c) $r_1 = (r(t)\cos(\phi),\ r(t)\operatorname{sen}(\phi))$ $\qquad r_2 = (d\cos(\omega t);\ d\operatorname{sen}(\omega t))$

$$r_L = r_1 - r_2 = (r(t)\cos(\phi) - d\cos(\omega t),\ r(t)\operatorname{sen}(\phi) - d\operatorname{sen}(\omega t))$$

$$|r_L| = \left((r(t)\cos(\phi) - d\cos(\omega t))^2 + (r(t)\operatorname{sen}(\phi) - d\operatorname{sen}(\omega t))^2\right)^{1/2}$$

$$= \left(r^2(t)\cos^2(\phi) - 2r(t)\cos(\phi)\, d\cos(\omega t) + d^2\cos^2(\omega t)\right.$$
$$\left. + r^2(t)\operatorname{sen}^2(\phi) - 2r(t)\operatorname{sen}(\phi)\, d\operatorname{sen}(\omega t) + d^2\operatorname{sen}^2(\omega t)\right)^{1/2}$$

$$= \left(r^2(t)\overbrace{\cancel{(\operatorname{sen}^2(\phi) + \cos^2(\phi))}}^{1} + d^2\overbrace{\cancel{(\cos^2(\omega t) + \operatorname{sen}^2(\omega t))}}^{1} - 2r(t)d\,\overbrace{\cancel{(\cos(\omega t)\cos\phi + \operatorname{sen}(\omega t)\operatorname{sen}(\phi))}}^{\cos(\omega t - \phi)}\right)^{1/2}$$

$$= \left(r^2(t) + d^2 - 2r(t)d\cos(\omega t - \phi)\right)^{1/2}$$

Punto 4.

a) $\vec{r} = r(t)\cos(\phi(t))\hat{i} + r(t)\sin(\phi(t))\hat{j}$

$\dot{\vec{r}} = -r(t)\sin(\phi(t))\phi(t) + r'(t)\cos(\phi(t))\hat{i} + r(t)\cos(\phi(t))\phi(t) + r'(t)\sin(\phi(t))\hat{j}$

$|\dot{\vec{r}}| = \left(\left(-r(t)\sin(\phi(t))\phi(t) + r'(t)\cos(\phi(t))\right)^2 + \left(r(t)\cos(\phi(t))\phi(t) + r'(t)\sin(\phi(t))\right)^2\right)^{1/2}$

$= \Big(r(t)^2\sin^2(\phi(t))\dot{\phi}^2(t) - \cancel{2r(t)\sin(\phi(t))r'(t)\cos(\phi(t))\phi(t)} + r'^2(t)\cos^2(\phi(t))$

$+ r(t)^2\cos^2(\phi(t))\dot{\phi}^2(t) + \cancel{2r(t)\cos(\phi(t))\phi(t)r'(t)\sin(\phi(t))} + r'(t)^2\sin^2(\phi(t)) \Big)^{1/2}$

$= \left(r(t)^2\dot{\phi}(t)^2(\sin^2(\phi(t)) + \cos^2(\phi(t))) + r'^2(t)(\sin^2(\phi(t)) + \cos^2(\phi(t)))\right)^{1/2}$

$= \left(r(t)^2\dot{\phi}^2(t) + r'(t)^2\right)^{1/2}$

$\Rightarrow \mathcal{L} = \frac{1}{2}m(r^2\dot{\phi}^2 + \dot{r}^2) + \frac{GmM_T}{r} + \frac{GmM_L}{r_L}$

$\Rightarrow H = p_r\dot{r} + p_\phi\dot{\phi} - \frac{1}{2}mr^2\dot{\phi}^2 - \frac{1}{2}m\dot{r}^2 - \frac{GmM_T}{r} - \frac{GmM_L}{r_L}$

$H = p_r\dot{r} - \frac{1}{2}p_r\dot{r} + p_\phi\dot{\phi} - \frac{1}{2}p_\phi\dot{\phi} - \frac{GmM_T}{r} - \frac{GmM_L}{r_L}$

$H = \frac{p_r\dot{r}}{2} + \frac{p_\phi\dot{\phi}}{2} - \frac{GmM_T}{r} - \frac{GmM_L}{r_L}$

$H = \frac{p_r^2}{2m} + \frac{p_\phi}{2}\frac{\overbrace{\dot{r}\cdot mr}^{p_\phi}}{r\cdot mr} - \frac{GmM_T}{r} - \frac{GmM_L}{r_L}$

$H = \frac{p_r^2}{2m} + \frac{p_\phi^2}{2mr^2} - \frac{GmM_T}{r} - \frac{GmM_L}{r_L}$

c) $\dot{r} = \frac{\partial}{\partial p_r}\left(\frac{p_r^2}{2m} + \cancel{\frac{p_\phi^2}{2mr^2}}^{0} - \cancel{\frac{GmM_T}{r}}^{0} - \cancel{\frac{GmM_L}{r_L}}^{0}\right) = \frac{p_r}{m}$

$\dot{\phi} = \frac{\partial}{\partial p_\phi}\left(\cancel{\frac{p_r^2}{2m}}^{0} + \frac{p_\phi^2}{2mr^2} - \cancel{\frac{GmM_T}{r}}^{0} - \cancel{\frac{GmM_L}{r_L}}^{0}\right) = \frac{p_\phi}{mr^2}$

$\dot{p}_r = -\frac{\partial H}{\partial r} = -\left(\frac{p_\phi^2}{2m}(-2r^{-3}) - GmM_T(-r^{-2})\dots\right.$

$\left.\dots - GmM_L\left(-\frac{1}{2}(r(t)^2 + d^2 - 2r(t)d\cos(\phi - \omega t))^{-3/2}(2r(t) - 2d\cos(\phi - \omega t))\right)\right)$

$= -\left(-\frac{p_\phi^2}{mr^3} + \frac{GmM_T}{r^2} + \frac{GmM_L}{2r_L^3}(2r(t) - 2d\cos(\phi - \omega t))\right)$

$$= \frac{P_\phi^2}{mr^3} - G\frac{mM_T}{r^2} - \frac{GmM_L}{r_L^3}\left(r(t) - 2d\cos(\phi - wt)\right)$$

$$\dot{P}_\phi = -\frac{\partial H}{\partial \phi} = -\left(-GmM_L\left(-\frac{1}{2}\left(r(t)^2 + d^2 - 2r(t)d\cos(\phi - wt)\right)^{-3/2}\left(+2r(t)d\sin(\phi - wt)\right)\right)\right)$$

$$= -\frac{GmM_L}{r_L^3}\left(r(t)d\sin(\phi - wt)\right)$$