MATH II Lecture Week I-III: Integrals, Substitutions, Integrals by Parts

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Main Objectives

- To understand Integration as an Anti-derivative
- To acknowledge Indefinite Integrals and Standard Integrals
- To understand and apply Integration by Substitutions
- To understand and apply Integration by Parts

Integrals as Anti-derivatives

$$if \ \frac{dy}{dx}F(x) = f(x),$$

$$then \int f(x)\,dx = F(x) + C$$

$$Where \int f(x)\,dx \ is\ an\ Indefinite\ Integral$$

U-Substitutions

Key Solving Techniques:

- Identify two parts
- Choose the part that is easier to differentiate as u
- Differentiate u as du/dx and move the dx to the right
- Move that du = u’ dx into the equation so it can substitute dx
- Occasionally dx = du/F(x)

Basic Rules of Integration

$$\int n dx = nx + C \qquad \int \frac{1}{x} dx = \ln|x| + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \qquad \int e^x dx = e^x + C$$

$$\int kf(x)dx = k\int f(x)dx \qquad \int a^x dx = \frac{a^x}{ln\ a} + C$$

Integration by Parts

Key Solving Techniques:

- Identify two Parts
- Choose the part that is easier to differentiate as u, easier to be integrated as dv
- Turn u to du = u’ dx and Integrate dv to v
- Sub into the formula
- $\int u\,dv = uv - \int v\,du$

Examples

U-Substitutions

$$\int 3e^{3x}\,dx$$

$$u = 3x$$

$$du = 3\,dx$$

$$\int e^u\,du = e^{3x} + c$$

Integration by Parts

$$\int x^2 e^x dx$$

$$u = x^2$$

$$du = 2x\,dx$$

$$dv = e^x dx$$

$$v = \int e^x dx$$

$$v = e^x$$

$$\int u\,dv = uv - \int v\,du$$

$$\int x^2 e^x dx = (x^2 - 2x + 2)e^x + c$$