LINEAR MODEL OF THE DC MOTOR

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Practical Report

Mathematical Modeling and simulation

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ON

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Abstract

In this document is analyzed the behavioral for a a linear motor using techniques controls to understand the stability implemented with MATLAB.

1 Introduction

In this report is analyzed the behavioral from the model of a Linear model of the DC motor, so that is present the results from a simulation implemented with MATLAB, in the first part is analyzed the equations from the model, in the state of space and in the space of frequency, after of that we show the code and the results with a step, the bode diagram and the Nyquist diagram to the end is presented a discussion and a conclusion about the results.

2 Lineal model from the DC motor

In this laboratory is analyzed the next differential equation:

$$U_M = R_\alpha I_\alpha + K_e W + L_\alpha \frac{dI_\alpha}{dt} \tag{1}$$

$$J\frac{dW}{dt} = M - M_L \tag{2}$$

Where U_M is the voltage of the motor[V], I_a is the motor armature current[A], W is the motor speed(angular velocity)[rad/sec], E_a is electromotive force[V], J is a moment of inertiakgm^2 and M_L a torque for external load [Nm].
So that in other form is possible have the next system:

$$\begin{cases} \dfrac{dI_\alpha}{dt} = \dfrac{1}{L_\alpha}(-R_\alpha I_\alpha - K_e W + U_M) \\ \dfrac{dW}{dt} = \dfrac{1}{J}(K_W I_\alpha - M_L) \end{cases} \tag{3}$$

Where $T_\alpha = \dfrac{L_\alpha}{R_\alpha}$ is the electrical time constant, [s], $T_M = \dfrac{JR_\alpha}{K_e Km}$ [s], so that the (3) could be expressed in the frequency state:

$$W_M(s) = \frac{W(s)}{U_M(s)} = \frac{1/K_e}{T_m T_a s^2 + T_m s + 1} \tag{4}$$

So that with the equation (4) is possible use to analyze the dynamic from the system.

3 Cases to analyze

After obtaining the Linear model expressed in the Frequency state is proposed analyze different types of motors and their parameters, so that is extracted the next table from the motors :

Parameter	(A-MAX 22-110149)	(A-MAX 22-110150)	(A-MAX 22-110152)
Terminal resistance $[\omega]$	9.09	14	33.3
Terminal inductance $[mH]$	0.585	0.891	2.1
Inv. of Speed const. $[\frac{V}{rad \cdot s^{-1}}]$	13.9×10^{-3}	17.1×10^{-3}	26.2×10^{-3}
Rotor Inertial$[kgm^2]$	4.2×10^{-7}	4.13×10^{-7}	4.09×10^{-7}

And the next condition are necessary for the analyze of the model:

$$K_m = K_e$$
$$T_a = \frac{L_a}{R_a}$$
$$T_m = JR_a/(K_e * K_m)$$

With the last algebraically expression and the table of parameters is possible analyze the behavioral of motors using the equation (4).

4 Implementation

So that in the next code is implemented the equation (4) and analyzed the response with a step, impulse, the bode diagram in the different cases and the Nyquist diagram:

Listing 1 Code from MATLAB

```
clear all; close all; clc;

%% Parameters of EC and DC motors from maxon motor Co.
% A-max 22 ?22 mm, Graphite Brushes, 6 Watt
% EC 90 flat ?90 mm, brushless, 260 Watt
Ra = 14; % Terminal resistance
La = 0.891; % Terminal inductance mH
Ke = 17.1e-3; % invers of Speed constant v/rad/s
Km = Ke; % Torque constant Nm/A
J = 4.13e-7; % Rotor inertia kgm^2
Ta = La/Ra; % Mechanical time constant s
Tm = J*Ra/(Ke*Km); % Mechanical time constant s
%% Transfer function
DC_Motor = tf([1/Ke], [Tm*Ta Tm 1])
%% graphs
% Frequency characteristics
figure(1);
```

```
nyquist(DC_Motor);
figure(2);
bode(DC_Motor,{10, 1000});
grid on;
 % Step responses
figure(3);
Tend = 0.2;
step(DC_Motor, Tend);
grid on;
figure(4);
impulse(DC_Motor, Tend);
grid on;
```

5 Results

In this section is presented the results from the experiment the linear system.

