Supplementary Information

Synchronized spread of COVID-19 in the cities of Bahia, Brazil

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- **Synchronization by Motif Method**

Motif synchronization is an association method used to construct networks based on association between pairs of time series [23]. It uses the sequence of micro patterns such as ups, downs, peaks and valleys, observed in time series to identify certain order of occurrences or patterns named motifs. They vary according to the number of points used in their construction (motif degree) or by their identification and the number of intervals (lag) between these points. For example, for degree $n = 3$ we have $n = 3! = 6$ types of motifs.

Figure 1 shows some grade 3 motifs for the lags, λ =1 and λ =2 [23].

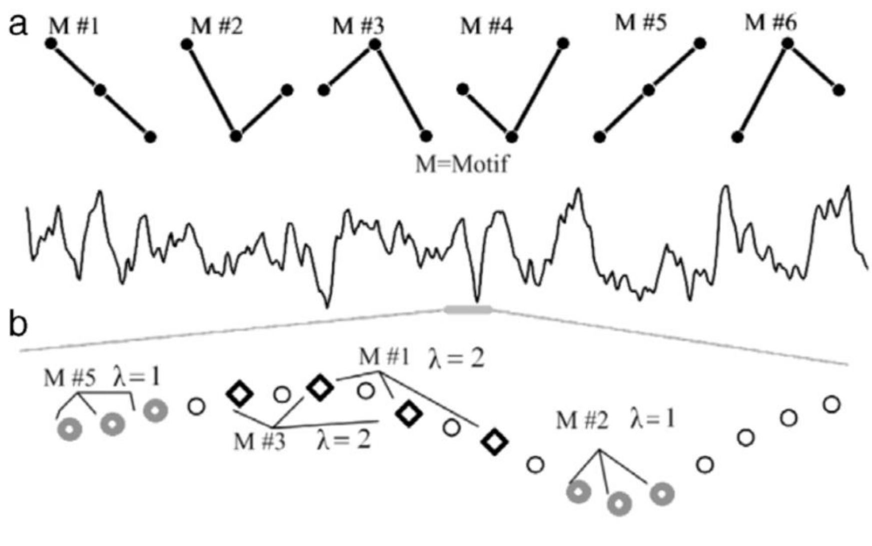

Figure 1: (a) All motifs of degree $n = 3$ (3!). (b) a signal and identification of some possible types of motifs of degree n=3 for a given segment of the signal (below). Black

circles represent time series points, gray circles represent $\lambda = 1$ lag motifs, and black diamonds represent $\lambda = 2$ lag motifs.

In general, motif synchronization consists of counting the quasi-simultaneous occurrence of these motifs [23].

For the description of the method, we are going to admit two time series X and Y, recorded simultaneously. The first step is to translate these time series into two new motif sequences X_M and Y_M series. For a degree=3, each X_{Mi} element can be defined as

$$X_{M_i} = \begin{cases} 1, & \text{if } X_i > X_{i+\lambda},\ X_{i+\lambda} > X_{i+2\lambda},\ X_i > X_{i+2\lambda} \\ 2, & \text{if } X_i > X_{i+\lambda},\ X_{i+\lambda} < X_{i+2\lambda},\ X_i > X_{i+2\lambda} \\ 3, & \text{if } X_i < X_{i+\lambda},\ X_{i+\lambda} > X_{i+2\lambda},\ X_i > X_{i+2\lambda} \\ 4, & \text{if } X_i > X_{i+\lambda},\ X_{i+\lambda} < X_{i+2\lambda},\ X_i < X_{i+2\lambda} \\ 5, & \text{if } X_i < X_{i+\lambda},\ X_{i+\lambda} < X_{i+2\lambda},\ X_i < X_{i+2\lambda} \\ 6, & \text{if } X_i < X_{i+\lambda},\ X_{i+\lambda} > X_{i+2\lambda},\ X_i < X_{i+2\lambda}. \end{cases}$$

Figure 2 shows an example of the procedure for converting time series into motif series [3].

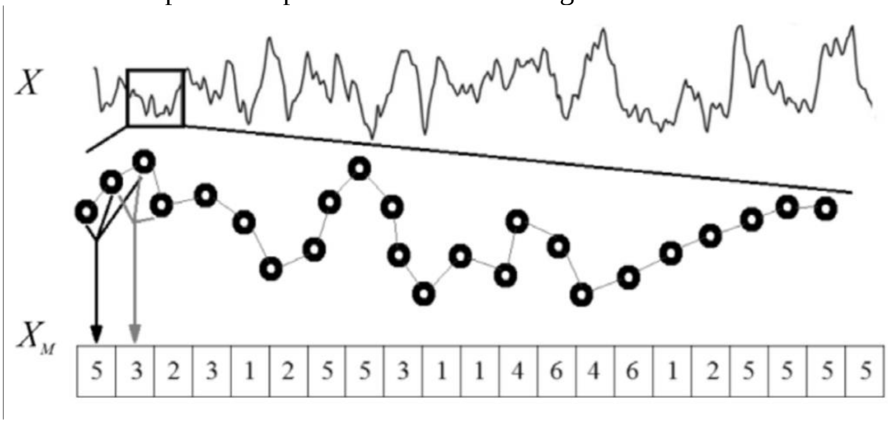

Figure 2: Transcription of the temporal series in the series of motifs of degree n=3.

