② $y_{n+1} = y_n + \frac{h}{12}\left(23 f_n - 16 f_{n-1} + 5 f_{n-2}\right)$

$$P_3(t) = \frac{(t - t_{n-1})(t - t_n)}{(t_{n-2} - t_{n-1})(t_{n-2} - t_n)} f_{n-2}$$

$$+ \frac{(t - t_{n-2})(t - t_n)}{(t - t_{n-2})(t_{n-1} - t_n)} f_{n-1} + \frac{(t - t_{n-2})(t - t_{n-1})}{(t_n - t_{n-2})(t_n - t_{n-1})}$$

$$P_3(t) = \frac{(t - t_{n-1})(t - t_n)}{2h^2} f_{n-2} + \frac{(t - t_{n-2})(t - t_n)}{-h^2} f_{n-1}$$

$$+ \frac{(t - t_{n-2})(t - t_{n-1})}{2h^2} f_n$$

* Integrar respecto a t:

$$\int_{y_n}^{y_{n+1}} P_3(t) = \frac{f_{n-2}}{2h^2} \int_{t_n}^{t_{n+1}} (t - t_{n-1})(t - t_n)\, dt + \frac{f_{n-1}}{-h^2} \int_{t_n}^{t_{n+1}} (t - t_{n-2})(t - t_n)\, dt$$

$$+ \frac{f_n}{2h^2} \int_{t}^{t_{n+1}} (t - t_{n-2})(t - t_{n-1})\, dt$$

* Integral respecto a t, límite h:

$$y_{n+1} = y_n + f_{n-2} \int_0^h \frac{(t+h)\, t\, dt}{2h^2} + f_{n-1} \int_0^h \frac{(t+2h)\, t}{-h^2}\, dt$$

$$+ f_n \int_0^h \frac{(t+2h)(t+h)}{2h^2}\, dt$$

$$K_3 : y_{n+1} = y_n + \frac{h}{12}\left(23 f_n - 16 f_{n-1} + 5 f_{n-2}\right)$$

$$P_4(t) = \frac{(t-t_{n-2})(t-t_{n-1})(t-t_n)}{(t_{n-3}-t_{n-2})(t_{n-3}-t_{n-1})(t_{n-3}-t_n)} f_{n-3}$$

$$+ \frac{(t-t_{n-3})(t-t_{n-1})(t-t_n)}{(t_{n-2}-t_{n-3})(t_{n-2}-t_{n-1})(t_{n-2}-t_n)} f_{n-2}$$

$$+ \frac{(t-t_{n-3})(t-t_{n-2})(t-t_n)}{(t_{n-1}-t_{n-3})(t_{n-1}-t_{n-2})(t_{n-1}-t_n)} f_{n-1}$$

$$+ \frac{(t-t_{n-3})(t-t_{n-2})(t-t_{n-1})}{(t_n-t_{n-3})(t_n-t_{n-2})(t_n-t_{n-1})} f_n$$

* Realizando las integraciones se llega a:

$$y_{n+1} = y_n + f_{n-3}\int_0^h \frac{(t+2h)(t+h)(t)\,dt}{-6h^3} + f_{n-2}\int_0^h \frac{(t+3h)(t+h)(t)}{2h^3}\,dt$$

$$+ f_{n-1}\int_0^h \frac{(t+3h)(t+2h)(t)}{-2h^3}\,dt + f_n\int_0^h \frac{(t+3h)(t+2h)(t+h)\,dt}{6h^3}$$

K_4:

$$y_{n+1} = y_n + \frac{h}{24}\left(55 f_n - 59 f_{n-1} + 37_{n-2} - 9 f_{n-3}\right)$$

③

$$P_3(t) = \frac{(t-t_n)(t-t_{n+2})}{(t_{n-1}-t_n)(t_{n-1}-t_{n+2})} f_{n-1}$$

$$+ \frac{(t-t_{n-1})(t-t_{n+1})}{(t_n-t_{n-1})(t_n-t_{n+1})} f_n + \frac{(t-t_{n-1})(t-t_n)}{(t_{n+1}-t_n)(t_{n+2}-t_{n-1})} f_{n+1}$$

$$y_{n+1} = y_n + f_{n-1}\int_0^h \frac{(t)(t-h)}{2h^2}\,dt + f_n \int_0^h \frac{(t+h)(t-h)}{-h^2}$$

$$+ f_{n+1}\int_0^h \frac{(t)(t+h)}{2h^2}\,dt$$

k_3 :

$$y_{n+1} = y_n + \frac{h}{12}\left(5 f_{n+1} + 8 f_n - f_{n-1}\right)$$

$$P_4(t) = \frac{(t-t_{n-1})(t-t_n)(t-t_{n+1})}{(t_{n-2}-t_{n-1})(t_{n-2}-t_n)(t_{n-2}-t_{n+1})} f_{n-2} + \frac{(t-t_{n-2})(t-t_n)(t-t_{n+1})}{(t_{n-1}-t_{n-2})(t_{n-1}-t_n)(t_{n-1}-t_{n+1})} f_{n-1}$$

$$+ \frac{(t-t_{n-2})(t-t_{n-1})(t-t_{n+1})}{(t_n-t_{n-2})(t_n-t_{n-1})(t_n-t_{n+1})} f_n + \frac{(t-t_{n-2})(t-t_n)(t-t_{n-1})}{(t_{n+1}-t_{n-2})(t_{n+1}-t_n)(t_{n+1}-t_{n-1})} f_{n+1}$$

$$y_{n+1} = y_n + f_{n-2}\int_0^h \frac{(t+h)(t)(t-h)}{-6h^3} + f_{n-1}\int_0^h \frac{(t+2h)(t)(t-h)}{2h^3}$$

$$+ f_n \int_0^h \frac{(t+2h)(t+h)(t-h)}{-2h^3} + f_{n+1}\int_0^h \frac{(t+2h)(t+h)(t)}{6h^3}$$

$$y_{n+1} = y_n + \frac{h}{24}\left(9 f_{n+1} + 19 f_n - 5 f_{n-1} + f_{n-2}\right)$$

④ $$\begin{pmatrix} \vec{x}_{n+1} \\ \vec{v}_{n+1} \end{pmatrix} = \begin{pmatrix} \vec{x}_n + \vec{v}_n h + \frac{1}{2} \vec{a}_n h^2 \\ \vec{v}_n + \frac{h}{2} \left(\vec{a}_n + \vec{a}_{n+1} \right) \end{pmatrix}$$

$$J = \begin{vmatrix} \frac{\partial \vec{x}_{n+1}}{\partial \vec{x}_n} & \frac{\partial \vec{x}_{n+1}}{\partial \vec{v}_n} \\ \frac{\partial \vec{v}_{n+1}}{\partial \vec{x}_n} & \frac{\partial \vec{v}_{n+1}}{\partial \vec{v}_n} \end{vmatrix} = \frac{\partial \vec{x}_{n+1}}{\partial x_n} \cdot \frac{\partial \vec{v}_{n+1}}{\partial v_n} - \underset{\uparrow}{0}$$

$$\frac{\partial \vec{v}_{n+1}}{\partial \vec{x}_n} = 0$$

$$J = (1 \cdot 1 - 0) = 1$$