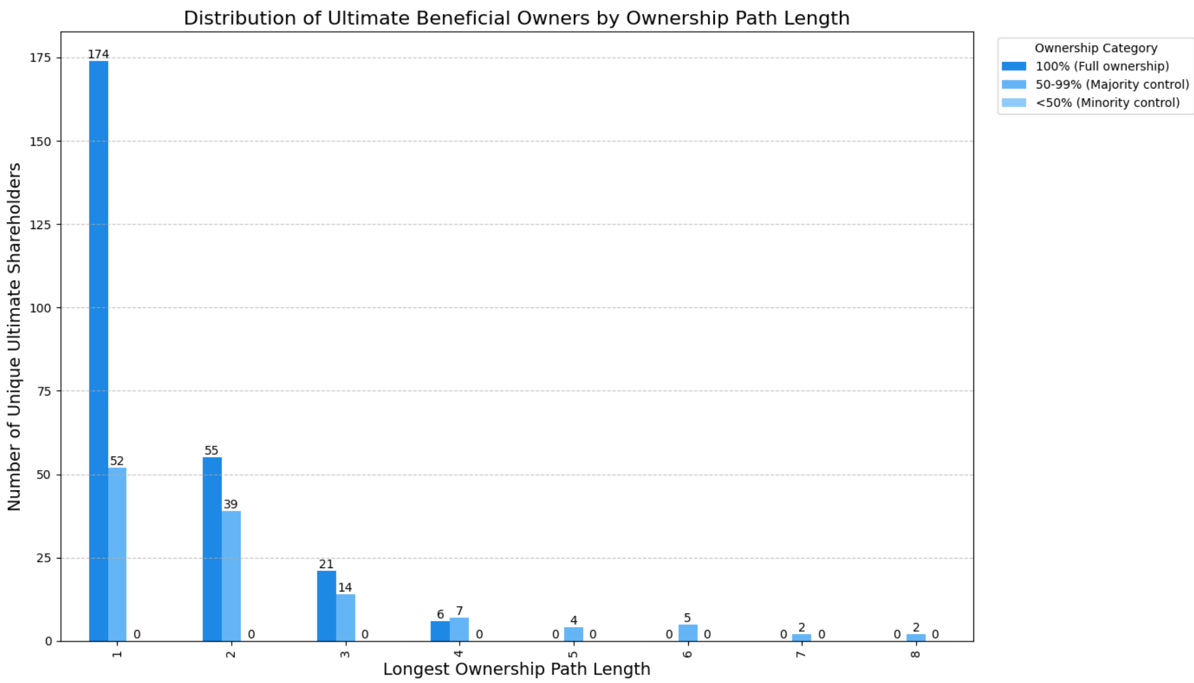

Figure 3: Distribution of Ultimate Beneficial Owners by Cumulative Matrix

3.4 Influence Ranking via Stochastic Matrix and Power Iteration

While structural ownership (as measured by BFS and DFS) determines who controls companies directly or indirectly, it does not fully reveal how influence propagates recursively through the network. A shareholder with modest direct ownership may have more influence through layers of companies, which can give them power that is not immediately visible. To capture this deeper form of power, we use a PageRank-style Power Iteration method to estimate systemic shareholder influence across the network. Figure 4 illustrates the computational workflow for deriving influence scores via power iteration.

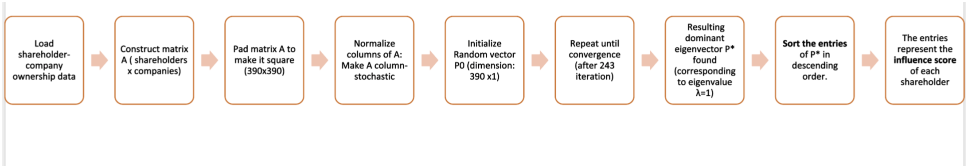

Figure 4: Power Iteration Process to Compute Shareholder Influence

3.4.1 Convergence & Stability

To validate the results, we track the L2 norm difference between successive iterations. As shown in Figure 5, convergence is achieved after approximately 240 iterations, supporting the numerical stability of the eigenvector solution.