5.1 Nyquist Diagram

The Nyquist diagram help to show the stability from the system then we need consider the point (-1,0) to have a perspective about the stability from the system, so that in the figure 1 is presented the three cases proposed. Is observable how this point is not in the area from the curve, then the means is the system don't is stable with a open loop.

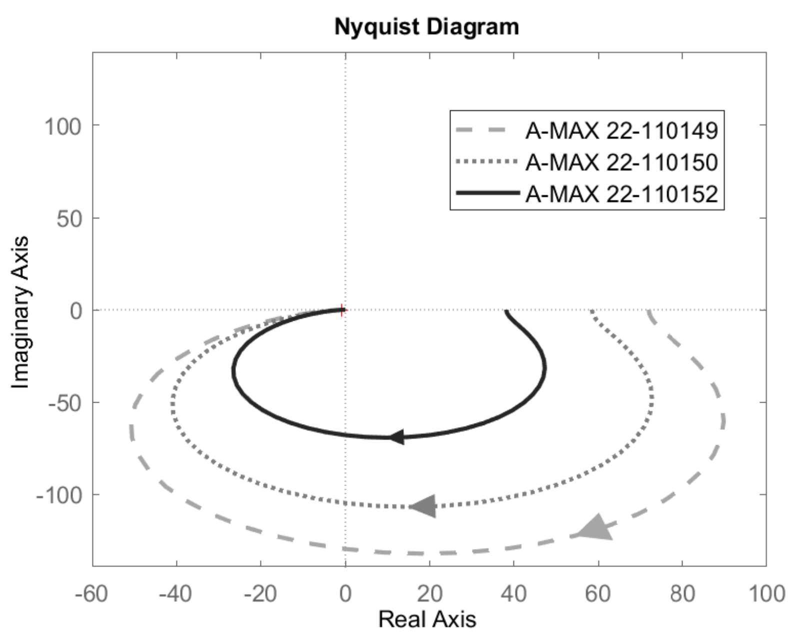

Fig. 1 Nyquist diagram for the system.

5.2 Bode diagram

In this case in the figure 2 Bode helps to provide a quickly stability criteria,so that is necessary observe how to *motor*1 the phase margin is 3.77 $[s]$, to *motor* the phase margin is similar to 3.77 *deg* and to motor3 the phase margin is to the same case similar to 3.77*deg* another important information is how the phase never crosser with 180 deg and the means is the gain margin is too high, so that with this graphic is necessary use root locus in order to optimize the stability from the system.

