blatt8 (2)

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Blatt 08 - Praktische Optimierung - Adrian Lentz, Robert Schönewald

Lösungen und Erklärungen für Blatt 08.

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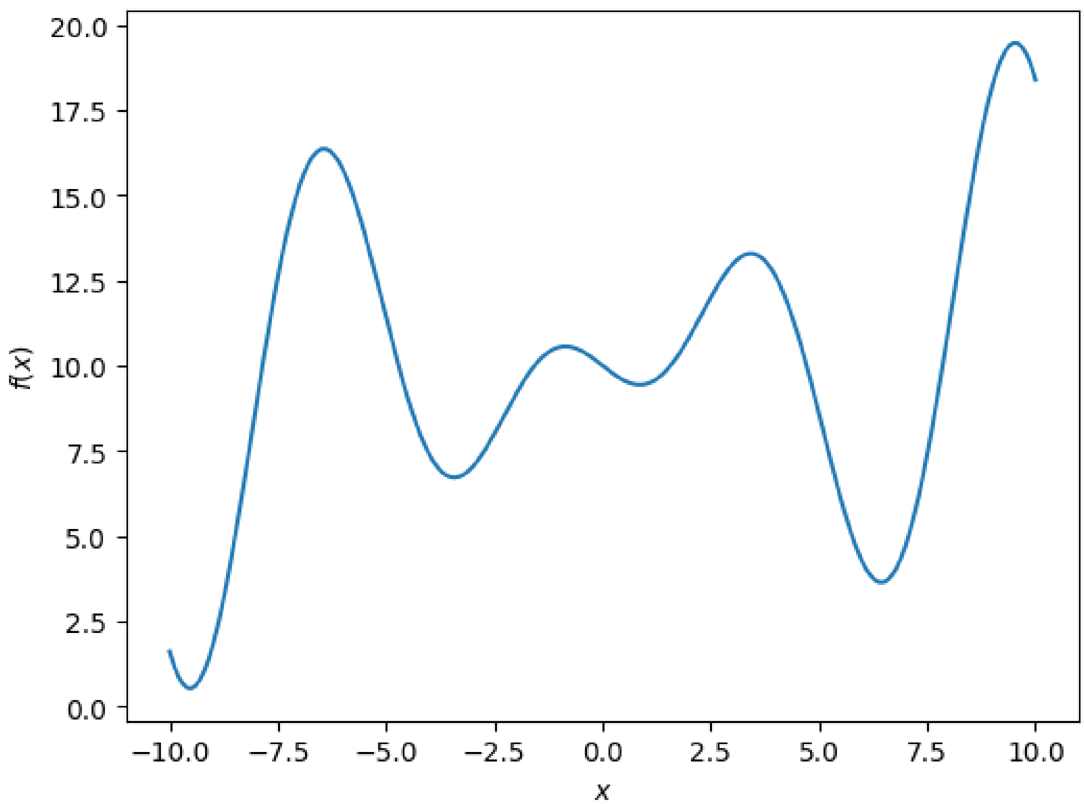
[1]:
```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.pipeline import Pipeline
from sklearn.preprocessing import PolynomialFeatures
from sklearn.linear_model import LinearRegression
from sklearn.metrics import mean_absolute_error, mean_squared_error
from gplearn.genetic import SymbolicRegressor

np.random.seed(0)
```

```
---------------------------------------------------------------------------
ModuleNotFoundError                       Traceback (most recent call last)
Cell In[1], line 7
      5 from sklearn.linear_model import LinearRegression
      6 from sklearn.metrics import mean_absolute_error, mean_squared_error
----> 7 from gplearn.genetic import SymbolicRegressor
      9 np.random.seed(0)

ModuleNotFoundError: No module named 'gplearn'
```

Teil a

[5]:
```
def true_fun(x):
    return 10 - x * np.cos(x)

LOWER = -10
UPPER = 10
```

```
N_Samples = 11
X_samples=np.linspace(LOWER,UPPER,N_Samples)
y_samples=true_fun(X_samples)

X = np.linspace(LOWER, UPPER, 201)  #201 äquidistante Stellen
plt.plot(X, true_fun(X), label="True function")
plt.xlabel("$x$")
plt.ylabel("$f(x)$")

plt.show()
```



Polynomial Modell

[3]:
```
def make_poly_model(f, k, X_samples, y_samples):
    # Erstellung der Polynomialfeatures und Lineare Regression
    polynomial_features = PolynomialFeatures(degree=k, include_bias=True)
    linear_regression = LinearRegression()
    pipe = Pipeline(
        [
            ("polynomial_features", polynomial_features),
```

```
            ("linear_regression", linear_regression),
        ]
    )
    # Modell an den gegebenen Samples fitten
    pipe.fit(X_samples.reshape(-1,1), y_samples.reshape(-1,1))

    return lambda x: pipe.predict(np.array(x).reshape(-1,1)).flatten()


#Test
#pred_fun = make_poly_model(true_fun, k=1) #k=1,2,5,10,15
#pred_fun(1)
```

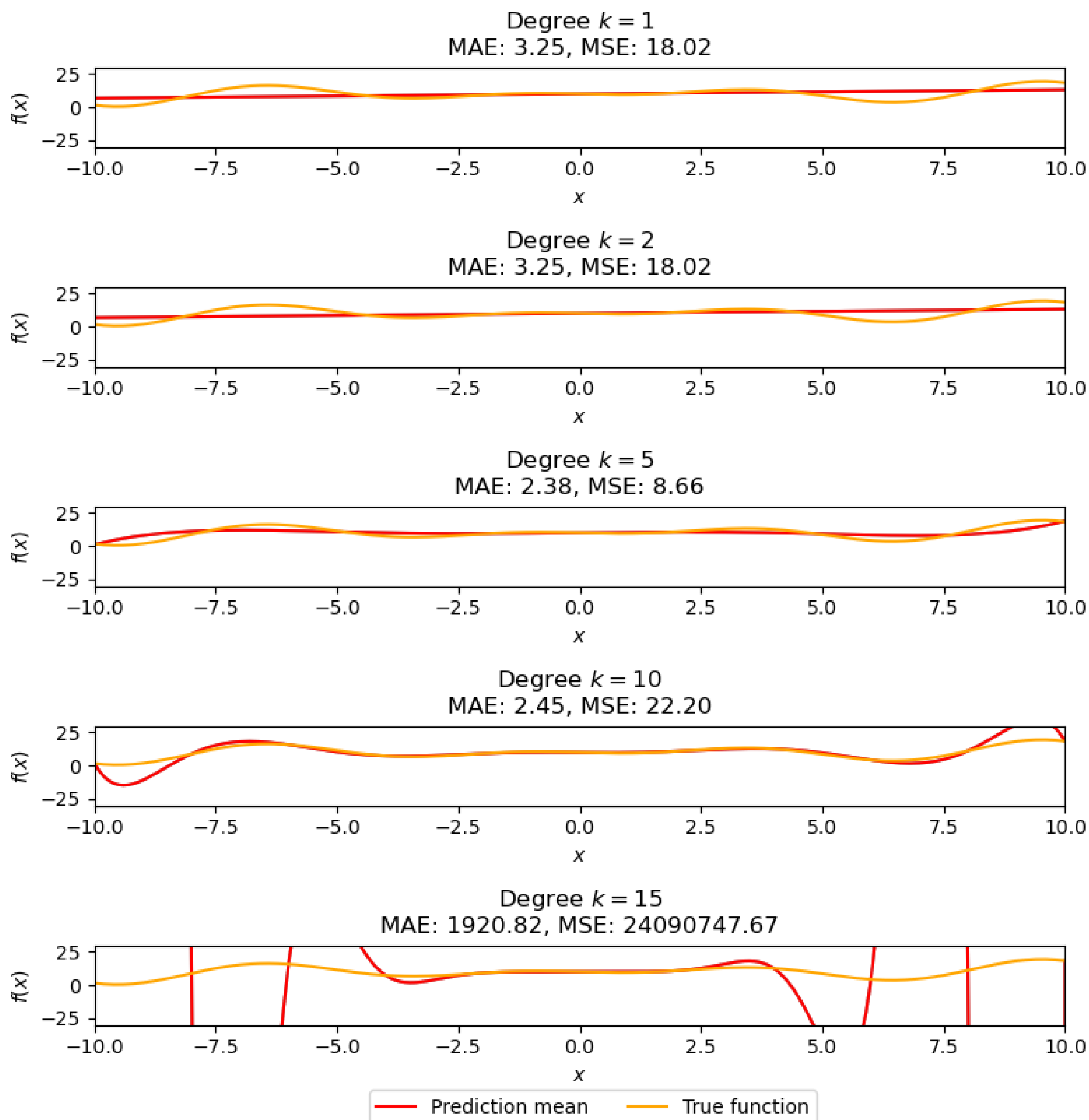
[4]:

```
degrees = [1,2,5,10,15] #k=1,2,5,10,15
NUM_OF_REPEATS = 1

pred_funcs = {}
for degree in degrees:
    pred_funcs[degree]=[]
    for _ in range(NUM_OF_REPEATS):
        pred_funcs[degree].append(make_poly_model(true_fun, k=degree,␣
 ↪X_samples=X_samples, y_samples=y_samples))
```

