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IPSAL: Implementation of the Morris Elementary Effects Method

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Abstract. *The importance of sensitivity analysis for understanding the impact of inputs on a model's output is crucial. The study identifies which inputs are influential in a model. Model sensitivity assessment can be performed locally, focusing around a nominal point in the input sample space, or globally, considering variations across the entire range of input variability. The Morris method is a "one-at-a-time" global analysis technique, examining one input at a time. It generates sets of model inputs using a random sampling strategy, achieved through what are known as trajectory matrices. The Morris method module in the Inverse Problem and Sensitivity Analysis Library developed in SciLab is applied to a practical case as the focus of this work.*

Keywords: *Elementary Effect; Sensitivity Analysis; Morris Method; Change Sensitivity; Sampling Strategy*

1. INTRODUCTION

The increase in complexity of mathematical models, driven by the growth of available data and expanding computational capabilities, demands deeper analyses. Such mathematical models require input data to feed them, and from these an output is expected (Borgonovo & Plischke, 2016). Understanding the impact of these input data on a model's output is of paramount importance, as it allows comprehending their influence (Saltelli et-al., 2019; Ferreira et-al., 2020). However, this knowledge is not always taken into consideration. A mathematical tool employed for this purpose is Sensitivity Analysis (SA). In a broader sense, SA is the study of how a model's outputs are related to and influenced by its inputs (Razavi et-al., 2021; Horita et-al., 2022). In the literature review on SA, there are several studies on this subject, in which decision-making is presented in hypothetical scenarios or on its effectiveness.

To a certain extent, SA is related to Design of Experiments (DOE), which utilizes a suite of statistical tools to systematically classify and evaluate cause-and-effect relationships between input and output factors in the studied model (Jankovic et-al., 2021). SA dates back to the 1970s and 1980s, with the wide availability of computers for computational modeling. Since then, DOE has been extended to design and analysis of computer experiments (DACE), which is typically "noise-free". Consistent with the SA standard, DOE provides a set of tools that permeate the study of the individual and combined effects of inputs on the model output.

SA can be viewed through two distinct approaches: local analysis and global analysis (Campolongo, 2011). The distinction between these approaches is the fact that in local analysis the sensitivity of the model is evaluated by setting a value for a given input, perturbing it with Δ, throughout the model domain (Razavi, 2015). On the other hand, in global analysis, sensitivity analysis evaluates the model's outputs induced by one input across various distinct values within a sample space (Pianosi et-al., 2016). Therefore, it is possible to work with dispersion measures to evaluate the quality of the input (Ge & Menendez, 2017).

The Morris method (1991) is a technique that generates sets of model inputs randomly stored in directional matrices. The method evaluates one input at a time (On-At-a-Time or OAT) and is considered a global SA analysis method (Franczyk, 2019), where it calculates the elementary effects (EE) of the model with respect to each input for each randomly generated set (Ruano et-al., 2012). This efficient and reliable method identifies and classifies important variables that are based on elementary effects (King & Perera, 2013).

The objective of this work is to introduce the implementation of the Morris method in the Inverse Problem and Sensitivity Analysis Library (IPSAL).

2. MATERIALS AND METHODS

Consider a mathematical and/or computational model $y(x_1, \ldots, x_k)$, where the inputs are components of the vector $\mathbf{x} = (x_1, \ldots, x_k)$. Here, k represents the total number of inputs for the model. Consider that each of these entries belongs to an edge of a unitary hypercub, Ω, i.e., $x_i \in [0, 1]$, where $i = 1, \ldots, k$, and are obtained through random directional matrices. So, the effect at each input, also called elementary elementary, given by

$$EE|_{x_i} = \frac{y(x_1, ..., x_{i-1}, x_i + \Delta, x_{i+1}, ..., x_k) - y(\mathbf{x})}{\Delta} \tag{1}$$

