$$V(x) = \alpha^{14} + \alpha^{9}x + \alpha^{0}x^{2} + \alpha^{2}x^{3} + \alpha^{3}x^{4} + \alpha^{6}x^{5} + \alpha x^{6} + \alpha^{4}x^{7} + \alpha^{15}x^{8}$$
$$+ \alpha^{10}x^{9} + \alpha^{4}x^{10} + \alpha^{2}x^{11} + \alpha x^{12} + \alpha^{3}x^{13} + \alpha x^{14}$$

$$R(x) = V(x) + E(x)$$

En $x^{2} = \alpha^{0} \oplus \alpha^{7} = \alpha^{9}$

$x^{8} = \alpha^{0} \oplus \alpha^{13} = \alpha^{6}$

$x^{10} = \alpha^{4} \oplus \alpha^{3} = \alpha^{7}$

$$R(x) = \alpha^{14} + \alpha^{9}x + \alpha^{9}x^{2} + \alpha^{2}x^{3} + \alpha^{3}x^{4} + \alpha^{6}x^{5} + \alpha x^{6} + \alpha^{4}x^{7} + \alpha^{6}x^{8}$$
$$+ \alpha^{10}x^{9} + \alpha^{7}x^{10} + \alpha^{2}x^{11} + \alpha x^{12} + \alpha^{3}x^{13} + \alpha x^{14}$$

$$S(x) = S_1 + S_2 x + \dots + S_{n-k} x^{n-k-1}$$

$$S(x) = S_1 + S_2 x + S_3 x^2 + S_4 x^3 + S_5 x^4 + S_6 x^5$$

$$S_i(x = \alpha^i) = R(x = \alpha^i)$$

$$S_1(x=\alpha) = \cancel{\alpha^{14}} + \alpha^{10} + \cancel{\alpha^{11}} + \alpha^{5} + \cancel{\alpha^{7}} + \cancel{\alpha^{11}} + \cancel{\alpha^{7}} + \alpha^{11} + \cancel{\alpha^{14}}$$
$$+ \alpha^{4} + \alpha^{2} + \cancel{\alpha^{13}} + \cancel{\alpha^{13}} + \alpha^{1} + \alpha^{15}$$
$$= \alpha^{10} + \alpha^{5} + \alpha^{11} + \alpha^{4} + \alpha^{2} + \alpha^{1} + \alpha^{15}$$

α^{10} 0111
+
α^{5} 0110
0001
+
α^{11} 1110
1111
+
α^{4} 0011
1100
+
α^{2} 0100
1000

$$+$$
$$\alpha^1 \quad \underline{0010}$$
$$1010$$
$$+$$
$$\alpha^{15} \quad \underline{0001}$$
$$1011 = \alpha^7$$

$$R(x) = \alpha^{14} + \alpha^9 x + \alpha^9 x^2 + \alpha^2 x^3 + \alpha^3 x^4 + \alpha^6 x^5 + \alpha x^6 + \alpha^4 x^7 + \alpha^6 x^8$$
$$+ \alpha^{10} x^9 + \alpha^7 x^{10} + \alpha^2 x^{11} + \alpha x^{12} + \alpha^3 x^{13} + \alpha x^{14}$$

$$S_2(x=\alpha^2) = \alpha^{14} + \cancel{\alpha^{11}} + \cancel{\alpha^{13}} + \alpha^8 + \cancel{\alpha^{11}} + \alpha^1 + \cancel{\alpha^{13}} + \alpha^3$$
$$+ \alpha^7 + \alpha^{13} + \alpha^{12} + \alpha^9 + \alpha^{10} + \cancel{\alpha^{14}} + \cancel{\alpha^{14}}$$

$$1001 + 0101 = 1100 + 0010 = 1110 + 1000 = 0110$$
$$+ 1011 = 1101 + 1101 = 0000 + 1111 = 1111 + 1010 = 0101$$
$$+ 0111 = 0010 = \alpha^1$$

$$S_3(x=\alpha^3) = \alpha^{11} \qquad S_4(x=\alpha^4) = \alpha^{13} \qquad S_5(x=\alpha^5) = \alpha^2$$

$$S_6(x=\alpha^6) = \alpha^{14}$$

$$S(X) = \alpha^7 + \alpha^1 x + \alpha^{11} x^2 + \alpha^{13} x^3 + \alpha^2 x^4 + \alpha^{14} x^5$$

i	$r_n = r_{n-2} / r_{n-1}$		$t_n = t_{n-2} + t_{n-1} \cdot q_n$
-1	$x^{n-k} = x^6$	q_n	0
0	$S(X)$		1
1	$x^6 / S(X)$		

$$r_1 = x^6 \quad \underline{|S(X)}$$
$$q_1$$
$$\overline{\quad r_1 \quad}$$

$$r_1 = \alpha^8 x^4 + \alpha^7 x^3 + \alpha^8 x^2 + \alpha^4 x + \alpha^{11}$$
$$q_1 = \alpha^1 x + \alpha^4$$