[5]:

```
fig = plt.figure(figsize=(8, 8))

true_y = true_fun(X)
mean_y_k, vars_y_k = {}, {}

mse_scores, mae_scores = {},{}

for idx,degree in enumerate(degrees):
    plt.subplot(5,1,idx+1)

    # plot predicted functions
    pred_y = np.zeros((NUM_OF_REPEATS,len(X)))

    for i, pred_f in enumerate(pred_funcs[degree]):
        pred_y[i] = pred_f(X)
        plt.plot(X, pred_y[i], c="gray")#, label="Predicted function")

    # calculate mean and variance
    mean_y_k[degree] = np.mean(pred_y, axis=0)
    vars_y_k[degree] = np.var(pred_y, axis=0)

    # calculate MAE and MSE
    mae_scores[degree] = mean_absolute_error(true_y, mean_y_k[degree])
    mse_scores[degree] = mean_squared_error(true_y, mean_y_k[degree])
```

```
    # plot mean prediction
    plt.plot(X, mean_y_k[degree], color="red", label="Prediction mean"\
                                if degree == degrees[0] else None)

    # plot true function
    plt.plot(X, true_y, color="orange", label="True function"\
                            if degree == degrees[0] else None)

    plt.xlabel("$x$")
    plt.ylabel("$f(x)$")
    plt.xlim((LOWER, UPPER))
    plt.ylim((-30,30))
    plt.title(f"Degree $k={degree}$\nMAE: {mae_scores[degree]:.2f}, MSE:␣
 ↪{mse_scores[degree]:.2f}")

fig.legend(loc='lower center', bbox_to_anchor=(0.54, -0.025), ncol=2)

plt.tight_layout()
plt.show()
```

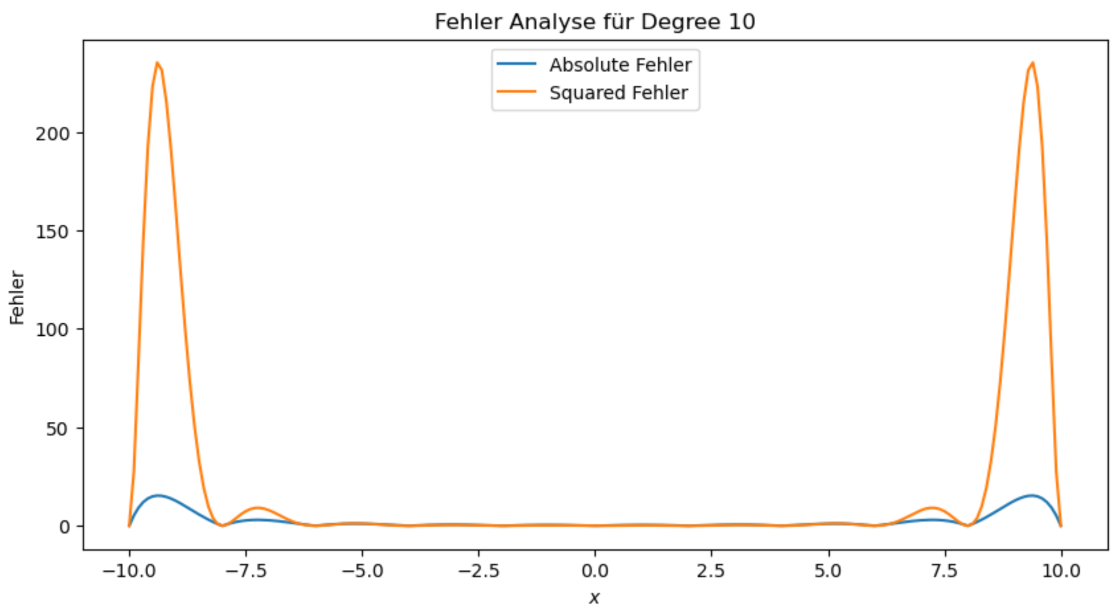


Werte des MSE können sehr stark varrieren, da MSE sehr empfindlich gegenüber Ausreißern ist. MAE (absolute Fehler) ist relativ robust, da nur absoluten Fehler betrachtet –> Außer für k=15 .

