

MATH III WK II: MUTUALLY EXCLUSIVE EVENTS AND CONDITIONAL PROBABILITY

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Main Objectives

- Understand Mutually Exclusive Events
- Interpret Addition rule of Mutually Exclusive Events with Venn Diagram
- Understand Conditional Probability
- Apply problems

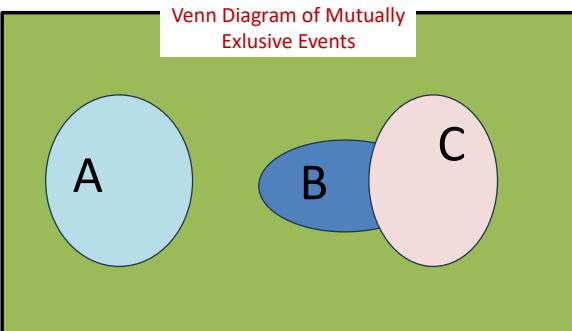
Mutually Exclusive Events

Event that can't happen at the same time

Conditional Probability

Probability of Event A happening given Event B happened
Ex: Odds of rain if ground is wet

Venn Diagram of Mutually Exclusive Events



A and C: Mutually Exclusive

B and C: Not Mutually Exclusive

$$P(A \cup C) = P(A) + P(C)$$

Conditional Probability

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Where $P(B|A)$ is B given A

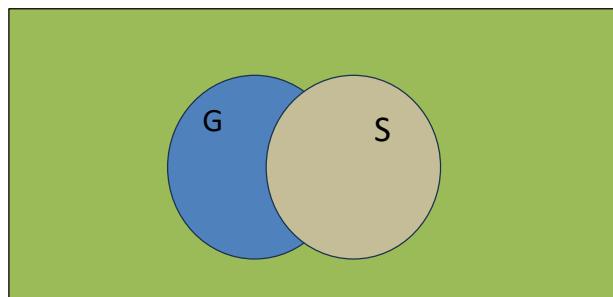
All the students in a class of 30 study at least one of the subject, Geography and Science. Of these, 20 study Geography and 21 study Science. Find the probability that a student chosen at random

(a) studies both Geography and Science,

$$\begin{aligned} n(G) &= 20 \\ n(S) &= 21 \\ n(G \cup S) &= 30 \\ n(G \cup S) &= n(G) + n(S) - n(G \cap S) \\ n(G \cap S) &= 20 + 21 - 30 = 11 \\ P(G \cap S) &= \frac{11}{30} \end{aligned}$$

(b) studies only Geography,

$$\begin{aligned} n(G \cup S) - n(S) &= n(G) \\ n(G) &= 30 - 21 = 9 \\ P(G) &= \frac{9}{30} \end{aligned}$$



Examples

(c) studies Geography, given that the student studies Science,

$$\begin{aligned} P(G|S) &= \frac{P(S \cap G)}{P(S)} \\ P(B|A) &= \frac{11}{30} = \frac{11}{21} \end{aligned}$$

(d) studies Science, given that the student studies Geography

$$\begin{aligned} P(S|G) &= \frac{P(G \cap S)}{P(G)} \\ P(B|A) &= \frac{11}{20} = \frac{11}{30} \end{aligned}$$

A machine shop has an experienced machinist and an apprentice. The experienced machinist who is faster and more skilful, produced 10 items of which only one is defective while the apprentice produced 8 items of which 2 are defective. Unaware of this, a buyer randomly selects one of these items. What is the probability that the item is machined by the apprentice?

$$\begin{aligned} P(A) &= \frac{8}{18} \\ P(A) &= \frac{4}{9} \end{aligned}$$

If the item is found to be defective, what is the probability that it is machine by the apprentice?

$$P(D|A) = \frac{2}{3}$$