Probability Problems and Solutions

1. Probability of Drawing a Numbered Card

In a 52-card deck, there are 4 aces, 4 kings, 4 queens, and 4 jacks, making a total of 16 face cards. The rest of the cards are numbered cards ranging from 2 to 10, with 4 cards for each number.

The total number of numbered cards is:

$$4 \times 9 = 36$$

Thus, the probability P of drawing a numbered card is:

$$P(\text{Numbered Card}) = \frac{36}{52} = \frac{9}{13}$$

2. Probability of Drawing Two Numbered Cards with Replacement

The probability P of drawing a numbered card on the first draw is:

$$P(\text{First Numbered Card}) = \frac{9}{13}$$

Since the card is put back into the deck and the deck is shuffled, the probability P of drawing a numbered card on the second draw is also:

$$P(\text{Second Numbered Card}) = \frac{9}{13}$$

The combined probability P of drawing two numbered cards with replacement is:

$$P(\text{Both Numbered Cards}) = \left(\frac{9}{13}\right) \times \left(\frac{9}{13}\right) = \left(\frac{9}{13}\right)^2 = \frac{81}{169}$$

3. Probability of Drawing Two Numbered Cards without Replacement

The probability P of drawing a numbered card on the first draw is:

$$P(\text{First Numbered Card}) = \frac{9}{13}$$

After drawing a numbered card, 35 numbered cards remain out of 51 total cards. The probability P of drawing another numbered card on the second draw is:

$$P(\text{Second Numbered Card}) = \frac{35}{51}$$

The combined probability P of drawing two numbered cards without replacement is:

$$P(\text{Both Numbered Cards}) = \frac{9}{13} \times \frac{35}{51} = \frac{315}{663} = \frac{105}{221}$$

4. Probability of Cutting to a Numbered Card in an Ordered Deck

In a deck arranged in order (Ace to King for Spades, Hearts, Diamonds, and Clubs), numbered cards (2 to 10) occupy positions 2 through 10 in each suit, totaling 36 numbered cards.

When cutting the deck at a random spot, there are 52 possible positions to make the cut (one between each pair of consecutive cards and one before the first card).

Therefore, the probability P that the bottom card of your cut is a numbered card is:

$$P(\text{Bottom Card is Numbered}) = \frac{36}{52} = \frac{9}{13}$$