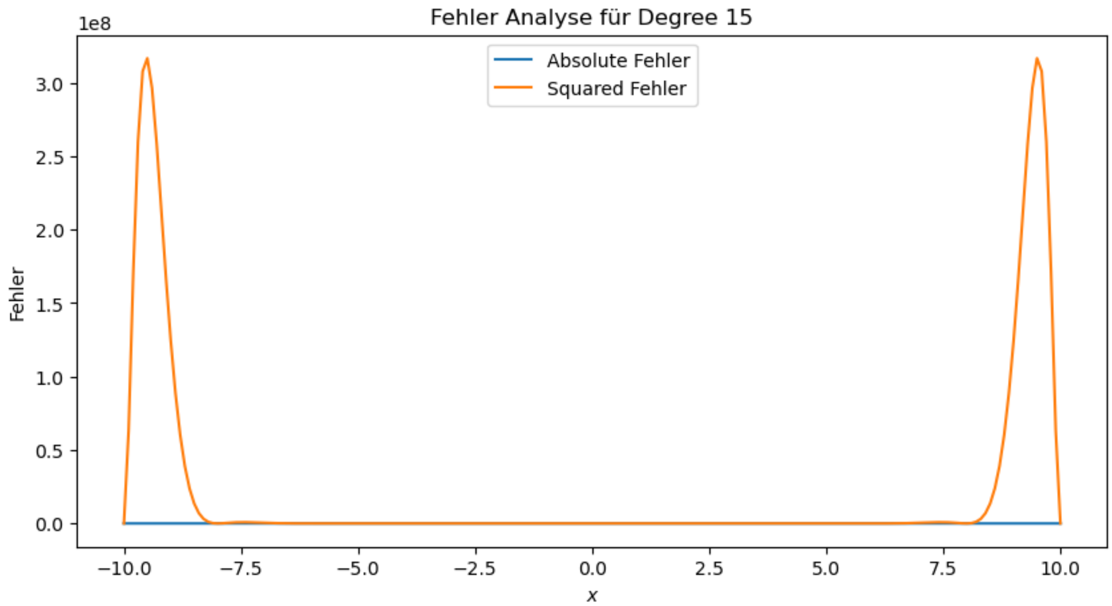
[6]:
```
for degree in [10, 15]:
    errors = np.abs(mean_y_k[degree] - true_y)
    squared_errors = (mean_y_k[degree] - true_y) ** 2

    print(f"Degree {degree}:")
    print(f"Max Absolute Fehler: {np.max(errors):.2f}")
    print(f"Max Squared Fehler: {np.max(squared_errors):.2f}")

    plt.figure(figsize=(10, 5))
```

```python
    plt.plot(X, errors, label="Absolute Fehler")
    plt.plot(X, squared_errors, label="Squared Fehler")
    plt.xlabel("$x$")
    plt.ylabel("Fehler")
    plt.title(f"Fehler Analyse für Degree {degree}")
    plt.legend()
    plt.show()
```

```
Degree 10:
Max Absolute Fehler: 15.33
Max Squared Fehler: 235.16
```




```
Degree 15:
Max Absolute Fehler: 17795.83
Max Squared Fehler: 316691515.42
```



Man erkennt das die größten Fehler in den Randbereichen des Definitionsbereich von x auftreten, wobei insbesondere für k=10 der absolute Fehler noch stark ist, für k=15 sind die Fehler noch deutlich größer, sodass in der Abbildung die Fehler-Skalierung deutlich größer ist ! Sowohl der absolute und der quadratische Fehler sind für k=15 sehr groß.

Kriging

[2]:
```
from sklearn.gaussian_process import GaussianProcessRegressor
from sklearn.gaussian_process.kernels import RBF, Matern, ExpSineSquared
```

[3]:
```
'Kernels'
kernels = [
    RBF(),  #Radiale Basisfunktion
    Matern(nu=0.5), #matern Kernel
    Matern(nu=1.5),
    Matern(nu=2.5),
    ExpSineSquared(periodicity=4), #Exp-sin-Square Kernel
    ExpSineSquared(periodicity=6),
    ExpSineSquared(periodicity=8)
]
```

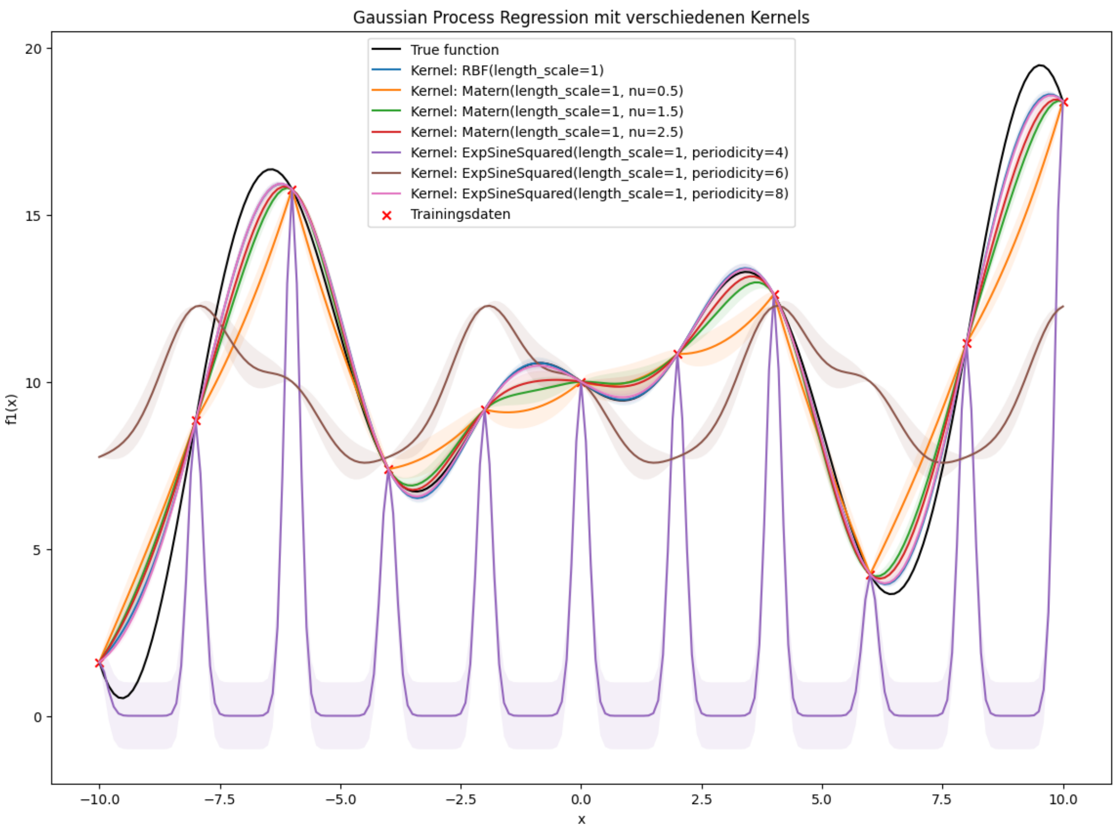
[6]:
```
'Test-Punkte'
X_test = np.linspace(-10, 10, 201).reshape(-1, 1)  # 201 Punkte
y_test = true_fun(X_test)

X_train = np.linspace(-10, 10, 11).reshape(-1, 1)  # 11 äquidistante Punkte
```

```
y_train = true_fun(X_train).ravel()  # Funktion an den Trainingspunkten␣
 ↪auswerten
```

[7]:

```
'Graphische Darstellung der verschiedenen Kernels'

plt.figure(figsize=(14, 10))
plt.plot(X_test, y_test, 'k', label='True function')

for kernel in kernels:
    # Modell initialisieren
    gpr = GaussianProcessRegressor(kernel=kernel)
    # Modell an Trainingsdaten anpassen
    gpr.fit(X_train, y_train)
    # Vorhersagen machen
    y_pred, sigma = gpr.predict(X_test, return_std=True)
    # Ergebnisse plotten
    plt.plot(X_test, y_pred, label=f'Kernel: {kernel}')
    plt.fill_between(X_test.ravel(), y_pred - sigma, y_pred + sigma, alpha=0.1)

plt.scatter(X_train, y_train, c='r', marker='x', label='Trainingsdaten')
plt.title('Gaussian Process Regression mit verschiedenen Kernels')
plt.xlabel('x')
plt.ylabel('f1(x)')
plt.legend()
plt.show()
```

```
c:\Users\Adria\AppData\Local\Programs\Python\Python312\Lib\site-
packages\sklearn\gaussian_process\kernels.py:445: ConvergenceWarning: The
optimal value found for dimension 0 of parameter length_scale is close to the
specified lower bound 1e-05. Decreasing the bound and calling fit again may find
a better value.
  warnings.warn(
c:\Users\Adria\AppData\Local\Programs\Python\Python312\Lib\site-
packages\sklearn\gaussian_process\kernels.py:455: ConvergenceWarning: The
optimal value found for dimension 0 of parameter periodicity is close to the
specified upper bound 100000.0. Increasing the bound and calling fit again may
find a better value.
  warnings.warn(
```



Man erkennt die deutlichen Unterschiede zwischen den verschiedenen verwendenten Kernel Varianten:

Der Exp-Sin-Kernel sind zwei der drei Varianten (p=4,6)(braun,violett) sehr schlecht, in Bezug auf die Funktion (schwarz).