A directional matrix of order $(k+1) \times k$ is formed, where for each column j, there are two rows differing only in their j-th entry. Denoted as B, the directional matrix of order $(k+1) \times k$ is given by

$$B_{k+1,k} = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & 1 & 1 & \cdots & 0 \\ 1 & 1 & 1 & \cdots & 1 \end{pmatrix} \tag{2}$$

The method involves constructing trajectory matrices $B_i^\star$, where $i = 1, \ldots, r$, and r is the total number of trajectories. Each trajectory matrix is obtained using the following expression:

$$B_i^\star = \left(J_{k+1,1}\mathbf{x}_{1,k}^\star + (\Delta/2)\left[(2B_{k+1,k} - J_{k+1,k})\, D_{k,k}^\star + J_{k+1,k}\right]\right) P_{k,k}^\star \tag{3}$$

where Δ takes the value $1/(p-1)$, p is referred to as the Morris method level, and $p-1$ represents the number of subintervals in the interval $[0, 1]$; $D^\star$ is a diagonal matrix of order k, with each diagonal element being randomly assigned as $+1$ or -1, each with equal probability; J is a matrix of dimensions $k+1$ by k with all entries equal to 1. It's notable that the matrix $(1/2)\left[(2B - J)\, D^\star + J\right]$ is a matrix of dimensions $k+1$ by k, where each defined column equals the corresponding column of B or is obtained by substituting 1 with 0 and vice versa; $P^\star$ is a random permutation matrix constructed from the identity matrix of dimensions k by k, where each column contains a single element equal to 1 while all other elements are set to 0, and no two columns have 1 in the same row; $\mathbf{x}^\star$ is a row matrix where each element $x_i^\star$, $i = 1, \ldots, k$, is randomly chosen from $\tilde{\mathbf{x}} = (0, 1/(p-1), 2/(p-1), \ldots, 1-\Delta)$. The number of components in $\tilde{\mathbf{x}}$ depends on the choice of calculating Δ.

Following the calculation of the r independent random orientations of B, the trajectories are concatenated to form the design matrix X for the entire experiment, given by

$$X = \begin{bmatrix} B_1^\star \\ B_2^\star \\ \vdots \\ B_r^\star \end{bmatrix} \tag{4}$$

After the construction of the matrix X, the elementary effects are calculated using equation (1) for each input x_i, where $i = 1, \ldots, k$, resulting in k elementary effects. For each elementary effect, the model is executed twice. Thus, to compute all the elementary effects for a single input, there will be a total of $2rk$ model executions.

Figure 1 illustrates the workflow in the computational code.

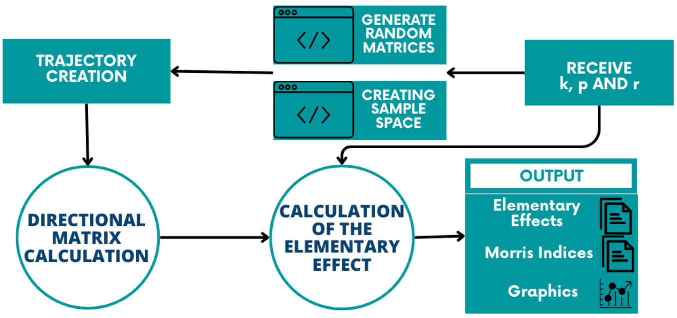


Figure 1: Flowchart for the Calculation of Elementary Effects..

To assess the effects of input parameters on the model's output, the mean absolute value of elementary effects will be used (Campolongo et al., 2007),

$$\mu_i^\star = \frac{1}{rk} \sum_{j=1}^{rk} |\mathrm{EE}i, j| \tag{5}$$

The standard deviation of the elementary effect is described as

$$\sigma i = \sqrt{\frac{1}{rk}\sum_{j=1}^{rk}(\mathrm{EE}_{i,j} - \mu_i^\star)^2} \tag{6}$$

where $i = 1, \ldots, k$ represents the input of the model, and j iterates through the elementary effects of that input.

