Geopotential

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This document describes the computations that are performed by the method `GeneralSphericalHarmonicsAcceleration` of class `Geopotential` to the determine the acceleration exerted by a non-spherical celestial on a point mass.

Let $\boldsymbol{r}$ be the vector going from the centre of the celestial to the point mass. Let $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ be a (direct) base whose $\boldsymbol{z}$ axis is along the axis of rotation of the celestial and whose $\boldsymbol{x}$ axis points toward a reference point on the celestial. In this base $\boldsymbol{r}$ has coordinates (x, y, z) which can be expressed in terms of the latitude β and the longitude λ:

$$\boldsymbol{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} r\cos\beta\cos\lambda \\ r\cos\beta\sin\lambda \\ r\sin\beta \end{pmatrix}$$

where r is the norm of $\boldsymbol{r}$.

The gravitational potential exerted by the celestial has the form:

$$U(\boldsymbol{r}) = -\frac{\mu}{r}\left(1 + \sum_{n=1}^{\infty}\sum_{m=0}^{n}\left(\frac{R}{r}\right)^{n} P_{mn}(\sin\beta)(C_{mn}\cos m\lambda + S_{mn}\sin m\lambda)\right)$$

where P_{mn} is the associated Legendre function defined as:

$$P_{mn} = (1-t^2)^{\frac{m}{2}}\frac{\mathrm{d}^m P_n(t)}{\mathrm{d}t^m}$$

The terms that are relevant for evaluating the acceleration exerted by the spherical harmonics on the point mass have the form:

$$V_{nm}(\boldsymbol{r}) = \frac{1}{r}\left(\frac{R}{r}\right)^{n} P_{mn}(\sin\beta)(C_{mn}\cos m\lambda + S_{mn}\sin m\lambda)$$

and the acceleration itself is proportional to $\nabla V_{nm}(\boldsymbol{r})$.

$V_{nm}(\boldsymbol{r})$ can be written as the product of three terms dependent of r, β, and λ, respectively:

$$V_{nm}(\boldsymbol{r}) = \mathfrak{R}(r)\mathfrak{B}(\beta)\mathfrak{L}(\lambda)$$

where:

$$\begin{cases} \mathfrak{R}(r) & = \frac{1}{r}\left(\frac{R}{r}\right)^{n} \\ \mathfrak{B}(\beta) & = P_{mn}(\sin\beta) \\ \mathfrak{L}(\lambda) & = C_{mn}\cos m\lambda + S_{mn}\sin m\lambda \end{cases}$$