Die Kernel Varianten der radialen Basisfunktionen und des Matern Kernel erzeugen visuell gute Ergebnisse, wobei keine direkten Unterschiede klar zu erkennen sind.

Um eine bessere Bewertung vorzunehmen, werden anschließend die mittlere absolute Abweichung und die mittlere quadratische Abweichung betrachtet, wodurch auch ein Vergleich zum Polynomiellen Modell möglich ist.

[8]:
```
results = {}

# Gaussian Process Modelle trainieren, vorhersagen und Abweichungen berechnen
for kernel in kernels:
    gpr = GaussianProcessRegressor(kernel=kernel)
    gpr.fit(X_train, y_train)
    y_pred, sigma = gpr.predict(X_test, return_std=True)

    mae = mean_absolute_error(y_test, y_pred)
    mse = mean_squared_error(y_test, y_pred)
```

```python
    results[str(kernel)] = {'MAE': mae, 'MSE': mse}
    print(f'Kernel: {kernel}\nMAE: {mae:.4f}\nMSE: {mse:.4f}\n')
```

```
Kernel: RBF(length_scale=1)
MAE: 0.4110
MSE: 0.4527

Kernel: Matern(length_scale=1, nu=0.5)
MAE: 1.2263
MSE: 2.7305

Kernel: Matern(length_scale=1, nu=1.5)
MAE: 0.7384
MSE: 1.0662

Kernel: Matern(length_scale=1, nu=2.5)
MAE: 0.6098
MSE: 0.7885

Kernel: ExpSineSquared(length_scale=1, periodicity=4)
MAE: 7.9667
MSE: 88.7055

Kernel: ExpSineSquared(length_scale=1, periodicity=6)
MAE: 3.4577
MSE: 18.1311

Kernel: ExpSineSquared(length_scale=1, periodicity=8)
MAE: 0.4122
MSE: 0.4359
```

```
c:\Users\Adria\AppData\Local\Programs\Python\Python312\Lib\site-
packages\sklearn\gaussian_process\kernels.py:445: ConvergenceWarning: The
optimal value found for dimension 0 of parameter length_scale is close to the
specified lower bound 1e-05. Decreasing the bound and calling fit again may find
a better value.
  warnings.warn(
c:\Users\Adria\AppData\Local\Programs\Python\Python312\Lib\site-
packages\sklearn\gaussian_process\kernels.py:455: ConvergenceWarning: The
optimal value found for dimension 0 of parameter periodicity is close to the
specified upper bound 100000.0. Increasing the bound and calling fit again may
find a better value.
  warnings.warn(
```

[9]:
```
#Bestes/Schlechtetes  Modell
best_kernel = min(results, key=lambda k: results[k]['MAE'])
worst_kernel = max(results, key=lambda k: results[k]['MAE'])
print(f'Bestes Modell basierend auf MAE: {best_kernel} mit MAE:␣
 ↪{results[best_kernel]["MAE"]:.4f} und MSE: {results[best_kernel]["MSE"]:.
 ↪4f}')
print(f'Schlechtestes Modell basierend auf MAE: {worst_kernel} mit MAE:␣
 ↪{results[worst_kernel]["MAE"]:.4f} und MSE: {results[worst_kernel]["MSE"]:.
 ↪4f}')
```

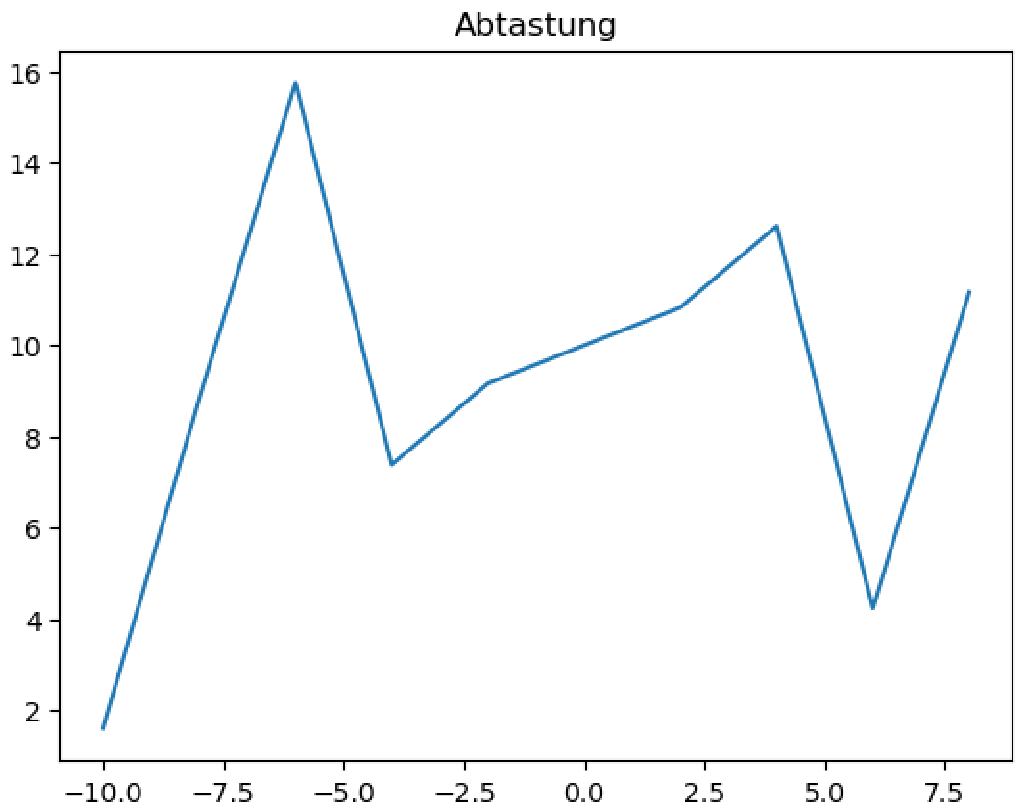
```
Bestes Modell basierend auf MAE: RBF(length_scale=1) mit MAE: 0.4110 und MSE:
0.4527
Schlechtestes Modell basierend auf MAE: ExpSineSquared(length_scale=1,
periodicity=4) mit MAE: 7.9667 und MSE: 88.7055
```