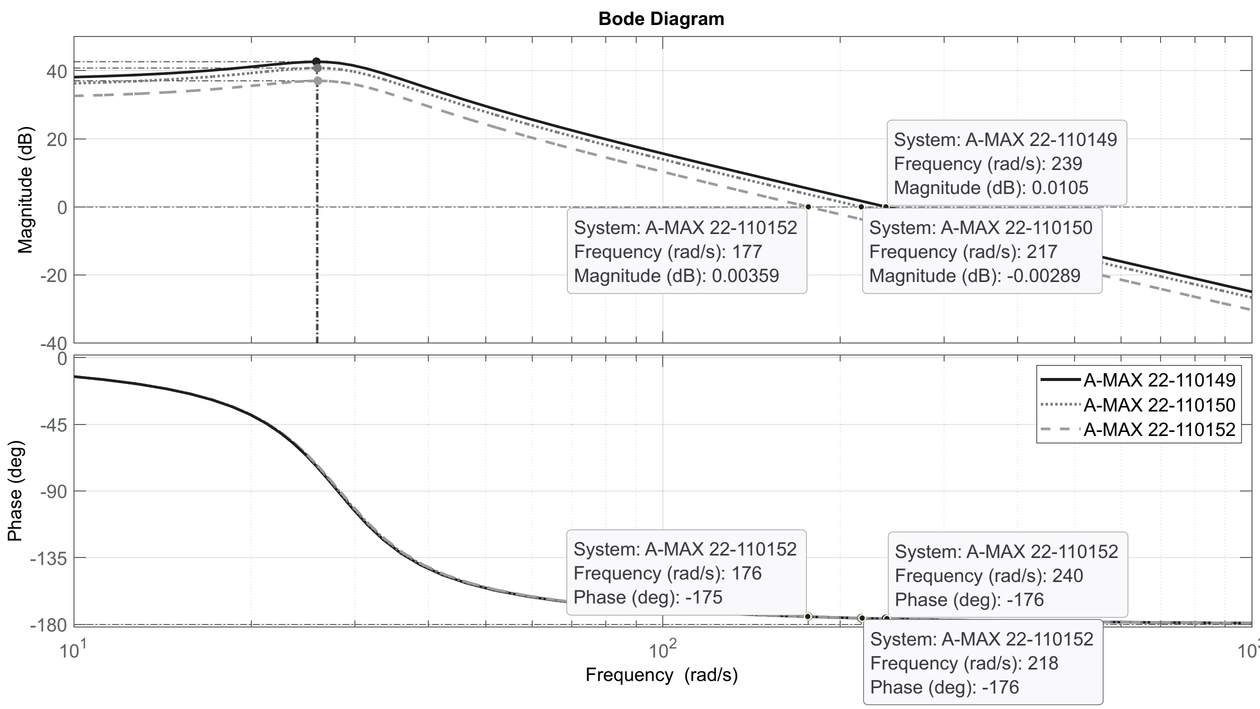

Fig. 2 Bode diagram for the system.

5.3 Impulse Response

The impulse response or Dirac Delta Input helps to understand the behavior from the system with a high and quickly excitation then in this case the graph show how to the case from *motor*2 the system is unstable before 600-1400 of amplitude which means in a real system can be translated to instability or failures by the high amplitude and the over damping, then in the case form *motor*1and *motor*3 the answer have a high setting time and in a personal observation this type of signal not necessarily good in real time systems, so is necessary a synchronization.

