1 Radius Calculation

To calculate the Radius out of the bowrange and the angle (endpoint angle – starting angle) the basic equation 1 for calculating the perimeter of a circle was settled as starting point. Equation 4 shows the final result of the formula transformations.

$$U = 2 \cdot r \cdot \pi \qquad U \text{ ... perimeter} \tag{1}$$
$$\frac{U}{l} = \frac{2 \cdot r \cdot \pi}{\phi \cdot r} \quad \Rightarrow \quad U = 2 \cdot l \cdot \frac{\pi}{\phi} \qquad r \text{ ... radius} \tag{2}$$
$$2 \cdot l \cdot \frac{\pi}{\phi} = 2 \cdot r \cdot \pi \qquad l \text{ ... radian measure} \tag{3}$$
$$r = \frac{l}{\phi} \qquad \phi \text{ ... angle} \tag{4}$$

2 End Point Calculation

The next formula transformations will show how the endpoint coordinates are calculated in all three cases (left curve, right curve and straight track).
Since every track piece exactly has one radius, it can be seen as an equiliteral triangle. The euqiliteral sides have the length of radius r, which calculation is shown in equation 4.
To calculate the difference between starting point and endpoint the sine and cosine functions will be used. So in addition to the length of the track between start and end point there is a need for the angle of that track. For any straight trails equation 5 and equation 6 can be used. Since it is a straight trail, the input angle and the output angle are the same.

$$x_{end} = x_{begin} + r \cdot \cos(\phi_{in}) \tag{5}$$
$$y_{end} = y_{begin} + r \cdot \sin(\phi_{in}) \tag{6}$$

For the slightly more complicated calculation of the end point of a curve first of all the coordinates of the center of the radius circle was calculated (as sort of reference point). To get a better idea of how this formulas come together have a look at figure 1.
Sine and cosine roles are switched due to the fact that the input and output angles are relative to the negative y-axis. So there is a rotation of $\frac{\pi}{2}$

$$x_R = x_{begin} - r \cdot \sin(\phi_{in}) \tag{7}$$
$$y_R = y_{begin} + r \cdot \cos(\phi_{in}) \tag{8}$$

Starting from this point and the help of the out angle the relative distance between radius center and endpoint can be calculated. The result of the left curve is shown in equation 11 and equation 12

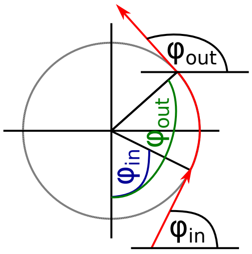

Abbildung 1: Left curve as a track piece

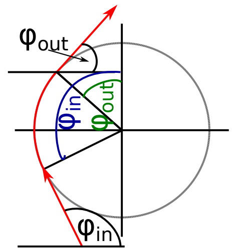

Abbildung 2: Right curve as a track piece

$$x_{end} = x_R + r \cdot \sin(\phi_{out}) \tag{9}$$
$$y_{end} = y_R - r \cdot \cos(\phi_{out}) \tag{10}$$
$$x_{end} = x_{begin} + r \cdot (\sin(\phi_{out}) - \sin(\phi_{in})) \tag{11}$$
$$y_{end} = y_{begin} + r \cdot (\cos(\phi_{in}) - \cos(\phi_{out})) \tag{12}$$

If we want to calculate the right curve just invert the + sign in front of r. This is shown in equation 13 and equation 14. To get a better idea of why this difference appears have a look at figure 2

$$x_{end} = x_{begin} - r \cdot (\sin(\phi_{out}) - \sin(\phi_{in})) \tag{13}$$
$$y_{end} = y_{begin} - r \cdot (\cos(\phi_{in}) - \cos(\phi_{out})) \tag{14}$$