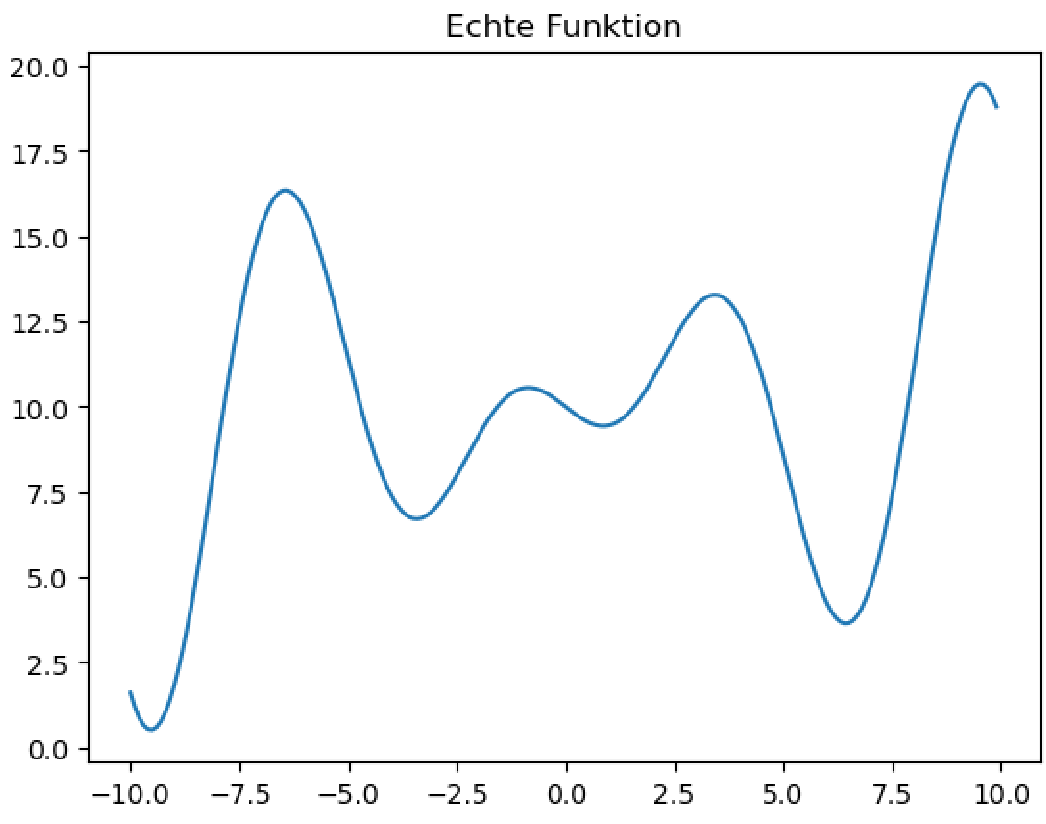
Durch die Betrachtung der mittleren Abweichung bestätigt sich die visuelle Vermutung, der radiale Basisfunktion Kernel schneidet am besten ab, hingegen der Exp-Sin Kernel mit p=4 am schlechtesten ist.

Symbolische Regression

[7]:
```
#ergebnisse=[[3.412096883468112, 3.411384119292495, 3.41139416429113, 3.
 ↪4144380500707427, 3.411589962370803, 3.411587573085578, 3.4123197802444296,␣
 ↪3.4123357068377245, 3.4123352027972116, 3.412225816492165], [3.
 ↪4310083553729425, 3.4461772054621775, 4.210850094556318, 3.4212849478562743,␣
 ↪3.4298376076071366, 3.4150936225491986, 3.438618800945967, 4.
 ↪229564625033519, 3.4233138889140546, 3.426234728198251], [2.569084276409377,␣
 ↪2.8461268152512726, 2.258520156416018, 2.83783902426103, 2.355779892079767,␣
 ↪2.1871537902850764, 2.483166645039068, 2.84198989547309, 2.8443472199543574,␣
 ↪2.847968630694601], [1.4467140095101552, 1.86489255061835, 2.
 ↪0769625127102262, 1.4455295143448579, 1.9123484317260617, 2.845316719842962,␣
 ↪1.8200045343759148, 1.8347978900227104, 2.206795898552612, 1.
 ↪1770640989925316], [9.000275539865292, 9.000322938444684, 9.000000271636575,␣
 ↪9.000026453618377, 9.000003017806371, 9.000010814467721, 9.000038031606294,␣
 ↪9.000035861688184, 9.000000000000002, 9.000041930961835], [2.
 ↪539735054230188, 0.4623621105103869, 3.2901547421202135, 0.8555423450398368,␣
 ↪0.995499502819521, 0.3552372235707458, 1.0729669018864543, 0.
 ↪8366159474659685, 0.4851271015154591, 2.7966886810863514], [0.
 ↪005132790373879794, 0.42489035092205135, 1.042522364710662, 0.
 ↪0022239493565253776, 0.06468732111599763, 0.5306720506911966, 0.
 ↪8386968462634403, 0.5318964825241479, 0.5162446806195534, 0.
 ↪1522034794703366], [1.5678714477165008, 0.23583224526432112, 0.
 ↪6925482369724189, 0.12969092310827274, 1.6591612681054635, 0.
 ↪011107242097641872, 1.133814469470137, 1.5571159174176648, 1.
 ↪9032059512600894, 0.20470373065054973]]
```

[9]:
```
x = np.arange(-10, 10, 2)
x_fine=np.arange(-10, 10, 0.1)
y_truth = 10 - x * np.cos(x)

ax = plt.figure().add_subplot()
plt.plot(x, y_truth)
plt.title("Abtastung")
ax = plt.figure().add_subplot()
plt.plot(x_fine, true_fun(x_fine))
plt.title("Echte Funktion")
plt.show()
```





[10]:
```
# Training samples
X_train = np.arange(-10, 11, 2).reshape(11, 1)
y_train = 10 - X_train * np.cos(X_train)

# Testing samples
X_test = np.arange(-10, 10.1, 0.1).reshape(201, 1)
y_test = 10 - X_test * np.cos(X_test)
```

[11]:
```
combs=[('add','sub'),('mul','div'),('add','mul'),('add','sub','mul','div'),('sin','cos'),('add
ergebnisse=[]
best_gp= SymbolicRegressor(population_size=500, generations=25,␣
 ↪stopping_criteria=0.
 ↪01,verbose=0,function_set=('sub','mul','cos'),n_jobs=-1,tournament_size=10)
best_gp.fit(X_samples.reshape(-1, 1),y_samples)
for y in combs:
    einzelergebnisse=[]
    for x in range(10):
        est_gp = SymbolicRegressor(population_size=500, generations=25,␣
 ↪stopping_criteria=0.01,verbose=0,function_set=y,n_jobs=-1,tournament_size=10)
        est_gp.fit(X_samples.reshape(-1, 1),y_samples)
        #print(est_gp._program.raw_fitness_)
```