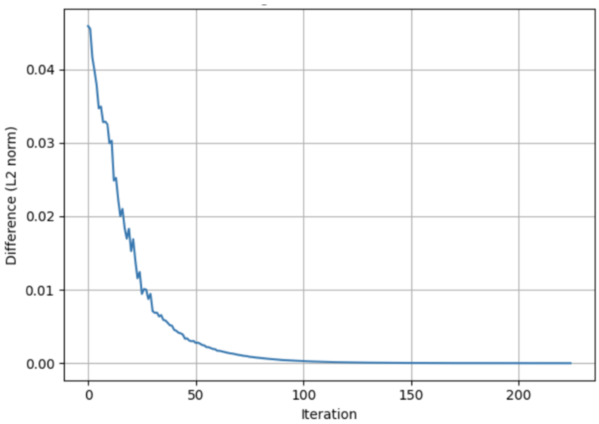

Figure 5: Convergence of Power Iteration for Stochastic Matrix Central Eigenvector

3.4.2 Most Influential Shareholders

The influence scores show a noticeable contrast between network centrality and direct cumulative ownership. Table 3 presents Shareholder 84, despite holding only 0.77% direct cumulative ownership, holds nearly 49% of the total influence — significantly more than peers with similar ownership levels.

Table 3: Top 10 Shareholders by Influence Score and Direct Cumulative Ownership

Rank	Shareholder ID	Influence Score (%)	Direct Cumulative Ownership (%)
1	84	49.10	0.7692
2	218	14.41	0.7692
3	605	10.50	0.5128
4	391	7.57	0.4872
5	773	7.49	0.5128
6	61	3.34	0.3081
7	755	2.15	0.2969
8	788	0.48	0.2564
9	143	0.44	0.2564
10	659	0.44	0.2564

A full ranking of all shareholders by their influence scores, as computed via power iteration, is provided in **Appendix C**.

3.4.3 Systemic Influence Distribution

To grasp the full distribution of influence, Figure 6 presents the influence scores of all shareholders in ranked order. Most shareholders have minor influence. A sharp drop-off reveals that only a small portion of shareholders dominate the network.

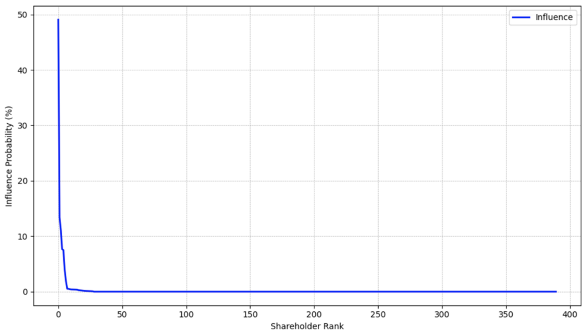

Figure 6: Influence Distribution for All Shareholders

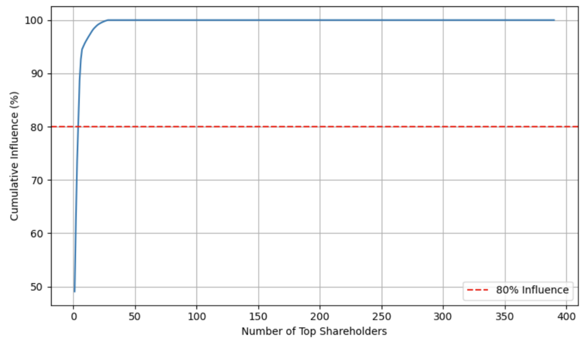

Figure 7: Pareto Analysis of Shareholder Influence

These findings are supported by the Pareto analysis in Figure 7, where fewer than 15 shareholders are shown to hold over 80% of the total influence.

