Advanced Math Week II: De Moivre's Theorem and Functions of Complex Numbers (Part I)

Jie Young
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Main Objectives

- Understand De Moivre's Theorem
- Understand Proof By Induction
- Solve problems Regarding De Moivre's Theorem
- Apply Euler's Formula and De Moivre's Theorem

De Moivre's Theorem

$$For\ Z^n,$$
$$z^n = (a+bi)^n$$
$$= r^n(\cos(n\theta) + i\sin(n\theta))$$
$$= r^n e^{i(n\theta)}$$

Proof By Induction

- Proof of every integer can be justified

Steps:

- Prove it's true for n=1
- Assume it's true for n=k
- Prove it's true for n=k+1 (More proofs at new paper)

Functions of Euler's Formula

$$e^{ix}e^{iy} = e^{(x+y)i}$$
$$= [\cos(x+y) + i\sin(x+y)]$$
$$e^{ix}e^{-iy} = e^{(x-y)}$$
$$= [\cos(x-y) + isin(x-y)]$$

Which we can solve further by using Trigonometric Identities:

$$\sin(x \pm y) = sinxcosy \pm cosxsiny$$
$$\cos(x \pm y) = cosxcosb \mp sinxsiny$$

Pascal Triangle

Usage:Binomial Expansion

for $(a+b)^n, n \in Z^+$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

Example:

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

How to use Pascal Triangle for Binomial Expansion:

1. Identify the power
2. Count from top(Starts with 0) and stop at the level of your power.
3. The value of the level will be the coefficient of your equation
4. Starts with a with the highest power and b with power of 0. Descend the power of a while increase b until b reaches highest power

Lists of Formulas

$$e^{inx} = \cos(nx) + i\sin(nx)$$

$$e^{inx}\text{-}e^{inx} = 2i\sin(nx)$$

$$e^{-inx} = \cos(nx)\text{-}i\sin(nx)$$

$$e^{inx} + e^{inx} = 2\cos(nx)$$