Punto 1)

$$\to S = \begin{cases} \frac{dx}{dt} = F(x,y) \\ \frac{dy}{dt} = G(x,y) \end{cases}$$

$\to$ Sea S un sistema autonomo con un punto critico en (x_0, y_0). Entonces, aproximando $z = F(x,y)$ y $z = G(x,y)$ por sus planos tangentes al punto (x_0, y_0),

$$F(x,y) \approx \frac{\partial F}{\partial x}(x_0,y_0)\cdot(x-x_0) + \frac{\partial F}{\partial y}(x_0,y_0)\cdot(y-y_0)$$

$$G(x,y) \approx \frac{\partial G}{\partial x}(x_0,y_0)\cdot(x-x_0) + \frac{\partial G}{\partial y}(x_0,y_0)(y-y_0)$$

$$\Rightarrow \begin{pmatrix} F(x,y) \\ G(x,y) \end{pmatrix} \approx \underbrace{\begin{pmatrix} \frac{\partial F}{\partial x}(x_0,y_0), & \frac{\partial F}{\partial y}(x_0,y_0) \\ \frac{\partial G}{\partial x}(x_0,y_0), & \frac{\partial G}{\partial y}(x_0,y_0) \end{pmatrix}}_{M} \begin{pmatrix} x-x_0 \\ y-y_0 \end{pmatrix}$$

$\to$ Si $X = x - x_0$ y $Y = y - y_0$

$$\Rightarrow \begin{pmatrix} X' \\ Y' \end{pmatrix} = M \begin{pmatrix} X \\ Y \end{pmatrix}$$

$$\frac{\partial E}{dt} = ME.$$

b)

Punto critico cuando $\frac{\partial x}{\partial t} = 0$ y $\frac{\partial y}{\partial t} = 0$

$$\begin{aligned} 2x - y &= 0 \\ x + 2y &= 0 \end{aligned} \Rightarrow x = 0 \text{ y } y = 0$$

$$p.c. = (0,0)$$

$$M = \begin{pmatrix} \frac{\partial(2x-y)}{\partial x}, & \frac{\partial(2x-y)}{\partial y} \\ \frac{\partial(x+2y)}{\partial x}, & \frac{\partial(x+2y)}{\partial y} \end{pmatrix}(0,0) = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$

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c) $\det(M - I\lambda) = \begin{vmatrix} 2-\lambda & -1 \\ 1 & 2-\lambda \end{vmatrix} = (2-\lambda)(2-\lambda)+1$

$4 - 4\lambda + \lambda^2 + 1 = 0$

$\lambda^2 - 4\lambda + 5 = 0$

$\lambda = \dfrac{4 \pm \sqrt{16 - 4(5)}}{2}$

$\lambda = 2 \pm i$

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