1 Part 2.2

Write the expression of the application-oblivious throughput bound TH for an all-to-all (among switches) traffic matrix. Assume $\bar{h}$ as known). The expression of TH must be a function of $\bar{h}$ and n only.

The application-oblivious throughput bounds is defined as

$$TH \leq \frac{m}{\bar{h}v_f} \tag{1}$$

Where m is the number of links of the network graph, $\bar{h}$ is the mean path length and v_f is the number of flows.
In this case, since we are assuming of working with an all-to-all traffic matrix, the expression for the number of flows is stated as

$$v_f = \frac{N(N-1)}{2}$$

So, we then have to consider two different expressions for the variable m, which directly depends by the kind of graphs we are working with.

Starting with the **Erdös-Rényi random graph**, which is defined by fixing the parameter p, since an arc is included in the graph with probability p independently of the other arcs. In this case, the number of links is itself a random variable with mean

$$m = \frac{pN(N-1)}{2}$$

The application-oblivious throughput bound for a p-ER random graph is obtained substituting m and v_f to the expression (1) above:

$$TH \leq \frac{p}{\bar{h}}$$

Instead, with an **r-regular random graph**, fixing the variable r to define it, we obtain a number of links equal to

$$m = \frac{Nr}{2}$$

Then, we know the number of servers N of an r-regular random graph is equal to $S(n-r)$, and the number of switches is $S \geq r+1$, so

$$\frac{1}{N-1} \leq \frac{1}{(r+1)(n-r)-1} \tag{2}$$

So, substituting into the equation of the application-oblivious throughput bound (1) the expressions v_f , m, and bounding with (2), we obtain

$$TH \leq \frac{1}{\bar{h}(r+1)(n-r)-1}$$

.