Venue ____________________

Student Number |__|__|__|__|__|__|__|__|__|

Family Name _____________________

First Name _____________________

Department of Computing

EXAMINATION

Semester 2 & Trimester 3B, 2016

COMP5001 Theoretical Foundations of Computer Science

This paper is for Bentley Campus students.

This examination has a total of 120 marks.

Examination Duration: 180 minutes

Reading Time: 10 minutes

Exam Conditions:

This is a CLOSED BOOK exam - no text books or written materials permitted

Students are permitted to write notes during reading time in the margins of the exam paper

This paper can be released to students after the exam

The students exam paper can be released to students after the exam

No calculators are permitted in this exam

Materials Permitted In The Exam Venue:

none

Materials To Be Supplied To Students:

1 x 16 page answer book

Instructions To Students:

For Examiner Use Only

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Section 1 - Classification

Problems 1 to 6 in this section each describes a problem in English, set notation or in terms of strings. It is your task to do the following, where applicable, for each of these problems:

1. Classify the problem into the appropriate category: Regular, Context-Free, Turing Decidable, or Undecidable (T1-T4). **(2 marks)**
2. For a problem that is Regular, prove that this is the case by constructing either a DFA, a NFA, or a Regular Expression that accepts the language of the problem. **(4 marks)**
3. For a problem that is Context-Free, do both of the following:
   a. Prove that the problem is not Regular using the pumping lemma, and **(5 marks)**
   b. Prove that the problem is Context-Free by constructing either a PDA or CFG that accepts the language of the problem. **(5 marks)**
4. For a problem that is Turing Decidable do all of the following:
   a. Prove that it is not Context-Free using the pumping lemma for Context-Free grammars, and **(7 marks)**
   b. Prove that the problem is Turing Recognizable by constructing a Turing Machine, and **(7 marks)**
   c. Prove that the Turing Machine constructed is a decider. **(4 marks)**

   Alternatively, instead of a-c above you may choose to prove that a problem is Turing Decidable by reduction to and from a known Turing Decidable problem. **(18 marks)**

   Also, some problems don't lend themselves to Pumping Lemma proofs as required for a. above. In that case you can argue why the problem is not context free AND why a pumping lemma proof isn't appropriate.
5. For a problem that is Undecidable, prove this using reduction from A_{TM}. **(18 marks)**

Problem 1:

The language of binary strings that consist of $a^ib^jc^k$ where $i>k>0$, $j>2$.

Problem 2:

After getting some strange results from a computer program, a research student realises that the program may be having problem with a method that is supposed to categorize an input by either membership or non-membership of a set. Every input she tries is accepted, including ones that aren't supposed to be. Curious, she decides to determine whether every possible input is accepted but also has problem with this.

Your job is to create a general way of checking a method that returns *true* or *false* based on classifying an input (for the sake of generality consider the input to be simply a binary string and ignore what that string may represent). You test should return *true* if the method always returns *true*, irrespective of input. If there is even a single input for which the method would return *false*, your test should return *false*.

Problem 3:

An insurance company plans to install a dashcam with built-in GPS in customer cars in order to make insurance claims less taxing for their legal division. Part of these devices will monitor the car's location, the speed limit at that location and the speed of the car. Practically speaking, the device has a component that will receive a 1 if the car is currently travelling over the speed limit or a 0 if travelling at the speed limit or below, with a new bit of information sent every second. If the car accelerates abruptly enough (*e.g.*, has an accident) the device will record whether the driver has been detected to be speeding for at two of the last three seconds. In this case the device should accept. If the driver was speeding for one or less of the last three seconds, the device should reject.

Your job is to design and build this component.

Problem 4:

The Transport Authority is planning to upgrade a traffic light controlling a crossing of two major roads so that its timing more accurately reflects accurate traffic flow. They plan to install two devices to check on traffic flow. Device A will return true if the number of cars passing along road A is at least two times as much as the number of cars passing along road B. Device B will return true if the number of cars passing along road B is at least two times as much as the number of cars passing along road A. The inputs to the devices are pressure plates on the road, that register a 0 if cars pass along road A and a 1 if cars pass along road B.