The values $\mu_j^\star$ and σ_j are utilized to analyze the inputs across various points in the sample space, as well as the distributional properties of these values. Thus, the global sensitivity of the model is assessed for each individual input. A higher value of $\mu_j^\star$ indicates higher sensitivity of the model with respect to the j-th input. The standard deviation of the EE, as in equation (6), indicates potential relationships with other variables and/or that the variable has a nonlinear effect on the output (Franczyk, 2019; Campolongo & Braddock, 1999).

2.1 Código computacional

The IPSAL library should be downloaded and saved in the user's working folder. Afterward, the library functions for the Morris method should be declared.

```
address  = get_absolute_file_path('MORRIS.sce');
exec(address+'\functions\modelo.sci');
exec(address+'\functions\funcaoprincipa.sci');
exec(address+'\functions\calculo.sci');
exec(address+'\functions\graph_distribuicao_entrada.sci');
exec(address+'\functions\graph_barra_horizontal.sci');
exec(address+'\functions\graph_scarter_media_desvio.sci');
exec(address+'\functions\matriz_desing.sci');
```

To perform sensitivity analysis using the Morris method, you should save the model to be analyzed as **modelo.sci**, define the values of r, k, and p, and call the function.

```
[B,x_] = matriz_desing(r,k,p)
```

to construct the design matrix. For the calculation of elementary effects, the function is used.

```
[EE,media,desvio]=calculo(k,r,B)
```

Returning the elementary effects, the mean of the absolute values of the elementary effects, and the standard deviation.

To generate the graphs, graphical functions are used.

```
graph_distribuicao_entrada(address,r,k,x_)
graph_barra_horizontal(address,media,desvio,r,k)
graph_scarter_media_desvio(address,media,desvio,k,r)
```

In Figure 2, the organizational structure of the code developed in SciLab is illustrated. The *results* folder will be responsible for storing all the outcomes obtained with the method.

2.2 Test Case

Consider a model described by the function:

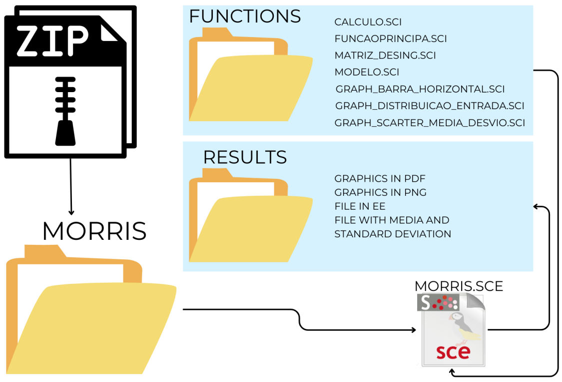


Figure 2: Organizational structure of the code.

$$f(\mathbf{x}^\star) = x_1 + x_2^2 + x_2 \sin(x_3) + x_4 \tag{7}$$

where $\mathbf{x}^\star = (x_1, x_2, x_3, x_4)$ represents the input data. The Morris method will be employed for r random orientations based on Equation 3, aiming to estimate the sensitivity of $f(\mathbf{x}^\star)$ (Equation 7) with respect to the inputs $\mathbf{x}^\star = (x_1, x_2, x_3, x_4)$. These inputs can be regarded as independent random variables uniformly distributed within the unit interval $[0, 1]$. The typical choice of p for SA (Sensitivity Analysis) is 4, 6, 8 due to their uniform distribution (Franczyk, 2019). In the current article, the value $p = 6$ will be utilized, leading to $\Delta = 1/5$, that is, $\tilde{\mathbf{x}} = (0, 1/5, 2/5, 3/5, 4/5)$.