3.4.4 Matrix Structure & Hidden Influence Paths

To understand how influence propagates in the power iteration process, we analyze the column-normalized ownership matrix (Figure 8). In this heatmap, rows represent shareholders, columns represent companies, and cell intensity indicates normalized ownership shares. The matrix is sparse and diagonally dominant. The strong diagonal structure emphasizes that most shareholders control distinct entities. On the other hand, the off-diagonal entries uncover indirect or shared ownership paths that contribute to influence propagation. These few indirect links form the channels through which control spreads across the network. Shareholders like ID 84, who have high influence, likely sit at the intersection of multiple such paths. This structure reveals why a small minority of shareholders accumulate disproportionate influence.

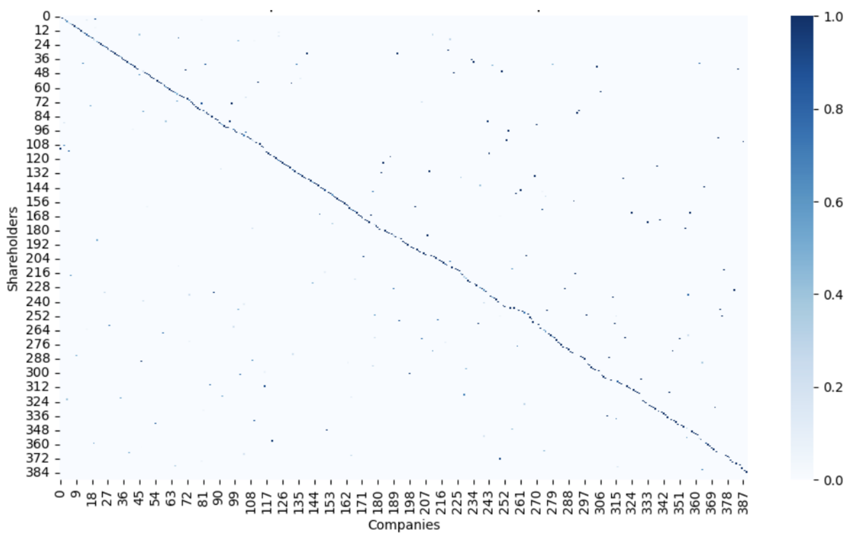

Figure 8: Heatmap of Column-Normalized Ownership Matrix. Each cell represents the normalized ownership share from a shareholder (row) to a company (column)

Although many shareholders are not structurally connected, those with strategic cross-links become central to the network's control. This validates the use of eigenvector-based methods, such as power iteration, for revealing hidden influence in complex ownership networks.

4 Comparison and Discussion

With all four analytical methods applied—BFS (ownership reach), DFS (control complexity), Cumulative Matrix (ownership strength), and Stochastic (influence)-we now compare their results to uncover where structural control aligns with or diverges from actual systemic influence. Table 4 summarizes the methods and their trade-offs.

Method	**What It Captures**	**Limitation**
BFS (Breadth-First Search)	Identifies how many companies a shareholder controls; provides a simple structural view of ownership.	Ignores indirect influence or control depth; doesn't reflect influence strength.
DFS (Depth-First Search)	Explores the depth and complexity of ownership chains; reveals multi-layered control paths.	Does not quantify influence magnitude; structural only.
Cumulative Matrix	Captures total ownership strength via matrix computation; reveals ultimate controllers with $> 50\%$ effective control.	Lacks path-level detail; does not show specific control chains; less useful for identifying influence distribution.
Stochastic Matrix	Quantifies total influence, including both direct and indirect control; identifies systemic power holders.	Produces abstract scores that are less intuitive; harder to interpret directly.

Table 4: Summary of Methods and Their Trade-offs

4.1 Alignment and Divergence of Results

According to findings, BFS and DFS identify who owns and how ownership is structured, Stochastic Matrix clarify how much that ownership actually contributes to influence in systematic network. We can observe this critical difference in the example of Shareholders 84 and 218, who both have 0.77% direct cumulative ownership — yet Shareholder 84 holds nearly 49% of network influence, compared to 14% for 218. This distinction is caused by 84's strategic position in the ownership network. In this kind of position, a shareholder might not hold a large ownership share but can indirectly control other shareholders through multiple layers, increasing their influence in the network. Applying all methods resulted in similar trends. BFS reveals that ownership is highly concentrated, with a few shareholders controlling many companies. DFS shows that most ownership chains are short, although a small portion of shareholder control involves deeper, more complex paths. Meanwhile, the Stochastic Matrix uncovers a noticeable imbalance in influence: just 15 shareholders account for 80% of total influence, while most others hold negligible systemic power. The Cumulative Matrix method further confirms these patterns by calculating formal control relationships through matrix-based aggregation, showing that the most effective control occurs through short paths and high ownership stakes. Although it lacks influence scoring, it strengthens structural insights and helps clarify potential control risks in multi-layered ownership chains.

