Theoretical Foundations of Computer Science

2017 Assignment

This assignment will not be submitted, but the questions in the assignment test will be based directly on the questions below. Note that while some questions may be identical, the general idea is that the concepts covered in the questions are the same. In other words, you need to understand the answers to these questions, not just memorize them.

Section 1 - Classification

For each of the following problems, classify it into a category (Regular, Context-Free, Decidable, Undecidable) and prove its membership of that category. You can use Tier 1-3 and 4+ instead of these categories if you wish.

To prove membership, do the following:

- Regular – provide a DFA, NFA or RE that solves the problem
- Context-Free – provide a proof (preferably using the pumping lemma) that it is not Regular <u>and</u> provide a PDA or CFG that solves the problem
- Turing Decidable – provide a proof (preferably using the pumping lemma for CFGs) that it is not Context-Free <u>and</u> provide a Turing Machine M <u>and</u> prove that M is a decider. If the format of the problem precludes the application of PDAs or CFGs the pumping lemma proof can be omitted and the format problems should instead be used as an argument.
  OR you can prove that a problem is Turing Decidable by reduction to and from a known Turing Decidable problem.
  OR some sensible combination of the two options above.
- Turing Undecidable – prove (using reduction from A_{TM}) that the problem is undecidable

If you cannot prove or disprove membership formally, give a convincing argument. If you need to make assumptions, state these clearly. If you are not sure of a term, make sure that you look up a definition from a reliable source.

PROBLEM 1:

For the language L = {🐼, 🏅, ⚖, ⚽} you are to detect all strings that have every panda getting a medal for football (in other words, the symbol 🐼 is always followed immediately by the symbol 🏅 and the symbol ⚽, in that order) and no reports involve pandas (in other words, the symbol ⚖, is never followed immediately by the symbol 🐼).

PROBLEM 2:

A scientist who has spent his life monitoring termites, decides to turn his attention to ant colony establishment. When a new queen hatches, a group of ants from the existing hive will migrate with her to form a new hive. After some initial observation, the scientist hypothesises that every such migration includes some number of soldier ants (S) and at least twice that number of worker ants (W), in addition to the queen herself (Q). He has never observed eggs

(E) being carried with a migration and hypothesises that this never happens.

You are to determine whether the hypothesis is correct.

PROBLEM 3:

An aerial drone is being used to conduct a study on congestion on the Kwinana Freeway. As it flies, it locates a line of ten or more cars that are in one lane and closely following one another. When such a group is found, it measures the speed and acceleration of each of the cars (as a pair of integers in $^{m}/_{s}$ and $^{m}/_{s^2}$)) and records this as a dataset. When the group of cars is reduced to less than 10 cars or the first of these cars leaves the Kwinana Freeway, the drone locates another group. The aim is to take one dataset per minute, although this isn't always possible.

Your job is to write software that will take a dataset as an input and accept if it meets a particular requirement. This requirement is met when a dataset contains one or more cars in the set (call the group Z) have a speed of zero, all of the cars in Z are adjacent to each other, all of the cars before Z have positive acceleration and all cars after Z have negative acceleration.

PROBLEM 4:

After much work, the Wiyrkomi Corporation has begun to produce software systems that are supposed to be able to learn from a user's use of devices such as their smartphone and computer, especially their viewing patterns, interests and online behaviour. The idea is that these learning systems will be improved until they can eventually become working copies of the person that they are learning from. Wiyrkomi is referring to these software systems as GPPs. They are currently implemented on a generic cloud platform (referred to as Heaven), but there some talk of creating customized hardware that incorporates non-Von-Neumann architecture to improve the process.

While this goal is still far away, the Wiyrkomi Corporation is employing your team to write some testing software. Recently, several of the GPPs have been showing worrying agreement on outputs, leading rise to a belief that the learning system has a basic fault that may cause multiple GPPs to converge to an equivalent 'personality'. The programmers in charge strongly deny this, which is why an outside group is needed.

The software should take two GPPs and determine whether, for any possible input, the two will produce identical outputs. For the sake of this question, you may treat each GPP as a black box that accepts finite binary strings and outputs a "yes" or "no" after a finite amount of time.

PROBLEM 5:

A mathematician asks you to design a program that accepts two graphs as input and tests whether these are isomorphic.

PROBLEM 6:

After getting some strange results from a computer program, a research student realises that the program may be having problem with a method that is supposed to categorize an input by either membership or non-membership of a set. Every input she tries is rejected, including ones that aren't supposed to be. Curious, she decides to determine whether every possible input is rejected but also has problem with this.

Your job is to create a general way of checking a method that returns *true* or *false* based on classifying an input (for the sake of generality consider the input to be simply a binary string and ignore what that string may represent). You test should return *true* if the method always returns *false*, irrespective of input. If there is even a single input for which the method would return *true*, your test should return *false*.

PROBLEM 7:

The controller for the cooling system for a cold storage area changes the temperature level by increasing or decreasing the level of cooling. A thermostat measures the temperature in the room, and sends one of the following signals to the controller:

i. 0 – the temperature is correct
ii. 1 – the temperature is too hot
iii. 2 – the temperature is too cold
iv. 3 – an error has occurred

Based on these signals, the state of the controller becomes one of the following:

i. INC – increase the level of cooling if the room is too hot
ii. DEC – decrease the level of cooling if the room is too cold
iii. MAIN – maintain the current level of cooling
iv. ERR – report an error and stop monitoring the thermostat output

The controller is considered working correctly (acceptable) when not in an error state. You may assume that the cooling can be increased or decreased as much as is required.