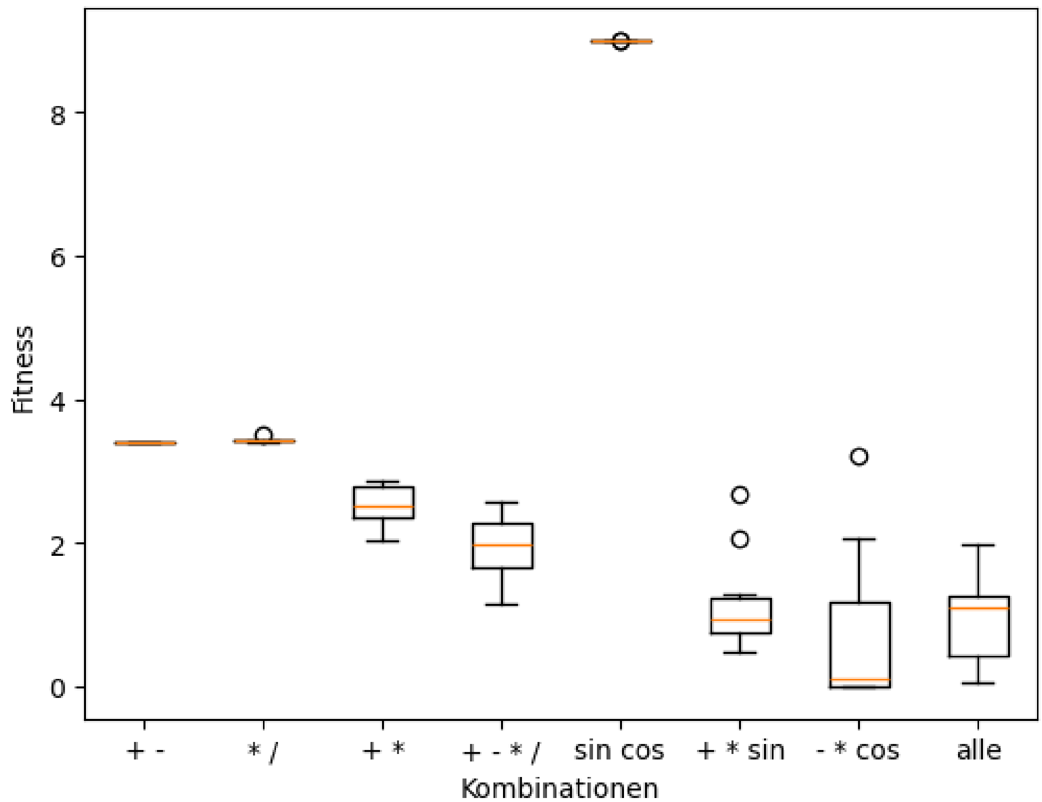
```
        einzelergebnisse.append(est_gp._program.raw_fitness_)
        if est_gp._program.raw_fitness_<best_gp._program.raw_fitness_:
            best_gp=est_gp
    ergebnisse.append(einzelergebnisse)
print(ergebnisse)
```

```
[[3.411708619911212, 3.411531942513665, 3.4136023305596854, 3.4113964343585277,
3.411880289400983, 3.4124747061813423, 3.4121998291115987, 3.413364997392431,
3.411383898561421, 3.4116094848917013], [3.4153887674049845, 3.442384873536167,
3.4352134078684458, 3.414670145568456, 3.418920173232552, 3.5116320663681746,
3.4200534511109066, 3.4384275561038447, 3.4285766773368405, 3.425679310208461 3],
[2.5723045122523973, 2.4280957416247513, 2.8564893337084247, 2.0277275456189163,
2.5361553977047144, 2.1356172803477236, 2.4947945165883922, 2.8418208239116867,
2.840672689295354, 2.3334363609686735], [1.5834125062464546, 2.51416616946715,
1.5097457699292258, 1.912848739555438, 2.349282325314984, 2.569750667191023,
2.0330740100516724, 2.0884158278891283, 1.1625828561599494, 1.9603730747778376],
[9.000005826805717, 9.000020684957786, 9.000051603009597, 9.000191336049134,
9.00004653311491, 9.000284307540108, 9.000009706278716, 9.000090796556465,
9.000012739565573, 9.000014034029956], [1.2767647113027072, 1.0862220481072755,
0.7677375947234069, 0.4832962361945347, 2.0645736736711084, 0.8801010094542672,
2.692766826548169, 0.734351593860925, 0.5594217941791109, 0.9745929500890579],
[0.0044192284898730335, 3.225095038688131, 0.2124658990998437,
0.019967904852415085, 0.00985381334777627, 2.071343789266626,
0.16909248279849365, 0.02987107811987869, 1.5039750904085503,
0.008688813248628267], [1.6126705722605104, 0.0474263866686591,
1.193224551301183, 0.4847290482066993, 1.297089833353825, 1.995552122142604,
1.0509625274290444, 1.1574105051161332, 0.41939507207302357,
0.2916669267763741]]
```

Funktion der Parameter:
Populationsgröße:
Das ist die Anzahl von Programmen, die sich in einer Generation befinden.
Turniergröße:
Die Anzahl an Programmen, die gegeneinander antreten, um Teil der nächsten Generation zu sein.
Generationsanzahl:
Wie viele Generationen durchlaufen werden, bevor gestoppt wird.
Stoppkriterium:
Falls vor Ende des Generationslimits ein bestimmter Wert erreicht wird, wird gestoppt.

[12]:
```
fig, ax = plt.subplots()
ax.boxplot(ergebnisse)
ax.set_xticklabels(['+ -',('* /'),('+ *'),('+ - * /'),('sin cos'),('+ *␣
 ↪sin'),('- * cos'),('alle')])
ax.set(xlabel='Kombinationen', ylabel='Fitness')
```