These differences emphasize a critical understanding: ownership does not always equal influence. Some shareholders may formally control a large number of companies but possess little power in the network, while others, through indirect control, hold key influence in the system. This has important implications for financial transparency, governance, and risk oversight.

Key Question	Method(s)
Who owns the most companies?	BFS
How complex is the control path?	DFS
Who holds the most systemic influence?	Stochastic Matrix
Who has formal control (ownership > 50)?	Cumulative Matrix
Who has control but limited influence?	BFS + Stochastic Matrix
Where are hidden influencers located?	DFS + Stochastic Matrix

Table 5: Why a Multi-Method Approach Matters: Questions and Corresponding Methods

By combining BFS, DFS, the Stochastic Matrix, and the Cumulative Matrix, we gain a clear view of the corporate control landscape: who owns, how they own, and how much that ownership actually matters. This multi-method approach highlights both visible power holders and hidden influencers, presenting a more realistic perspective on ownership and control in complex company–shareholder networks.

5 Conclusion

In this project, we combined structural and network-based methods to identify two things. (1) the ultimate owners in a corporate ownership network and (2) the shareholders who exert the most systemic influence. Using Breadth-First Search (BFS), we tracked direct and indirect control paths and found that ownership is highly centralized—meaning a small number of top-level shareholders fully or majority-own large portions of the network. Depth-First Search (DFS) added insight into ownership complexity, revealing that minority control often originates from deeper, multi-layered chains. To measure influence beyond formal control, we applied a PageRank-style power iteration to a column-normalized stochastic matrix. This method revealed that both direct and indirect influence, highlighting a sharp discrepancy i.e shareholders with small direct cumulative ownership as Shareholder ID 84 can hold dominant systemic power due to their strategic position in the network. Across all methods, the results were consistent: a small minority of shareholders control most of the network—whether in terms of depth of control or network centrality. A Pareto analysis demonstrated that fewer than 15 shareholders hold over 80% of the total influence, underscoring the necessity of combining structural and influence-based approaches to recognize real corporate power dynamics.

By incorporating graph-theoretic algorithms (BFS, DFS) with cumulative matrix and stochastic matrix-based centrality analysis (PageRank-style power iteration), this work demonstrates how formal ownership and functional influence can differ significantly in modern corporate structures. These methods are essential for regulators, analysts, and policymakers seeking to identify hidden power and map the true architecture of control in complex ownership networks.

Appendix A: BFS generated initial table

Note: The BFS generated initial table is available at: https://uob-my.sharepoint.com/:x:/g/personal/zw24834_bristol_ac_uk/Ed96-joN13BEpIE6_sNGMqYBrSVAlnIisHFNbXb2K7vsbw?e=E98viI

Appendix B: Cumulative Matrix generated initial table

Note: The Cumulative Matrix generated initial table is available at: https://uob-my.sharepoint.com/:x:/g/personal/nv24222_bristol_ac_uk/EajY1yC5iLxOt0ODi5pvyO0BflUfJPDZRID7enXCHO2KBA?e=AaYNZ5&nav=MTVfe0IwM0M3QjI0LUFGMzMtNDE3Qi05NTY2LTk4NUVGNDc5OTdGN30

Appendix C: Full Influence Scores

Note: The full influence score dataset is available at: https://uob-my.sharepoint.com/:x:/g/personal/bz24793_bristol_ac_uk/EaCV7ux1Rc5Alp8RzKtUygEB8WZeQEkeihZO6vG0c85tbQ?e=Od5emF

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