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Subgroup 5

Probability and Statistics – Assignment 1

You ask your neighbour to water a secret plant while you're on holiday. There is an 80% chance of the plant dying without it being watered and a 15% chance of it dying if it's watered.

You're 90% certain that the neighbour will remember to water the plant.

- a) Compute the probability of the plant being alive when you return.
- b) Compute the probability of the plant not being watered if you return home to a dead plant.

A: The plant is alive given the neighbour watered

B: The plant is alive given the neighbour did not water it

C: The plant is alive when you return

$$P(A) = 1 - 0.15 = 0.85$$

$$P(B) = 1 - 0.80 = 0.20$$

By the law of total probability:

$$P(C) = P(C|A) * P(A) + P(C|B) * P(B) \Rightarrow P(C) = 0.85 * 0.90 + 0.20 * 0.10 = 0.785$$

- b) We want to figure out the probability that the plant wasn't watered given that it died.

A: The plant was watered

B: The plant was not watered

C: The plant died

Using Bayes theorem we calculate $P(B|C)$

$$P(B|C) = P(C|B) * P(B) / P(C)$$

$$P(C|B) = 0.80 \text{ (as given in the problem)}$$

$$P(B) = 1 - P(A) = 1 - 0.90 = 0.10$$

$$P(C) = P(C|A) * P(A) + P(C|B) * P(B) \Rightarrow P(C) = 0.15 * 0.90 + 0.80 * 0.10 = 0.215$$

$$P(B|C) = (0.80 * 0.10) / 0.215 = 0.372$$

Extra from lab 4:

$$x = \begin{pmatrix} -2 & 0 & 1 & 2 \\ \frac{1}{3} & \frac{1}{12} & \frac{1}{4} & \frac{1}{3} \end{pmatrix} \quad y = \begin{pmatrix} -1 & 1 & 2 \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$$

$$\text{Var}(x-y) = \text{Var}(x) + \text{Var}(y)$$

$$\text{Var}(x) = E[x^2] - E[x]^2$$

$$\text{Var}(y) = E[y^2] - E[y]^2$$

$$\text{Var}(x-y) = E[x^2] - E[x]^2 + E[y^2] - E[y]^2$$

$$E[x] = \frac{-2}{3} + \frac{1}{4} + \frac{2}{3} = \frac{1}{4} \Rightarrow E[x]^2 = \frac{1}{16}$$

$$E[x^2] = \frac{4}{3} + \frac{1}{4} + \frac{4}{3} = \frac{8}{3} + \frac{1}{4} = \frac{35}{12}$$

$$E[y] = \frac{-1}{3} + \frac{1}{2} + \frac{2}{6} = \frac{-2}{6} + \frac{1}{2} + \frac{2}{6} = \frac{1}{2} \Rightarrow E[y]^2 = \frac{1}{4}$$

$$E[y^2] = \frac{1}{3} + \frac{1}{2} + \frac{4}{6} = \frac{2}{6} + \frac{3}{6} + \frac{4}{6} = \frac{9}{6} = \frac{3}{2}$$

$$\text{Var}(x-y) = \frac{35}{12} - \frac{1}{16} + \frac{3}{2} - \frac{1}{4} = \frac{197}{48}$$