MAT 140 6.3.10-Planes # 4af, 8ab, 12, 24, 32, 36c, 40, 49, 64, 70, 74, 94

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4.a.

Vector: $X=(1,-1,0)+t\begin{bmatrix}0\\2\\1\end{bmatrix}+s\begin{bmatrix}2\\-1\\-2\end{bmatrix}$

parametric $\begin{cases}x=1+2s\\y=-1+2t-s\\z=t+2s\end{cases}$

normal: $-3x+2y-4z=-5$

$\vec{d}=Q-P \quad \vec{D}=R-Q$

$\vec{d}=\begin{bmatrix}1-1\\1+1\\1-0\end{bmatrix} \quad \vec{D}=\begin{bmatrix}3-1\\0-1\\-1-1\end{bmatrix}$

$\vec{d}\begin{bmatrix}0\\2\\1\end{bmatrix} \quad \vec{D}\begin{bmatrix}2\\-1\\-2\end{bmatrix}$

$\vec{n}=\begin{bmatrix}0\\2\\1\end{bmatrix}\times\begin{bmatrix}2\\-1\\-2\end{bmatrix}$

$\vec{n}=\begin{bmatrix}2\cdot-2-1\cdot-1\\1\cdot2-0\cdot-2\\0\cdot-1-2\cdot2\end{bmatrix}$

$\vec{n}=\begin{bmatrix}-4+1\\2-0\\0-4\end{bmatrix}$

$\vec{n}=\begin{bmatrix}-3\\2\\-4\end{bmatrix}$

$-3x+2y-4z=k$

$1(-3)+1(2)+1(-4)=k$

$-3+2-4=k$

$-5=k$

f.

Vector: $X=(3,2,-3)+t\begin{bmatrix}3\\1\\4\end{bmatrix}+s\begin{bmatrix}1\\1\\1\end{bmatrix}$

parametric $\begin{cases}x=3+3t+s\\y=2+t+s\\z=-3+4t+s\end{cases}$

normal: $3x+7y-4z=35$

$3x+7y-4y=k$

$3(3)+7(2)-4(-3)=k$

$9+14+12=k$

$35=k$

f. (cont.)

$3(d_1)+7(d_2)-4(d_3)=0$

$3(3)+7(1)-4(d_3)=0$

$9+7-4d_3=0$

$-4d_3=-16$

$d_3=4 \qquad d=\begin{bmatrix}3\\1\\4\end{bmatrix}$

$3(D_1)+7(D_2)-4(D_3)=0$

$3(D_1)+7(1)-4(1)=0$

$3D_1+7-4=0$

$3D_1=3 \qquad D=\begin{bmatrix}1\\1\\1\end{bmatrix}$

$D_1=1$

8.a. $\begin{bmatrix}2\\9\\12\end{bmatrix}$

$\begin{bmatrix}3\\-2\\0\end{bmatrix}\times\begin{bmatrix}3\\2\\-2\end{bmatrix}$

$\begin{bmatrix}-2\cdot-2-0\cdot2\\0\cdot3-3\cdot-2\\3\cdot2--2\cdot3\end{bmatrix}$

$\begin{bmatrix}4-2\\3+6\\6+6\end{bmatrix}$

$\begin{bmatrix}2\\9\\12\end{bmatrix}$

b. 14

$\begin{bmatrix}3-3\\2+2\\-2-6\end{bmatrix}\times\begin{bmatrix}3-6\\-2-0\\0-0\end{bmatrix}$

$\begin{bmatrix}0\\4\\-2\end{bmatrix}\times\begin{bmatrix}-3\\-2\\0\end{bmatrix}$

$\begin{bmatrix}4\cdot0-2\cdot-2\\-2\cdot3-0\cdot0\\0\cdot-2-4\cdot3\end{bmatrix}$

$\begin{bmatrix}0+4\\-6-0\\0-12\end{bmatrix}$

$\begin{bmatrix}4\\-6\\-12\end{bmatrix}$

$\sqrt{4^2+(-6)^2+(-12)^2}$

$\sqrt{16+36+144}$

$\sqrt{196}$

14

12. 6

$\dfrac{|1(3)+2(-1)+-2(6)-7|}{\|\vec{n}\|}$

$\dfrac{|3-2-12-7|}{\sqrt{1^2+2^2+(-2)^2}}$

$\dfrac{|-18|}{\sqrt{1+4+4}}$

$\dfrac{18}{\sqrt{9}}$

$\dfrac{18}{3}$

6

24. $\dfrac{2\sqrt{6}}{3}$

$\vec{n}=\begin{bmatrix}1\\3\\1\end{bmatrix}\times\begin{bmatrix}1\\-1\\-1\end{bmatrix}$

