**Exercise 1** Given a two-input, two-output system with transfer function matrix

$$G = \begin{bmatrix} \frac{k_1}{T_1 s+1} & -\frac{0.05}{0.1s+1} \\ \frac{0.1}{0.3s+1} & \frac{k_2}{T_2 s-1} \end{bmatrix}$$

where the coefficients $k_1$ and $k_2$ have nominal values 12 and 5, respectively, and relative uncertainty 15 %, and the time constants $T_1$ and $T_2$ have nominal values 0.2 and 0.7, respectively, and relative uncertainty 20 %.

(a) Build uncertainty model of the given system;
(b) design a loop shaping controller that fulfills the following requirements:

- robustness requirements: roll off $-20$ dB/decade and attenuation of $-20$ dB for the frequency 100 rad/s,
- performance requirements: maximize $1/\sigma(S)$ in the low frequency range;

(c) obtain the singular value plot of the open-loop system and compare with the desired frequency response;
(d) obtain the singular value plots of the sensitivity matrix and complementary sensitivity function of the uncertain closed-loop system;
(e) obtain the transient response of the uncertain closed-loop system.

**Exercise 2** For the system from Exercise 11.1 do the following.

(a) Design a mixed sensitivity $\mathcal{H}_\infty$ controller with performance and robustness bounds specified by

$$W_1 = \frac{s+10}{2s+0.3}, \qquad W_3 = \frac{s+10}{0.05s+20}$$

(b) obtain the singular value plots of the sensitivity matrix and complementary sensitivity matrix and compare them with the performance and robustness bounds;
(c) obtain the singular value plots of the sensitivity matrix and complementary sensitivity matrix of the uncertain closed-loop system;
(d) obtain the transient responses of the uncertain closed-loop system.

**Exercise 3** For the system from Example 12.1 do the following.

(a) Design a $\mu$-controller minimizing the performance index

$$\left\| \begin{bmatrix} W_p S G \\ W_u K S G \end{bmatrix} \right\|_\infty$$

where

$$W_p = \begin{bmatrix} w_p & 0 \\ 0 & w_p \end{bmatrix}, \qquad W_u = \begin{bmatrix} w_u & 0 \\ 0 & w_u \end{bmatrix}$$

and

$$w_p = \frac{0.95(s^2 + 2000s + 4000)}{s^2 + 1900s + 10}, \qquad w_u = \frac{10^{-6}(0.1s + 1)}{0.001s + 1}$$

(b) Analyze the robust performance of the closed-loop system.

(c) Obtain the singular value plots of the sensitivity function and complementary sensitivity function of the uncertain closed-loop system.

(d) Obtain the transient response of the uncertain closed-loop system.