[12]:
```
[Text(0.5, 0, 'Kombinationen'), Text(0, 0.5, 'Fitness')]
```

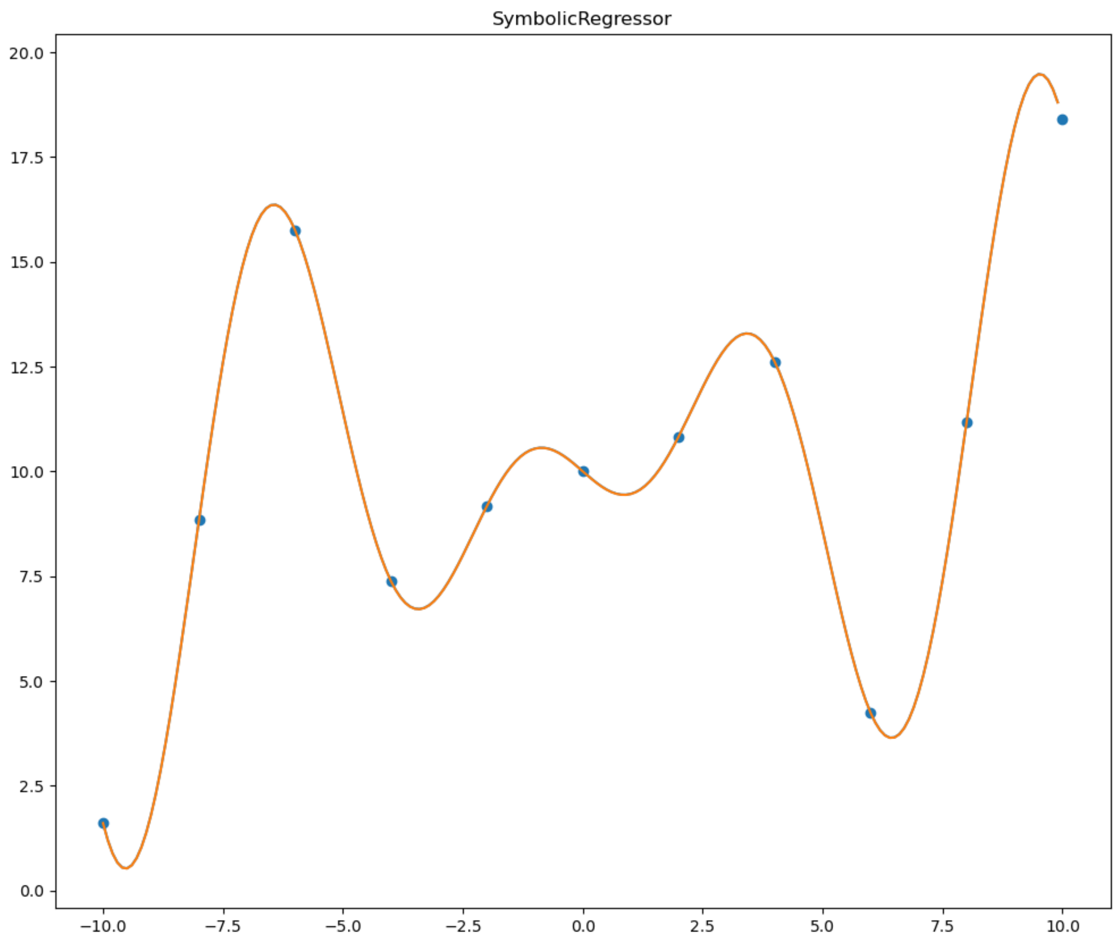


Es scheint die Kombination "- * cos" konstant die beste Fitness zu erreichen.

[13]:
```
print(best_gp._program)
```

```
sub(cos(cos(cos(mul(0.922, -0.248)))), sub(sub(sub(sub(0.122, sub(cos(0.740),
sub(mul(-0.505, sub(cos(-0.022), sub(X0, X0))), sub(sub(sub(0.809, -0.298),
mul(X0, cos(X0))), sub(0.610, 0.793))))), sub(cos(cos(mul(0.922, -0.248))),
sub(sub(sub(sub(sub(sub(cos(-0.148), cos(-0.555)), cos(mul(cos(-0.793),
cos(-0.148)))), 0.793), sub(sub(0.928, X0), sub(-0.708, X0))),
cos(cos(mul(-0.058, 0.772)))), 0.740))), cos(sub(X0, X0))),
cos(cos(cos(mul(0.922, -0.248)))))
```

[16]:
```
y_gp = best_gp.predict(np.c_[x_fine.ravel()]).reshape(x_fine.shape)
score_gp = best_gp.score(X_test, y_test)
fig = plt.figure(figsize=(12, 10))

ax = fig.add_subplot(1, 1, 1)
plt.plot(x_fine, y_gp)
plt.plot(x_fine, true_fun(x_fine))
points = ax.scatter(X_train, y_train)
plt.title("SymbolicRegressor")
plt.show()
```



Beide Kurven überdecken sich, man kann vermuten, dass die Abweichungen sehr gering sein werden.

[22]:
```
yAbw=true_fun(x_fine)
absAbw=np.sum(np.abs(yAbw-y_gp))/201
print(absAbw)
quaAbw=np.sum((yAbw-y_gp)**2)/201
print(quaAbw)
```

```
0.004397242278481326
1.9432418353941956e-05
```

Vergleich der Abweichung

Das polynomielle Modell zeigt eine maximale Abweichung von Max Absolute Fehler: 17795.83 ; Max Squared Fehler: 316691515.42, welches besondes groß sind. Jedoch ist die minimale Abweichung für ein Polynom mit Grad k=5 nur MAE:2,38 und ein MSE: 8,66.

Das Kriging Modell zeigt eine maximale Abweichung von ExpSineSquared(length_scale=1, periodicity=4) mit MAE: 7.9667 und MSE: 88.7055. Die minimale Abweichung ergibt sich für die radiale

Basisfunktion mit MAE: 0.4110 und MSE: 0.4527.

Die Symbolische Regression zeigt eine maximale Abweichung von MAE:0.004397242278481326 und MSE: 1.9432418353941956e-05.

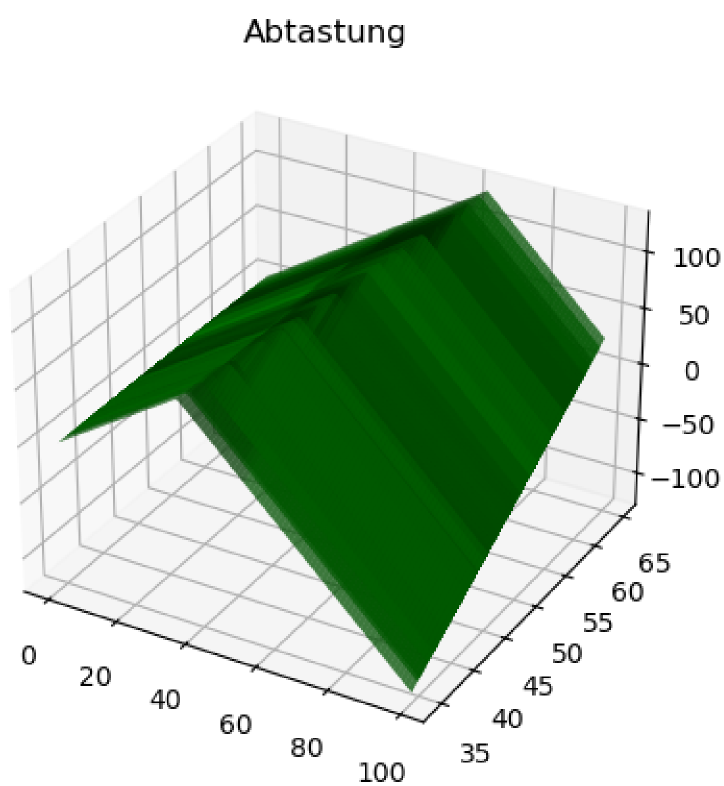
Im gesamten Vergleich ist somit die symbolische Regression am besten geeignet, jedoch liefert auch das Kriging Modell mit einer radialen Basisfunktion, eine relativ gute Abbildung der Funktion.