PROBLEM 8:

The language of binary strings that contain the sub-string 01^n0 and the sub-string 10^n1 where $0<n$.

PROBLEM 9:

The language of binary strings that contain 1^n0^m but not 1^m0^n where $m > n > 2$.

PROBLEM 10:

For a ternary code ($\Sigma = \{0,1,2\}$), find all strings of the form $0^i1^{i+1}2^{2i}$, $i \geq 0$.

PROBLEM 11:

ES_{TM}, which is the problem of testing a TM *M* to see whether M accepts the empty string.

Section 2 - Complexity

For each of the following problems, classify the problem into one or more time-complexity categories (P, NP, NP-complete, NP-hard). If you cannot prove membership formally, give a convincing argument. If you need to make assumptions, state these clearly. Note that if there is no known classification for an underlying problem I don't expect you to create one, but I would expect you to be able to determine that this is the case and provide references. You may have to look up terms in order to understand some of the questions fully, but I expect you to be able to do this.

When using reduction to show a problem is NP-Complete or NP-Hard, relate the problem to SAT, 3-SAT, or 3-COLOR.

PROBLEM 1:

As part of the roll-out of the NBN you have been asked to check that the plans for the coverage of a Perth suburb that happens to have a person of some importance living in it. You will be given a map of the suburb that shows every junction and the connections between them. The idea is to install monitoring devices at enough junctions that it's possible to ensure that the connection speed through every connection meets the required standard. Due to financial concerns, it is required that the least number of such monitoring devices be used. For the same reasons, it is not likely that this project will be applied to other suburbs in the future.

PROBLEM 2:

As part of an election campaign, a Western Australian government senator plans to lead a charity bicycle ride. While plans for this ride are made quickly, the senator's office contacts your company to develop software to plan future rides, which may be in Perth or in the eastern states. The bike ride is to start and end at the Parliament House, although future rides are likely to use other locations of political significance.

Your software will be provided details on the bike paths and dual use paths (those that can be used by both bicycles and pedestrians) in the area that the ride is to take place, and may specify the format of the information provided.

PROBLEM 3:

A network specialist is planning to work on methods of routing communications within a large organization that uses a private WAN. The plan is to build hard-coded routing tables to direct packets through the large-capacity connections that connect the various LANs, and then to also have at least one backup. This means that for every cable in the WAN that links two LANs, there must be a third LAN that is connected by a cable to each of the first two. This would allow a link along a broken cable to be replaced by routing the traffic through the third LAN.

He asks you to write an initial program that simply checks whether a company's network has an appropriate connectivity structure for this strategy. Any network that doesn't, will have to be given special attention, and this will help him with future designs.

PROBLEM 4:

Your friend has recently obtained employment as a minion of Evil Acquisitions Inc, which is a company working on overthrowing the world order. The three owners of the Perth-based company have decided to start small – with the southern parts of Perth. They do plan to adhere to the company's motto though: "…and tomorrow, the world!"

Your friend has been tasked with writing software to randomly split up the acquired suburbs between the three owners. The trick is that the owners want to spread their personal gains as much as possible, so the suburbs must be assigned in such a way that no owner gains control of a suburb adjacent to another that they already control. It has been agreed that uninhabitable places – those not part of an official suburb, oceans, etc – will not be counted and will be labelled "hunting preserves".

PROBLEM 5:

You have been asked to design software for a company that designs, builds and runs enclosed communities for the survival conscious. Each community will be outfitted with several layers of security facing the outside world as well as bunkers and supply depots, in case the unimaginable happens and Tony Abbot becomes prime minister again.

Your software should be able to be fed data regarding the layout of a community, specifically including the roads and how they connect. You may specify the format of the data. It should also accept specifications for communication systems – specifically, the range that broadcasts from such a system can be heard. Your software should then calculate the best way to place communication systems so that a broadcast can be heard from any point in the community. Houses tend to interfere with sound transmissions due to the special equipment built into them to ward against sonic attacks, so you are asked to only consider the distance along roads, not across houses.

Additional PROBLEM:

This is an especially challenging problem that is probably too difficult for most students to solve themselves, especially given current time pressures. As such, it is presented as a challenge and there will be no requirement to solve the problem fully. It is expected that students will have done their best to source a solution and will understand some of the concepts involved.

You are part of a team designing software for an automated vehicle that will be tasked with keeping Google Street View images updated. Of course, one of the objectives is to do so as efficiently as possible. You are to design the software that plans the route for the car.

Given the map of an area, your software should find a route for the car that has it travel along each stretch of road exactly once. The map you are given for any area will represent intersections as vertices and roads as edges. You may ignore all properties of intersections, but must note that some roads may only allow traffic to flow in one direction. Roads (edges) also have a length.

Your route should start and end at the same spot (forming a circuit) and should be minimal (*i.e.*, have the minimum total length). Ideally the route would go along each section of road exactly once, but repeating sections is fine if necessary.

You may assume that road lengths are in integers.