$t_1 = 0 + 1 \; q_1 = 0 + 1 \cdot (\alpha^1 x + \alpha^4) = \alpha^1 x + \alpha^4$

i	$r_n = r_{n-2} / r_{n-1}$	q_n	$t_n = t_{n-2} + t_{n-1} \cdot q_n$
-1	$x^{n-K} = x^6$		0
0	$S(x)$		1
1	$\alpha^8 x^4 + \alpha^7 x^3 + \alpha^8 x^2 + \alpha^4 x + \alpha^{11}$	$\alpha^1 x + \alpha^4$	$\alpha^1 x + \alpha^4$

$r_2 = \alpha^{14} x^3 + x + \alpha^{12}$

$q_2 = \alpha^6 x + \alpha^6$

$t_2 = \alpha^7 x^2 + \alpha^6 x + \alpha^5$

i	$r_n = r_{n-2} / r_{n-1}$	q_n	$t_n = t_{n-2} + t_{n-1} \cdot q_n$
-1	$x^{n-K} = x^6$		0
0	$S(x)$		1
1	$\alpha^8 x^4 + \alpha^7 x^3 + \alpha^8 x^2 + \alpha^4 x + \alpha^{11}$	$\alpha^1 x + \alpha^4$	$\alpha^1 x + \alpha^4$
2	$\alpha^{14} x^3 + x + \alpha^{12}$	$\alpha^6 x + \alpha^6$	$\alpha^7 x^2 + \alpha^6 x + \alpha^5$

$r_3 = \alpha^{12} x^2 + \alpha^9 x + \alpha^3$

$q_3 = \alpha^9 x + \alpha^8$

$t_3 = \alpha^1 x^3 + \alpha^1 x + \alpha^{11}$

i	$r_n = r_{n-2} / r_{n-1}$	q_n	$t_n = t_{n-2} + t_{n-1} \cdot q_n$
-1	$x^{n-K} = x^6$		0
0	$S(x)$		1
1	$\alpha^8 x^4 + \alpha^7 x^3 + \alpha^8 x^2 + \alpha^4 x + \alpha^{11}$	$\alpha^1 x + \alpha^4$	$\alpha^1 x + \alpha^4$
2	$\alpha^{14} x^3 + x + \alpha^{12}$	$\alpha^6 x + \alpha^6$	$\alpha^7 x^2 + \alpha^6 x + \alpha^5$
3	$\alpha^{12} x^2 + \alpha^9 x + \alpha^3$	$\alpha^9 x + \alpha^8$	$\alpha^1 x^3 + \alpha^1 x + \alpha^{11}$

Como el grado de r_3, es menor a $\frac{n-K}{2} - 1 = \frac{15-9}{2} - 1 = 2$

Normalizamos t_3, tal que el coeficiente de mayor grado de x es α^1, por lo que dividimos t_3 en α^1

t [illegible] $\alpha^1 x^3 + \alpha^1 x + \alpha^{11} = \alpha^0 x^3 + \alpha^0 x + \alpha^{10} = x^3 + x + \alpha^{10}$

es α^1, por lo que dividimos t_3 en α^1

$$t_{n\ normalizado} = \frac{\alpha^1 x^3}{\alpha^1} + \frac{\alpha^1 x}{\alpha^1} + \frac{\alpha^{11}}{\alpha^1} = \alpha^0 x^3 + \alpha^0 x + \alpha^{10} = x^3 + x + \alpha^{10}$$

Y ahora buscamos las raíces:

$x = \alpha^1 \rightarrow t_n = \alpha^0 \alpha^3 + \alpha^0 \alpha^1 + \alpha^{10} = \alpha^2 \neq \emptyset$

$x = \alpha^2 \rightarrow t_n = \alpha^6 + \alpha^2 + \alpha^{10} = \alpha^5 \neq \emptyset$

$x = \alpha^3 \rightarrow t_n = \alpha^9 + \alpha^3 + \alpha^{10} = \alpha^8 \neq \emptyset$