We then define $c(X_M\ ;\ Y_M)$ as the greatest number of times the same motif was found in XY_M soon after being found in X_M, for different delay times τ, that is,

$$C(X_M; Y_M) = C_{XY} = max\left(\sum_{i=1}^{L_m} J_i^{\tau_0}, \sum_{i=1}^{L_m} J_i^{\tau_1}, \ldots, \sum_{i=1}^{L_m} J_i^{\tau_n}\right)$$

Where

$$J_i^{\tau} = \begin{cases} 1 & if\ \ M\#_{X_i} = M\#_{Y_{i+\tau}} \\ 0 & otherwise \end{cases}$$

Where $M\#X_i$ is the time series of motif X position i and L_m is the total size of the motif series. The delay times τ vary between $\tau_0 = 0$ and τ_n, where τ_n is the maximum value considered. Similarly, we define c_{XY}.

Finally, we define the degree of synchronization Q_{XY} and the direction of synchronization q_{XY}, given by

$$Q_{XY} = \frac{max(\,C_{XY}, C_{YX})}{L_m}$$

and

$$q_{XY} = \begin{cases} 0 \;\; if \;\; C_{XY} = C_{YX} \\ signal(C_{XY} - C_{YX}) \times 1 \;\; otherwise \end{cases}$$

The degree of synchronization is between $0 \leq Q_{XY} \leq 1$, and the index q_{XY} takes the value zero for a synchronization without preferential direction between X and Y, takes the positive value of 1 when X precedes Y, and takes the value -1 when Y precedes X.

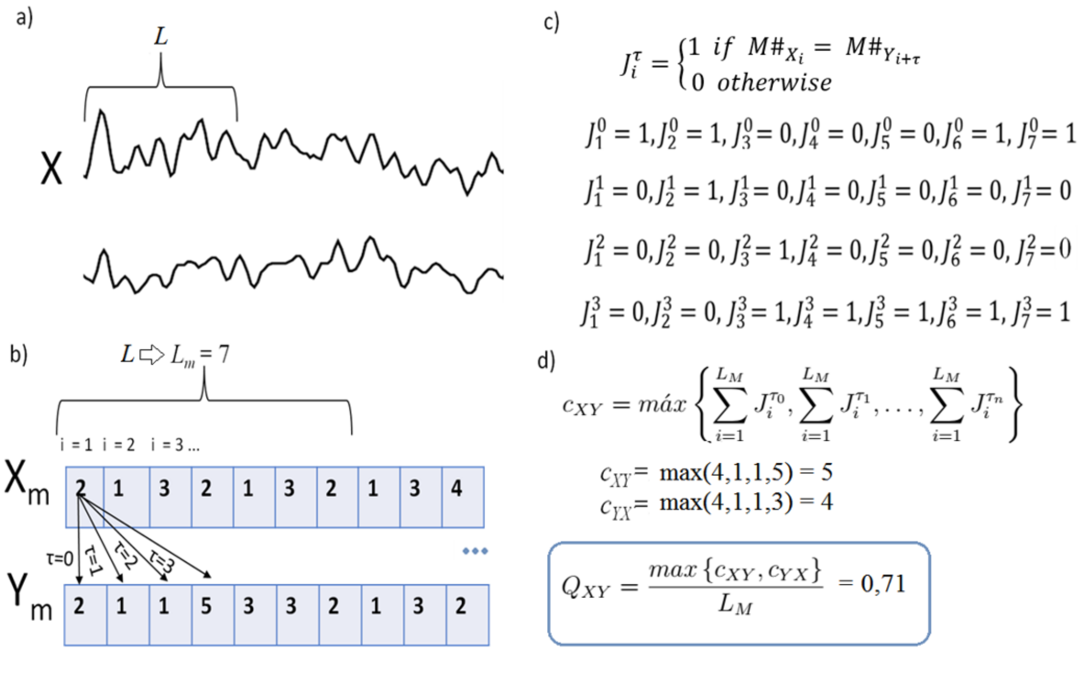

Figure 3: Diagram representing the motif synchronization method.

a) Pair of original time series.

b) Transformation of time series into motif series (purely illustrative example).

c) The function J_i^τ for the series shown in b).

d) Calculation of Q_{XY}, note that c_{XY} can be obtained similarly by just changing the direction of association from X_m to X_m.