$$U^{l+1}_{ij} = V^2\left[U^l_{i+1j} - 2U^l_{ij} + U^l_{i-1j} + \frac{\Delta\rho}{\rho[i]}\cdot\left(U^l_{ij} - U^l_{i-1j}\right)\right.$$

$$\left. + \left(\frac{\lambda^2}{\rho[i]}\right)\cdot\left(U^l_{ij+1} - 2U^l_{ij} + U^l_{ij-1}\right)\right] + 2U^l_{ij} - U^{l-1}_{ij}$$

$$\lambda = \frac{\Delta\rho}{\Delta\phi} \quad y \quad V = \frac{\alpha\,\Delta t}{\Delta\rho}$$

* Pasar a coordenadas polares:

$x = \rho\cos(\phi)$ $\qquad$ $y = \operatorname{sen}(\phi)\,\rho$

con

$\rho = \sqrt{x^2 + y^2}$ $\qquad$ $\phi = \arctan(y/x)$

* Las nuevas derivadas para el laplaciano serán:

$$\frac{\partial\rho}{\partial x} = \frac{x}{\sqrt{x^2 + y^2}} = \frac{\rho\cos\phi}{\rho} = \cos\phi$$

así mismo para y: $\partial\rho/\partial x = \operatorname{sen}\phi$

* Derivadas de ϕ:

$$\frac{\partial\phi}{\partial x} = -\frac{y}{x^2 + y^2} = \frac{-\rho\sin\phi}{\rho^2} = \frac{-\sin\phi}{\rho}$$

$$\frac{\partial\phi}{\partial y} = \frac{x}{x^2 + y^2} = \frac{\rho\cos\phi}{\rho^2} = \frac{\cos\phi}{\rho}$$

$$\nabla^2 U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}$$

Donde

$$\frac{\partial^2 U}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial U}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial\rho}\frac{\partial\rho}{\partial x} + \frac{\partial u}{\partial\phi}\frac{\partial\phi}{\partial x}\right) = \frac{\partial^2 u}{\partial\rho^2}\cos^2\phi + \frac{\partial^2 u}{\partial\phi^2}\frac{\sin^2\phi}{\rho^2}$$

$$-\frac{\partial^2 u}{\partial\rho\partial\phi}\frac{\sin(2\phi)}{\rho} + \frac{\partial u}{\partial\phi}\frac{\sin(2\phi)}{\rho^2} + \frac{\partial u}{\partial\rho}\frac{\sin^2\phi}{\rho}$$

* si se realiza el mismo proceso para $\partial^2 U/\partial y^2$

$$\frac{\partial u^2}{\partial^2 y} = \frac{\partial^2 u}{\partial \rho^2}\sin^2\phi + \frac{\partial^2 u}{\partial \phi^2}\frac{\cos^2\phi}{\rho^2} + \frac{\partial^2 U}{\partial \rho \partial \phi}\frac{\sin(2\phi)}{\rho} - \frac{\partial U}{\partial \phi}\frac{\sin(2\phi)}{\rho^2}$$

$$+ \frac{\partial u}{\partial \rho}\frac{\cos^2\phi}{\rho}$$

* sumando $\partial^2 U/\partial x^2$ y $\partial U^2/\partial y^2$:

$$\nabla^2 U = \frac{\partial^2 u}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2 u}{\partial \phi^2} + \frac{1}{\rho}\frac{\partial u}{\partial \rho}$$

La ecuación de onda puede ser escrita como:

$$\frac{\partial^2 u}{\partial t^2} = \alpha^2 \nabla^2 u$$

si se discretiza la ecuación:

$$\frac{u^{l+1}_{i,j} - 2u^{l}_{i,j} + u^{l-1}_{i,j}}{(\Delta t)^2} = \alpha^2 \cdot \left[\frac{U^l_{i+1,j} - 2U^l_{i,j} + U^l_{i-1,j}}{(\Delta \rho)^2} \right.$$

$$\left. + \frac{1}{\rho[i]^2}\frac{U^l_{i,j+1} - 2U^l_{i,j} + U^l_{i,j-1}}{(\Delta\phi)^2} + \frac{1}{\rho[i]}\frac{U^l_{i,j} - U^l_{i-1,j}}{\Delta \rho} \right]$$

* Realizando Álgebra:

$$U^{l+1}_{i,j} = \left(\frac{\alpha^2 \Delta t}{\Delta \rho}\right)^2 \cdot \left[U^l_{i+1,j} - 2U^l_{i,j} + U^l_{i-1,j} + \left(\frac{\Delta \rho}{\rho[i]\Delta\phi}\right)^2 \left(U^l_{i,j+1} - 2U^l_{i,j} + U^l_{i,j-1}\right) \right.$$

$$\left. + \frac{\Delta\rho}{\rho[i]}\left(U^l_{i,j} - U^l_{i,j-1}\right) \right] + 2U^l_{i,j} - U^{l-1}_{i,j}$$

Reemplazando:

$$U^{l+1}_{i,j} = \nu^2 \left[U^l_{i+1,j} - 2U^l_{i,j} + U^l_{i-1,j} + \left(\frac{\lambda}{\rho[i]}\right)^2 \cdot \left(u^l_{i,j+1} - 2u^l_{i,j} + u^l_{i,j-1}\right) + \frac{\Delta\rho}{\rho[i]}\left(u^l_{i,j} - u^l_{i-1,j}\right) \right]$$

$$+ 2u^l_{i,j} - U^{l-1}_{i,j}$$