Problem 5:

A conspiracy theorist has decided that Garfield cartoons form a secret code. The idea is that each panel in each cartoon is assigned a ‘character number’, which is the number of main characters (Garfield, Odie, Jon, and Nermal) who are clearly visible in that panel. The character numbers then form a string, which is then translated into binary and hence into English.

A linguistics expert has pointed out that if such a code were to exist, certain groupings would naturally occur more than any other. In particular, the sequence 001 would occur roughly four times as often as 013, while the sequence 102 occurs roughly three times as often as 013.

The problem is to write a program that checks a string of ‘character numbers’ to see if the expected frequency is upheld.

Problem 6:

A property developer wishes to design an innovative three-sided high-rise building. As such he asks you to develop software that searches the map of a city (he hasn’t decided which one yet) and returns a possible location. Such a location is determined by three streets that form a triangle. He is willing to provide the map as a graph file - when he decides on the city.

END OF SECTION

Section 2 - Complexity.

Choose one of the problems in this section. For this problem, classify it as being a member of P, NP, NP-Complete or NP-Hard and then prove this classification. When proving membership of NP-Complete or NP-Hard by reduction, use SAT, 3-SAT or 3-COLOR as the problem to reduce to and/or from.

Attempt only one of the questions in this section. If you attempt more than one, only the first question attempted will be marked and any others will be ignored. **(20 marks)**

Problem 1:

The government is making plans for enacting martial law in the case of a serious occurrence such as riots by theoretical mathematicians arguing over the nature of pi. They plan to station army or police vehicles at intersections such that they have visibility over every street in the area to be locked down. Your task is, given a street map of the area and the number of vehicles available, to check whether there are sufficient vehicles to have every street watched.

Problem 2:

A student is working on a final-year project involving an online gaming scenario called 'Wumpus World'. In this scenario, an intelligent agent must explore a world (which can be modelled as a graph with each graph node representing one location in the world and each legal movement being an edge in the graph) to locate gold that is visible from only one location away. The first part of the project – identifying dangers – has been completed, and all parts of the world that are to be avoided have been removed from the world (*i.e.*, the graph). The next step is to write a function that plots a course for the agent through the world. Because reducing the amount of moving that the agent needs to do is a requirement, the agent should visit as few locations as possible. Because the gold is visible from one location away, not all locations remaining need to be visited.

Your task is to first check the remaining graph for connectivity (in particular, find all locations connected by a path of one or more moves to the starting point, forming what is called a "connected sub-graph") and then finding a path through the connected area such that as few locations as possible are actually visited by the agent, but that every location in the connected area is viewed.

END OF SECTION

Section 3 – Short Answer.

For each of the questions in this section, do the following:

- If asked whether a statement is true or false, state this clearly. You should specifically use one of these two words. If you believe that the statement is both (perhaps with conditions) write "both true and false" or if you believe that it is neither write "neither true nor false".
- If asked to give any other opinion or classification, state it clearly.
- Justify your answer or opinion. Answers without justification will be awarded zero (0) marks.

Question 1:

An eager theoretical mathematician has an idea while struck in the head during a riot. He creates a new machine by combining an NFA with a tape – due to the head injury he is a little shaky on exactly how long the tape actually is, but it's definitely finite. When he wakes up in the morning, he contacts you to ask your opinion on what Tier of problems can be solved by this machine.

(4 marks)

Question 2:

A researcher has proved that a problem is in both P and NP. Does this mean that P=NP?

(2 marks)

Question 3:

A computing student has developed software that uses heuristic measures and advanced memory monitoring techniques to detect whether a program that is running is caught in an infinite loop or not. She demonstrates the software in a presentation and shows that it catches three types of infinite loops almost as soon as the program gets "stuck". She claims that this provides proof that computability theory is incorrect, although she uses somewhat stronger and more colourful vocabulary.

Can you determine whether her claim is true of false, assuming the demonstrations were not faked? If it is not possible to be certain, what is more likely? Explain your reasoning.

(4 marks)

END OF EXAMINATION