

MATH III WK I: COUNTING RULES AND PROBABILITY

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Main Objectives

- Understand Multiplication Rule
- Interpret Permutation arrangement in distinct, repetition and restriction
- Apply Permutation in a circle
- Interpret Combination
- Understand the different between Permutation and Combination

Multiplication Rule

How many category to choose from multiply by how many choices from each category

Permutation

- Number of arrangement of an object (**ORDER IS IMPORTANT**)
- E.g. AB does not equals BA

Combination

- Number of Combination(**ORDER IS NOT IMPORTANT**)
- E.g. AB = BA

Categories of Permutation

- When Numbers are distinct:

$$P = n!$$

- When Choosing r out of n distinct:

$$P = \frac{n!}{(n - r)!}$$

- When Choosing out of repeating:

$$P = \frac{n!}{a! b! c! \dots}$$

Where a,b,c is the number of repeating objects

- n objects in a circle(go both way):

$$P = \frac{(n - 1)!}{2}$$

Combination Formulas

$$C = \frac{n!}{r! (n - r)!}$$

Examples

How many ways can “GLOBE” be arranged?

$$P = 5!$$

$$P = 120 \text{ ways}$$

4 digit pin is selected. What's the probability if repeated is allowed and not allowed

(Repeat allowed)

$$10 \times 10 \times 10 \times 10 = 10000$$

(Repeat not allowed)

$$P = \frac{10!}{(10 - 4)!}$$

$$P = 5040$$

How many ways can “STATISTICS” be arranged?

$$P = \frac{10!}{3! 3! 2!}$$
$$P = 50400$$

8 Members should stand together in a circle. How many arrangement if A and B refuses to stand together?

P = Total – Together

$$P = 7! - \frac{6!}{2} = 2160$$

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Main Objectives

- Understand the theory of probability
- Application on deck of cards
- Two-way table
- Understand probability of compound events

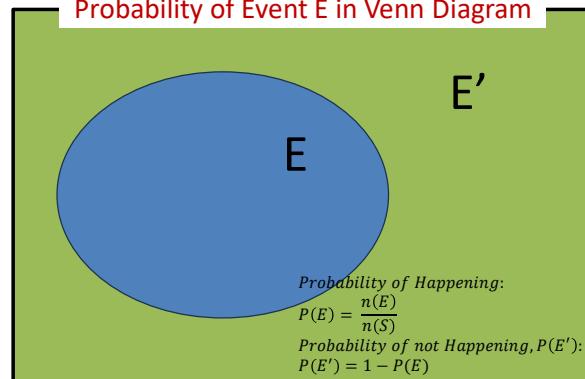
Theoretical Probability

$$P = \frac{\text{Total Number of Success}}{\text{Total Number of Possible Outcome}}$$

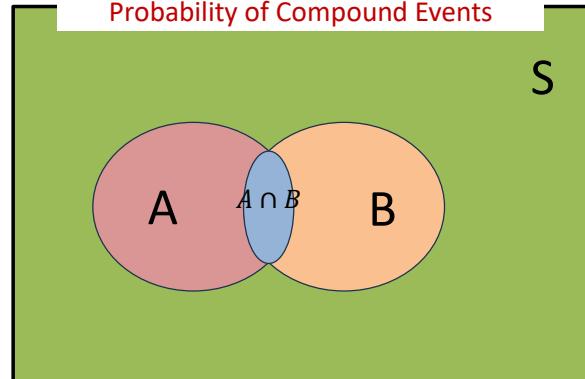
Sample Space

Set of all possible outcome in a probability experiment

Probability of Event E in Venn Diagram



Probability of Compound Events



Examples

5 member is chosen randomly from Groups of 4 boy and 6 girl. Find probability of:
a) 3 Boys and 2 Girls

Combination of choosing 5 member from 10:

$$C = \frac{10!}{5!(10-5)!} = 252$$

Combination of choosing Girls and Boys:

$$C_{\text{girls}} = \frac{6!}{2!4!} = 15 \quad C_{\text{boys}} = \frac{4!}{3!} = 4$$

$$P = \frac{\text{Ways to choose girls and boys}}{\text{ways of choosing 5 members}}$$

$$P = \frac{60}{252}$$

b) More Boys than Girls

More boys than girls: 3B2G, 4B1G

Combination of choosing 4 Boys 1 Girls:

$$C_{4B1G} = \frac{4!}{4!} \times \frac{6!}{1!5!} = 1 \times 6 = 6$$

Total Combination: $60 + 6 = 66$

$$P = \frac{66}{252}$$

c) 3 Boys 2 Girls, and one particular Girl refuses to be with one particular Boy

Combination of choosing 3 Boys and 2 Girls: 60

$$\text{Combination where Girl is with Boy: } \frac{5!}{1!(5-1)!} \times \frac{3!}{2!(3-2)!} = 15$$

Total Combination: $60 - 15 = 45$

$$P = \frac{45}{252}$$

Number is selected from the first 250 Positive int, what's the probability of it being divisible by 2 or 7?

A = Numbers Divisible by 2

B = Numbers Divisible by 7

$$n(A) = 125$$

$$n(B) = 35$$

$$n(A \cup B) = 17$$

Apply Addition Rule of Combined Event:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{125}{250} + \frac{35}{250} - \frac{17}{250} = \frac{143}{250}$$