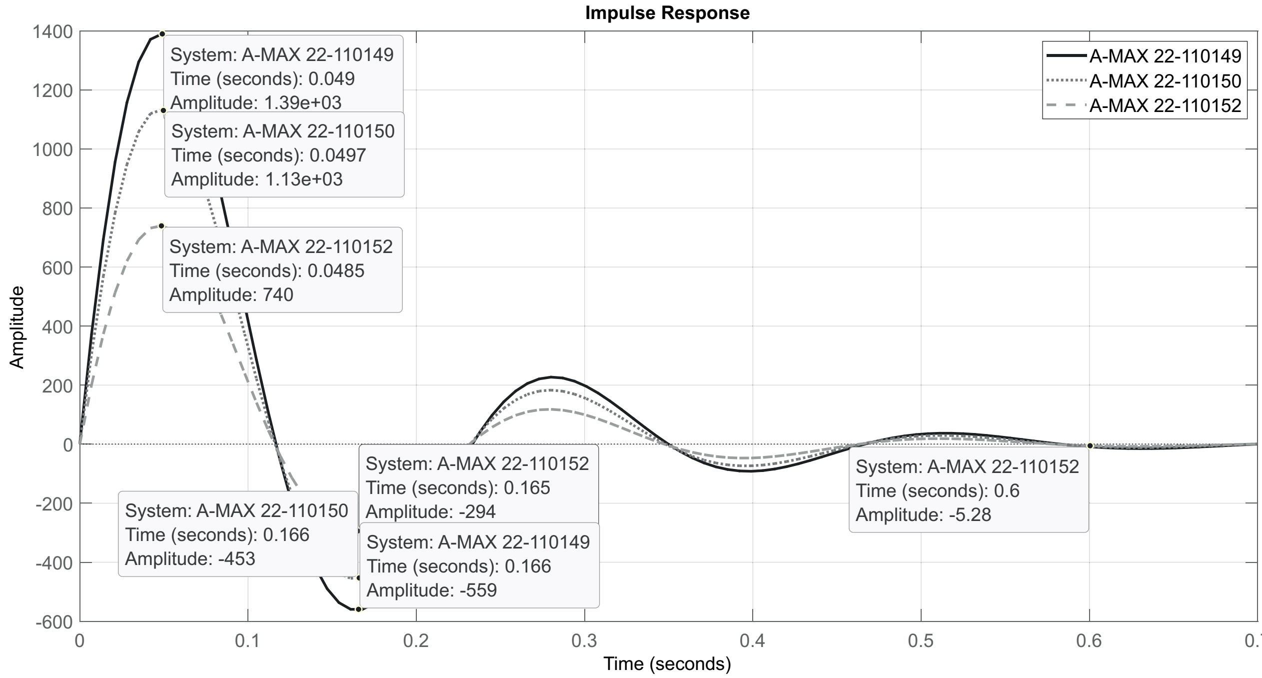

Fig. 3 Impulse response.

5.4 Step Response

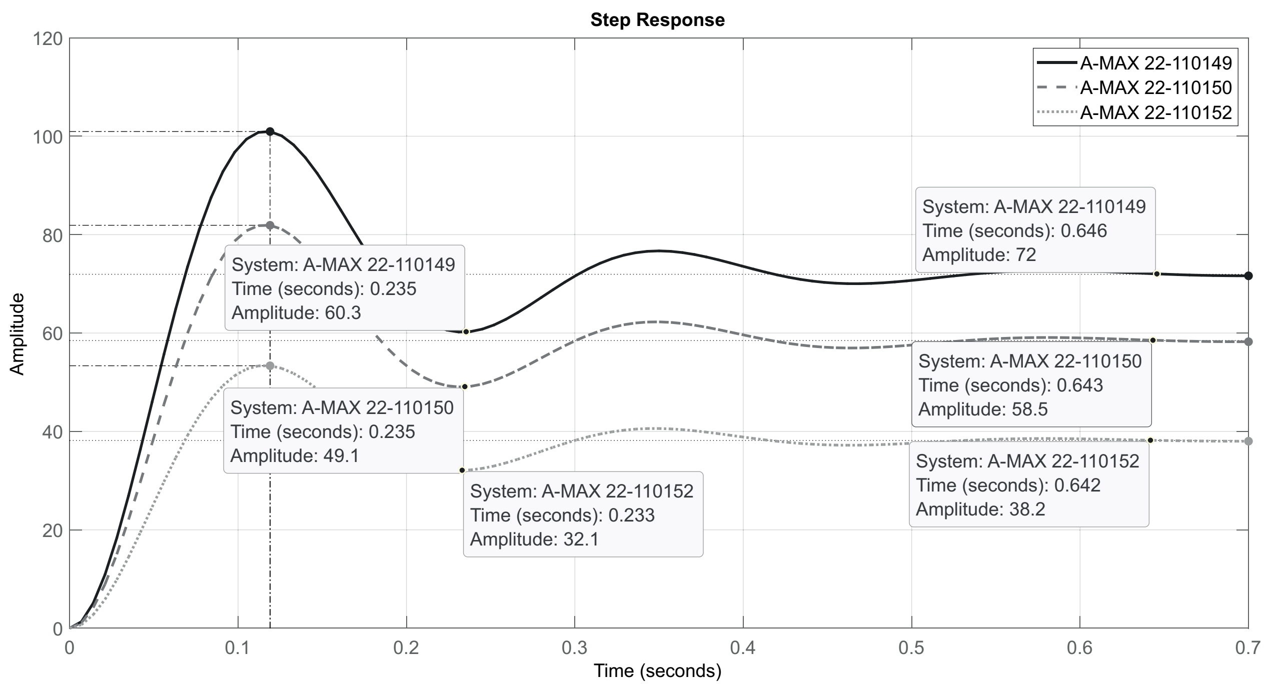

Fig. 4 Step response.

The step response to the system have a interesting answer, the most relevant response is from the case $motor11$ the overshoot is not too high respect with the other cases,but the rise time could be better, in the case from $motor2$ and $motor3$ the problems is a high overshoot and a high rise time.

6 Conclusion

To conclude the response of the three previously proposed cases is observed, then according to the stability and response graphs of the system for an impulse signal and step type it is necessary to perform a closed loop and tuning to optimize the response of the system in this way this laboratory served to observe the behavior of open loop systems and in the same way in the future it is necessary to apply a control system to have a better system response