**Exercise 1** For a two-channel system with plant and controller transfer matrices

$$G = \begin{bmatrix} \frac{1}{s-1} & 0 \\ 0 & \frac{1}{s+1} \end{bmatrix}, \qquad K = \begin{bmatrix} \frac{s-1}{s+1} & 1 \\ 0 & 1 \end{bmatrix}$$

obtain the sensitivity matrices $S_i$, $S_o$, complementary sensitivity matrices $T_i$, $T_o$ and loop transfer matrices $L_i$, $L_o$ and compute the singular value plots of the corresponding frequency responses.

**Exercise 2** For the system with plant and controller transfer matrices

$$G = \begin{bmatrix} \frac{2}{0.015s^2+0.8s+1} & \frac{0.02}{0.3s+1} & \frac{-0.08}{0.4s+1} \\ \frac{0.04}{0.1s+1} & \frac{1}{0.04s^2+1.2s+1} & \frac{-0.03}{s+1} \end{bmatrix}$$

$$K = \begin{bmatrix} \frac{20(0.3s+1)}{0.1s+1} & 0 \\ 0 & \frac{30(0.5s+1)}{0.2s+1} \\ \frac{s+1}{0.2s+1} & \frac{-0.4}{0.5s+1} \end{bmatrix}$$

obtain the closed-loop transient responses due to step reference $r$ and step input and output disturbances $d_i$, $d$.

**Exercise 3** For the system in the previous problem analyze the influence of controller elements $K_{11}$ and $K_{22}$ gains on the closed-loop frequency responses and transient responses. What is the influence of these gains on the magnitude of the steady-state errors in both channels?

**Exercise 4** Build an uncertain model for the system

$$G = \begin{bmatrix} \frac{K}{10s+1} \\ \frac{1}{Ts+1} \end{bmatrix}$$

where the nominal values of the parameters $K$, $T$ are 5 and 0.5, respectively, and the uncertainty in $K$ is $\pm 30$ %, and in $T$—$\pm 10$ %.

**Exercise 5** Given is a two-input/two-output plant with transfer function matrix

$$G = \begin{bmatrix} \frac{k_1}{T_1 s+1} & \frac{k_2}{T_2 s+1} \\ \frac{k_3}{T_3 s+1} & \frac{k_4}{T_4 s-1} \end{bmatrix}$$

The nominal parameter values are $k_1 = 7.2$, $T_1 = 0.9$, $k_2 = -3$, $T_2 = 1.2$, $k_3 = 2$, $T_3 = 3$, $k_4 = 5$ and $T_4 = 0.7$, the relative parameter uncertainty being 45 %.

The controller transfer matrix is

$$K = \begin{bmatrix} \frac{10(s+1)}{0.3s+1} & 0 \\ 0 & \frac{15(s+2)}{s+1} \end{bmatrix}$$

Analyze the robust stability of the closed-loop system using the function `robuststab`.

**Exercise 6** Given is a two-input/two-output plant with nominal transfer function matrix

$$G = \frac{1}{s^2 + 2s + 4} \begin{bmatrix} -s+2 & 2s+1 \\ -3 & -s+2 \end{bmatrix}$$

and controller

$$K = \frac{2(s+2)(s^2+2s+4)}{s(s+1)(s^2+2s+7)} \begin{bmatrix} -s+2 & -2s-1 \\ 3 & -s+2 \end{bmatrix}$$

The plant has input uncertainties that do not exceed 2 % in the low frequency range, increasing gradually to 100 % at frequency 10 rad/s and reaching 200 % in high frequency range.

Construct a plant model with input multiplicative uncertainty and analyze the robust stability of the closed-loop system.

**Exercise 7** For the systems presented in Exercises **5** and **6** , analyze the closed-loop robust performance using the function `robustperf`.

**Exercise 7** Given is a plant with uncertain parameters and input uncertainty whose transfer function is

$$G = \frac{k}{Ts + 1}$$

The gain $k$ and time constant $T$ have nominal values 1 and 2, respectively, and relative uncertainty 25 %. The input plant uncertainty is 5 % in the low frequency range, increasing gradually to 100 % at frequency 2.5 rad/s and reaching 500 % in the high frequency range.

The controller transfer function is

$$K = \frac{k_c(T_c s + 1)}{T_c s}$$

where $k_c = 0.15$, $T_c = 0.4$.

(a) Construct a plant model representing the input uncertainty as a multiplicative uncertainty.
(b) Determine the worst case gain of the closed-loop system by using the function `wcgain`;
(c) Obtain the magnitude responses and transient responses of the closed-loop system for the nominal gain and worst-case gain.
(d) Obtain the magnitude responses and transient responses of the closed-loop system for certain number of random uncertainty values and compare them with the corresponding responses for the worst-case gain.