3. RESULTS AND DISCUSSION

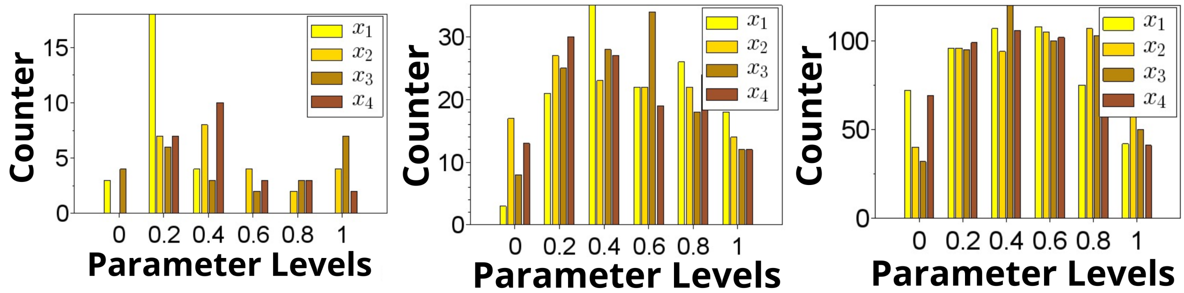
Through the test case presented in Section 2.2, the design matrix, as given by equation (4), is constructed with r different trajectory matrices. Table 1 presents the first trajectory matrix $B_1^\star$ for various values of r. For the parameter x_4 and $r = 5$, it is observed that nearly all the values are 0.4. Similarly, the values 0.2 and 0.8 appear repeatedly for the parameter x_2 in the cases of $r = 25$ and $r = 100$, respectively. Since the selection of x_i values is random, this emphasizes the necessity of choosing a suitable value for r" that ensures all possible **x** values are covered by the algorithm.

Table 1: First calculated trajectory matrix for $r = 5$, $r = 25$ e $r = 100$.

$r = 5$				$r = 25$				$r = 100$			
x_1	x_2	x_3	x_4	x_1	x_2	x_3	x_4	x_1	x_2	x_3	x_4
0.0	0.4	0.2	0.4	0.8	0.2	0.8	0.4	0.6	0.8	1.0	0.2
0.2	0.4	0.2	0.4	0.8	0.2	0.8	0.6	0.6	0.8	0.8	0.2
0.2	0.2	0.2	0.4	0.8	0.2	1.0	0.6	0.6	0.8	0.8	0.0
0.2	0.2	0.0	0.4	0.6	0.2	1.0	0.6	0.4	0.8	0.8	0.0
0.2	0.2	0.0	0.6	0.6	0.4	1.0	0.6	0.4	0.6	0.8	0.0

In Figure 3, the random distribution of values from the set $\{0, 0.2, 0.4, 0.6, 0.8, 1\}$ can be observed, which each component x_i of the vector $\mathbf{x}^\star = (x_1, x_2, x_3, x_4)$ is forced to assume. For

$r = 5$, there is a sparsity of values for the parameters x_3 and x_4. However, for $r = 25$ and $r = 100$, all components of $\mathbf{x}^\star$ encompass the complete set of values.



(a) $r = 5$ (b) $r = 25$ (c) $r = 100$

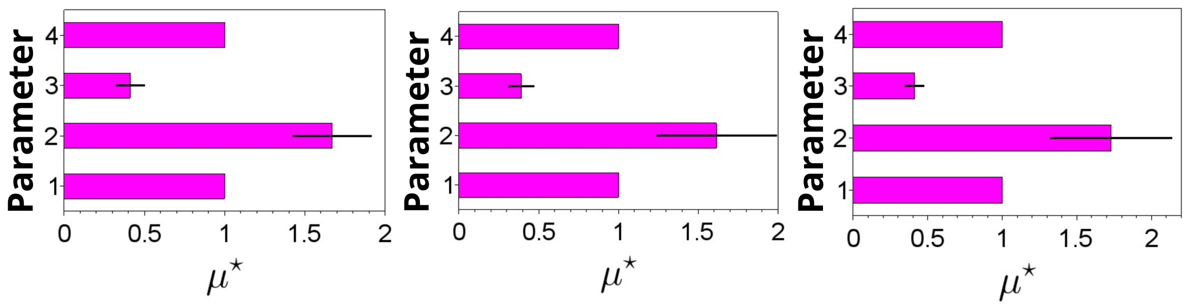
Figure 3: Number of times each value was randomly chosen for each input of the model.