$x = \alpha^4 \rightarrow t_n = \alpha^{12} + \alpha^4 + \alpha^{10} = \alpha^5 \neq \emptyset$

$x = \alpha^5 \rightarrow t_n = \alpha^{15} + \alpha^5 + \alpha^{10} = \emptyset \rightarrow \alpha^5$ es raíz

$x = \alpha^7 \rightarrow t_n = \alpha^6 + \alpha^7 + \alpha^{10} = \emptyset \rightarrow \alpha^7$ es raíz

$x = \alpha^{13} \rightarrow t_n = \alpha^9 + \alpha^{13} + \alpha^{10} = \emptyset \rightarrow \alpha^{13}$ es raíz

Ya obtuvimos las 3 raíces de los errores.

Ahora obtenemos las posiciones del error:

$\frac{\alpha^5}{\alpha^{15}} = \alpha^{-10} \rightarrow x^{10}$ hay error

$\frac{\alpha^7}{\alpha^{15}} = \alpha^{-8} \rightarrow x^8$ hay error

$\frac{\alpha^{13}}{\alpha^{15}} = \alpha^{-2} \rightarrow x^2$ hay error

Ya tenemos las posiciones, ahora buscamos los valores de error

tal que $\frac{r(x = \text{raíz del } t_n)}{t'_n(x \text{ en raíz})}$

$$t'_n = x^3 + x + \alpha^{10} = 3x^2 + 1 = x^2 + 1$$

$\sqrt{x} = r_n \cdot \alpha^{14} = \alpha^2 + \alpha^8 x + \alpha^{11} x^2$

Para α^5

$r_n(x = \alpha^5) = \alpha^2 + \alpha^{13} + \alpha^6 = \alpha \rightarrow \frac{\alpha^8}{\alpha^5} = \alpha^3$

$t'_n(x = \alpha^5) = \alpha^{10} + \alpha^0 = \alpha^5$

Error en $x^{10} \rightarrow \alpha^4$ tal que $\alpha^7 + \alpha^3$

$\frac{\alpha^3}{\alpha^4} \rightarrow$ Valor original

$$\frac{\alpha^3}{\alpha^4} \rightarrow \text{Valor original}$$

Para α^7

$r_n(x=\alpha^7) = \alpha^2 + \alpha^{15} + \alpha^{10} = \alpha \longrightarrow \frac{\alpha^7}{\alpha^3} = \alpha^{-2} \rightarrow \alpha^{-2} \cdot \alpha^{15} = \alpha^{13}$

$t'_n(x=\alpha^7) = \alpha^{14} + \alpha^0 = \alpha^3$

Error en $x^8 \rightarrow \alpha^3$ tal que

$$\begin{array}{r} \alpha^6 \\ + \;\alpha^{13} \\ \hline \alpha^{15} \end{array}$$

Para α^7

$r_n(x=\alpha^{13}) = \alpha^2 + \alpha^6 + \alpha^7 = \alpha^4 \longrightarrow \frac{\alpha^4}{\alpha^{12}} = \alpha^{-8} \rightarrow \alpha^{-8} \cdot \alpha^{15} = \alpha^7$

$t'_n(x=\alpha^{13}) = \alpha^{11} + \alpha^0 = \alpha^{12}$

Error en $x^2 \rightarrow \alpha^7$ tal que

$$\begin{array}{r} \alpha^9 \\ + \;\alpha^7 \\ \hline \alpha^0 \end{array}$$

Entonces con los errores obtenidos, corregimos $R(x)$ y obtenemos $V(x)$

$$R(x) = \alpha^{14} + \alpha^9 x + \alpha^9 x^2 + \alpha^2 x^3 + \alpha^3 x^4 + \alpha^6 x^5 + \alpha x^6 + \alpha^4 x^7 + \alpha^6 x^8 + \alpha^{10} x^9 + \alpha^7 x^{10} + \alpha^2 x^{11} + \alpha x^{12} + \alpha^3 x^{13} + \alpha x^{14}$$

Corregimos en x^2 con α^{15}, en x^8 con α^{15} y en x^{10} con α^4 y obtenemos

$$V(x) = \alpha^{14} + \alpha^9 x + \alpha^{15} x^2 + \alpha^2 x^3 + \alpha^3 x^4 + \alpha^6 x^5 + \alpha x^6 + \alpha^4 x^7 + \alpha^{15} x^8 + \alpha^{10} x^9 + \alpha^4 x^{10} + \alpha^2 x^{11} + \alpha x^{12} + \alpha^3 x^{13} + \alpha x^{14}$$