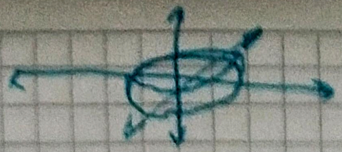
a) $I_z = \frac{mR^2}{2}$ } Inercia de disco

① $I_z = I_x + I_y$ pero $I_x = I_y$ por simetría

② $\frac{I_z}{2} = I_{xy}$ } $I_{xy} = \frac{mR^2}{4}$

Finalmente, por ejes paralelos, como centro está en d.

$$I_0 = \frac{mR^2}{4} + md^2$$



b) $\iiint (x^2 + y^2)\,\delta\,dV$ $\quad\Big|\quad \delta = \frac{M}{V} = \frac{M}{2\pi R^2 h}$

$x = r\cos\theta$
$y = r\,\mathrm{sen}\,\theta$ $\qquad z = r^2$

$$= \delta \int_0^R \int_0^h \int_0^{2\pi} r^2 \cdot r\, d\theta\, dz\, dr$$

$$= \delta\, h \cdot 2\pi \int_0^R r^3 = \frac{\pi h R^4}{2}$$

$$I = \frac{M}{\pi R^2 h} \cdot \frac{2\pi h R^4}{2} = \frac{M R^2}{2}$$

c) $L = \frac{I_0}{2}(\dot\theta^2 + \dot\phi^2 \sin^2\theta) + \frac{I_z}{2}(\dot\phi\cos\theta + \dot\psi)^2 - mgd\cos\theta$

$$\frac{\partial L}{\partial \dot\phi} = \frac{I_0}{2}\left(2\dot\phi\sin^2\theta\right) + \frac{I_z}{2}\left(2\dot\phi\cos\theta\right) + \frac{I_z}{2}\left(2\dot\psi\cos\theta\right) \checkmark$$

$$\frac{\partial L}{\partial \dot\psi} = \frac{I_z}{2}\left(2\dot\phi\cos\theta + 2\dot\psi\right) = I_z(\dot\phi\cos\theta + \dot\psi) \checkmark$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta}\right) - \frac{\partial L}{\partial \dot\theta} = 0$$

$$\frac{d}{dt}\left(I_0\dot\theta\right) = \frac{2}{2}\dot\phi^2 I_0 \sin\theta\cos\theta + \frac{2}{2}\dot\phi^2 I_z(-\sin\theta\cos\theta)$$
$$- \dot\phi\dot\psi I_z \sin\theta - (-mgd\sin\theta)$$

$$I_0\ddot\theta = \dot\phi^2 \sin\theta\cos\theta(I_0 - I_z) - \dot\phi\dot\psi I_z\sin\theta + mgd\sin\theta \checkmark$$