The Morris method consists of analyzing and relating the mean and standard deviation, equations (5) and (6), of the elementary effects, equation (1), for each input. Table 2 presents these results. It can be observed that the parameters x_1 and x_4 have constant means, $\mu^\star = 1$, and zero standard deviation. These results will be further evaluated later on.

Table 2: Mean and standard deviation of elemental effects for different values of r, for each input.

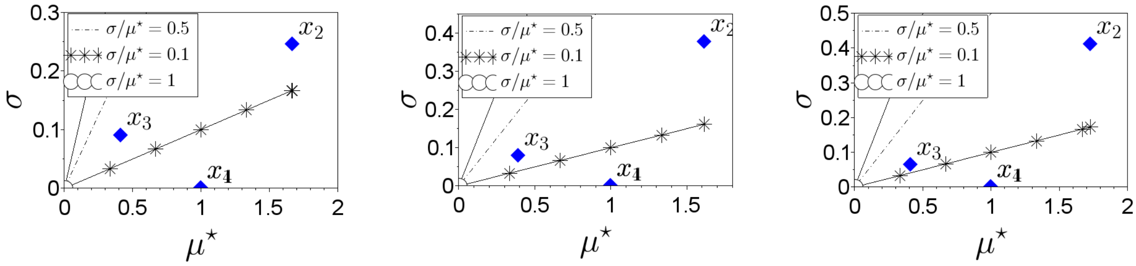
	$r = 5$			$r = 25$			$r = 100$		
$\mathbf{x}$	$\mu_j^\star$	μ_j	σ_j	$\mu_j^\star$	μ_j	σ_j	$\mu_j^\star$	μ_j	σ_j
x_1	1.000	1.000	0.000	1.000	1.000	0.000	1.000	1.000	0.000
x_2	1.670	1.670	0.246	1.615	1.615	0.301	1.730	1.730	0.423
x_3	0.412	0.412	0.090	0.391	0.391	0.080	0.412	0.412	0.064
x_4	1.000	1.000	0.000	1.000	1.000	0.000	1.000	1.000	0.000

There are two graphical ways to represent the indices calculated by the Morris method, as shown in figures 4 and 5.



(a) $r = 5$ (b) $r = 25$ (c) $r = 100$

Figure 4: Mean and standard deviation for each model entry.



(a) $r = 5$ (b) $r = 25$ (c) $r = 100$

Figure 5: Morris method results for inputs x_1, x_2, x_3 and x_4.

Influential inputs exhibit high average elementary effects. As can be observed in figures 4 and 5, inputs x_1, x_2, and x_4 are more influential than input x_3, as variations in these inputs result in a significant change in the model. The small standard deviation of the elementary effects for inputs x_1 and x_4 indicates a linear effect. This linear effect is evident in Equation (7). A high standard deviation, especially if it's of the same order of magnitude as the elementary effect, suggests nonlinearity or interactions with other inputs. As observed in Equation (7) and in figures 4 and 5, inputs x_2 and x_3 exhibit nonlinearity within the model, with input x_2 showing a stronger nonlinear interaction compared to x_3.

In the Table 3, the evaluations of the entries are presented in summary form.

Table 3: Summary of Effects on Model Inputs.

x	$\mu_j^\star$	Assessment	σ_j	Assessment
x_1	1.000	Influential	0.000	Linear Effect
x_2	1.653	Most Influential	0.423	Nonlinear effect or Iteration with another input
x_3	0.390	Less Influential	0.065	Low Nonlinear Effect or Iteration with another input
x_4	1.000	Influential	0.000	Linear Effect

4. CONCLUSIONS

The obtained results align with the model used in the test case. Linear inputs yielded a standard deviation of zero, while nonlinear inputs had nonzero values. All inputs are influential, some more than others, but each is important in the model. The SciLab code developed is well-structured and organized, enabling the sensitivity analysis using the Morris method for any model with just a slight adaptation to the model's code.

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