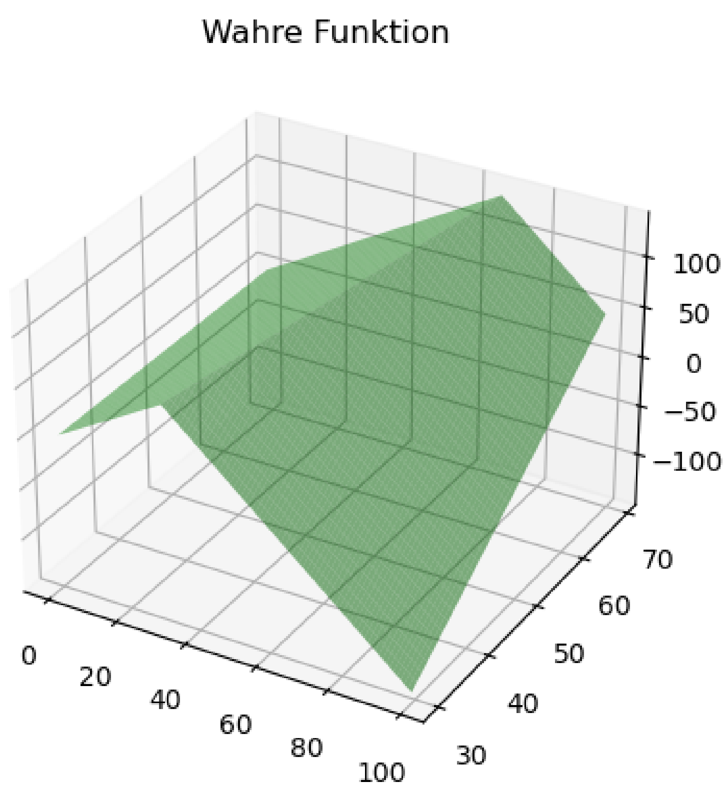
Teil b

Symbolische Regression

[2]:
```
def true_fun2(x):
    return 5*np.minimum(x[0],x[1]) - 3 * x[0]
```

[19]:
```
x0 = np.arange(0, 100, 1)
x1 = np.random.normal(50, 7, 100)
x0, x1 = np.meshgrid(x0, x1)
y_truth = 5*np.minimum(x0,x1) - 3 * x0

x0_fine=np.arange(0, 100, 1)
x1_fine=np.arange(30, 70, 1)
x0_fine, x1_fine = np.meshgrid(x0_fine, x1_fine)
y_truth_fine = 5*np.minimum(x0_fine,x1_fine) - 3 * x0_fine

ax = plt.figure().add_subplot(projection='3d')
surf = ax.plot_surface(x0, x1, y_truth, rstride=1, cstride=1,
                       color='green', alpha=0.5)
plt.title("Abtastung")
plt.show()
ax = plt.figure().add_subplot(projection='3d')
surf = ax.plot_surface(x0_fine, x1_fine, y_truth_fine, rstride=1, cstride=1,
                       color='green', alpha=0.5)
plt.title("Wahre Funktion")
plt.show()
```


Abtastung
100
50
0
−50
−100
0
20
40
60
80
100
35
40
45
50
55
60
65


Wahre Funktion
100
50
0
−50
−100
0
20
40
60
80
100
30
40
50
60
70

[64]:
```python
# Training samples
a=[]
b=[]
for j in range (100):
    k=np.random.normal(50, 7)
    for i in range(101):
        a.append(i)
        b.append(k)
X_train=np.stack([a,b],axis=1)
print(X_train)
y_train = 5*np.minimum(X_train[:, 0],X_train[:, 1]) - 3 * X_train[:, 0]
```

```
[[  0.          37.22853389]
 [  1.          37.22853389]
 [  2.          37.22853389]
 …
 [ 98.          53.69217396]
 [ 99.          53.69217396]
 [100.          53.69217396]]
```

[66]:
```python
est_gp = SymbolicRegressor(population_size=1000, generations=25,␣
↪stopping_criteria=0.
↪01,verbose=1,function_set=('min','mul','min'),n_jobs=-1,tournament_size=10)
est_gp.fit(X_train, y_train)
```

```
    |   Population Average    |             Best Individual              |
---- ------------------------- ------------------------------------------
----------
 Gen   Length          Fitness   Length          Fitness      OOB Fitness  Time
Left
   0    38.93      3.89979e+12       15          31.8957              N/A
1.27m
   1    19.61      2.20773e+07       15          31.8957              N/A
15.00s
   2    11.31          27390.1       47          30.3462              N/A
16.65s
   3    13.78           170531       51          29.9901              N/A
18.21s
   4    14.00           6765.7       55          29.9901              N/A
18.54s
   5    24.63          45414.4       55          29.1404              N/A
18.60s
   6    46.01           573908       45          28.3967              N/A
19.24s
   7    52.41      1.29288e+08       43          28.3967              N/A
```

```
20.01s
   8    56.77          11982.2        31          28.3967              N/A
18.77s
   9    52.91           127387        47          28.3967              N/A
29.55s
  10    47.50           147611        57          28.3967              N/A
16.28s
  11    39.60          56495.1        59          28.3967              N/A
14.22s
  12    30.51            11962        53          28.3967              N/A
13.67s
  13    23.83          64509.4        63          28.3967              N/A
11.34s
  14    20.82          48525.9        15          28.3967              N/A
9.98s
  15    17.79          19651.8        25          28.3967              N/A
8.80s
  16    16.73           643303        15          28.3967              N/A
7.66s
  17    15.87          77391.7        15          28.3967              N/A
6.59s
  18    15.47      2.30984e+06        21          28.3967              N/A
5.50s
  19    13.99           7242.1        11          28.3967              N/A
4.36s
  20    12.74          10152.4        11          28.3967              N/A
3.55s
  21    11.87            21745        11          28.3967              N/A
2.33s
  22    11.73          11332.8        15          28.3967              N/A
1.62s
  23    11.50          13418.1        11          28.3967              N/A
0.83s
  24    11.32           619636        11          28.3967              N/A
0.00s
```

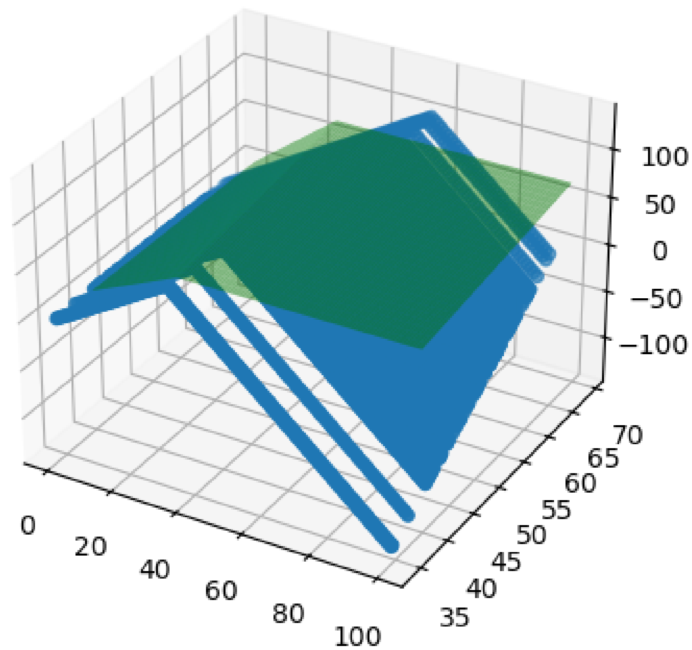
[66]:
```
SymbolicRegressor(function_set=('min', 'mul', 'min'), generations=25, n_jobs=-1,
                  stopping_criteria=0.01, tournament_size=10, verbose=1)
```

[67]:
```
print(est_gp._program)
```

```
min(X1, mul(mul(X0, X1), mul(mul(-0.592, -0.232), 0.273)))
```

[68]:
```
x0 = np.arange(0, 100, 1)
x1 = np.arange(40, 70, 1)
x0, x1 = np.meshgrid(x0, x1)
y_gp = est_gp.predict(np.c_[x0.ravel(), x1.ravel()]).reshape(x0.shape)
fig = plt.figure(figsize=(12, 10))
```

```
ax = fig.add_subplot(2, 2, 1, projection='3d')
surf = ax.plot_surface(x0, x1, y_gp, rstride=1, cstride=1, color='green',␣
  ↪alpha=0.5)
points = ax.scatter(X_train[:, 0], X_train[:, 1], y_train)
plt.show()
```



[76]:
```
yAbw=true_fun2([x0,x1])
absAbw=np.sum(np.abs(yAbw-y_gp))/4141
print(absAbw)
quaAbw=np.sum((yAbw-y_gp)**2)/4141
print(quaAbw)
```

```
19.22553588632645
1008.7567837194632
```

Im Gegensatz zu dem 2D Fall entstehen hier deutlich größere Fehler

[ ]:
```

```