

ν -sim

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1 Numerical description

In general:

$$i \frac{d\psi}{dt} = \mathcal{H}\psi \Rightarrow \frac{d}{dt} \begin{bmatrix} \Re \\ \Im \end{bmatrix} = \begin{bmatrix} \mathcal{H}_{\Im} & \mathcal{H}_{\Re} \\ -\mathcal{H}_{\Re} & \mathcal{H}_{\Im} \end{bmatrix} \begin{bmatrix} \Re \\ \Im \end{bmatrix},$$

where $\psi = \Re + i\Im = (\Re_1 + i\Im_1, \dots, \Re_n + i\Im_n)$, $\mathcal{H} = \mathcal{H}_{\Re} + i\mathcal{H}_{\Im}$ being $\Re, \Im, \mathcal{H}_{\Re}, \mathcal{H}_{\Im}$ real.

The original complex system of n equations becomes a real system with $2n$ equations.

In the case of Neutrino Oscillations in matter, our hamiltonian is always of the form:

$$\mathcal{H} = \mathcal{H}^0 + \text{diag}(V(L), 0, \dots, 0),$$

where \mathcal{H}^0 is constant and $V(L) = \sqrt{2}G_F N_e(L)$ is the only parameter that depends on the traveled distance L (time also, because $L = ct = t$). G_F is the Fermi constant and $N_e(L)$ is the solar electron density. The hamiltonian \mathcal{H} is so simple because the only neutrino that interacts with solar matter is the neutrino ν_e of the electron.

The matrix \mathcal{H}^0 is simply given by the sandwich:

$$\mathcal{H}^0 = U M U^\dagger,$$

where U is the neutrino mixing matrix (it's only a unitary matrix, and it has standard [parametrization](#)), and M is the diagonal matrix corresponding to the mass eigenvalues of the neutrinos in vacuum (it can be simplified, making one of its entries zero).

Now, the algorithm is very simple:

1. Assume U, M and a table $L \times N_e(L)$ of data as input.
2. Using the following structures of the [GSL Library](#):

```
gsl_complex, gsl_matrix_complex, gsl_matrix;
```

we easily calculate $\mathcal{H}^0 = \mathcal{H}_{\Re}^0 + i\mathcal{H}_{\Im}^0$ by complex matrix operations and then obtain $\mathcal{H}_{\Re}^0, \mathcal{H}_{\Im}^0$ by taking the real and imaginary parts with the C macros:

```
GSL_REAL, GSL_IMAG.
```

3. Interpolate $N_e(L)$ using [Bahcall's data](#) and compute $V(L) = \sqrt{2}G_F N_e(L)$ for $-R_\odot \leq L \leq R_\odot$, where R_\odot is the solar radius. Here we use the following header and type from GSL:

```
#include <gsl/gsl_spline.h>
gsl_interp_steffen,
```

which by the last [1D Interpolation Example](#) seems to be the best spline for the case.

4. Now we simply solve the following ODE numerically and print the results.

$$\frac{d}{dt} \begin{bmatrix} \Re \\ \Im \end{bmatrix} = \begin{bmatrix} \mathcal{H}_{\Im} & \mathcal{H}_{\Re} \\ -\mathcal{H}_{\Re} & \mathcal{H}_{\Im} \end{bmatrix} \begin{bmatrix} \Re \\ \Im \end{bmatrix},$$

where $\mathcal{H}_{\Re} = \mathcal{H}_{\Re}^0 + \text{diag}(V(L), 0, \dots, 0)$ and $\mathcal{H}_{\Im} = \mathcal{H}_{\Im}^0$. For this we use the header

```
#include <gsl/gsl_odeiv2.h>.
```