Advanced Math II W1: Complex Numbers and Euler's Formula

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Learning Objectives

- To find the Definition, Arithmetic Operations and Algebraic Operations of Complex Numbers
- To Understand and apply methods to find the Roots of Complex Numbers
- To Understand Polar Form of Complex Numbers and Perform Multiplication and Division
- To Apply Euler's Formula and understand the concept of modulus and arguments
- To Perform Multiplication and Division of Complex Numbers in Exponential Form

Complex Numbers

$z = a + bi$
$where,$
$r = \sqrt{a^2 + b^2}$
$\theta/\arg(z) = tan^{-1}(\frac{b}{a})$

$\#Quadratic\ roots\ when$
$b^2 - 4ac < 0$

Euler's Formula

$z = re^{i\theta}$
$re^{i\theta} = r(cos\theta + i\ sin\theta)$
$where,$
$r = modulus$
$\theta = argument$

Finding roots of Complex Number

Key Techniques:

- Perform square roots on x $+yi$, where you will sub in your complex number, a + bi based on real and imaginary component.
- The real component is usually $x^2 - y^2$,and the imaginary component is usually 2xy.
- Get your y value in the imaginary component, and sub into it to do the system of equation in two variables.
- You will form a quartic equation, which you will turn into two quadratic equation
- As X and Y must be real number, only use the equation that returns real number
- Use the x value you get to sub into the equation where you derived y value based on x from the imaginary component, and it will be the imaginary component
- X will be real component and y will be imaginary component of the roots.

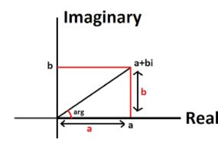
Example：

$(x + yi)^2 = 5 + 12i$
$x^2 + 2xyi - y^2 = 5 + 12i$
$x^2 - y^2 = 5\ - (1)$
$2xyi = 12i\ - (2)$
$(2): y = \frac{6}{x}$
$x^2 - \frac{6^2}{x} = 5$
$x^4 - 36 = 5x^2$
$x = 3, -3$
$y = 2, -2$
$z = 3 + 2i, -3 - 2i$

Relationship between Argand Diagram

$$z = a + bi$$

In imaginary Plane, a will be the x axis and b will be the y axis.



Polar form is converted based on The argand Diagram with Argument(Angle) derived from arctan and Modulus(r) derived based on distance between two points.

Same thing can be imposed to Exponential form by substituting into the correct positions

Multiplication and Division of Polar Number

Multiplication:
$z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$

Division:
$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$

Arithmetic of Imaginary Number

$i^0 = 1$	$i^1 = i$	$i^2 = -1$	$i^3 = -i$
$i^4 = 1$	$i^5 = i$	$i^6 = -1$	$i^7 = -1$

Polar form of Complex Numbers

$$z = r(cos\theta + i\ sin\theta)$$