$=\begin{bmatrix}3\cdot-1-1\cdot-1\\1\cdot1-1\cdot-1\\1\cdot-1-3\cdot1\end{bmatrix}$

$=\begin{bmatrix}-3+1\\1+1\\-1-3\end{bmatrix}$

$\vec{n}=\begin{bmatrix}-2\\2\\-4\end{bmatrix}=\begin{bmatrix}-1\\1\\-2\end{bmatrix}$

$-3+4-2(3)=k$

$1-6=k$

$-5=k$

$\dfrac{|-1(2)+1(-1)-2(-1)+5|}{\|\vec{n}\|}$

$\dfrac{|-2-1+2+5|}{\sqrt{(-1)^2+1^2+(-2)^2}}$

$\dfrac{|4|}{\sqrt{1+1+4}}$

$\dfrac{4}{\sqrt{6}}$

$\dfrac{2\sqrt{6}}{3}$

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32. $\sqrt{174}/6$

$$\vec{d} = \begin{bmatrix}1-5\\1+1\\1-3\end{bmatrix} = \begin{bmatrix}-4\\2\\-2\end{bmatrix} = \begin{bmatrix}-2\\1\\-1\end{bmatrix}$$

$$\frac{\|(P-Q)\times\vec{d}\|}{\|\vec{d}\|}$$

$$\frac{\left\|\begin{bmatrix}1-1\\3-1\\5-1\end{bmatrix}\times\begin{bmatrix}-2\\1\\-1\end{bmatrix}\right\|}{\sqrt{(-2)^2+1^2+(-1)^2}}$$

$$\frac{\left\|\begin{bmatrix}2\cdot-1-4\cdot1\\4\cdot-2-0\cdot-1\\0\cdot1-2\cdot-2\end{bmatrix}\right\|}{\sqrt{4+1+1}}$$

$$\frac{\left\|\begin{bmatrix}-2-4\\-8-0\\0+4\end{bmatrix}\right\|}{\sqrt{6}}$$

2 6
1 2
-2 -4

$$\frac{\left\|\begin{bmatrix}-6\\-8\\4\end{bmatrix}\right\|}{\sqrt{6}}$$

$$\frac{\sqrt{(-3)^2+(-4)^2+2^2}}{\sqrt{6}}$$

$$\frac{\sqrt{9+16+4}}{\sqrt{6}}$$

$$\frac{\sqrt{29}}{\sqrt{6}}$$

$$\frac{\sqrt{174}}{6}$$

36. 72.2341°

$$\begin{bmatrix}2\\4\\-3\end{bmatrix}\cdot\begin{bmatrix}5\\1\\2\end{bmatrix} = \sqrt{2^2+4^2+(-3)^2}\cdot\sqrt{5^2+1^2+2^2}\cdot\cos\theta$$

$$2\cdot5+4\cdot1+-3\cdot2 = \sqrt{4+16+9}\cdot\sqrt{25+1+4}\ \cos\theta$$

$$10+5-6 = \sqrt{29}\cdot\sqrt{30}\ \cos\theta$$

$$\frac{9}{\sqrt{29}\cdot\sqrt{30}} = \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{9}{\sqrt{29}\cdot\sqrt{30}}\right)$$

$$\theta = 72.2341$$

40. 123.0476 or 56.9124

$$\theta = \cos^{-1}\left(\frac{\vec{v}\cdot\vec{n}}{\|\vec{v}\|\|\vec{n}\|}\right)$$

$$\vec{v}\cdot\vec{n} = 2\cdot2+-3\cdot4+-1\cdot3$$
$$= 4-12-3$$
$$= -11$$

$$\|\vec{v}\| = \sqrt{2^2+(-3)^2+(-1)^2}$$
$$= \sqrt{4+9+1}$$
$$= \sqrt{14}$$

$$\|\vec{n}\| = \sqrt{2^2+4^2+3^2}$$
$$= \sqrt{4+16+9}$$
$$= \sqrt{29}$$

$$\theta = \cos^{-1}\left(\frac{-11}{\sqrt{14}\sqrt{29}}\right)$$

$$\theta = 123.0876 \text{ or } 56.9124$$

49. 130.1197 or 49.8803

$$n_1 = \begin{bmatrix}1\\2\\6\end{bmatrix} \quad n_2 = \begin{bmatrix}1\\-1\\2\end{bmatrix}\times\begin{bmatrix}-1\\0\\1\end{bmatrix}$$

$$= \begin{bmatrix}-1\cdot1-2\cdot0\\2\cdot-1-1\cdot1\\1\cdot0-1\cdot-1\end{bmatrix}$$

$$= \begin{bmatrix}-1\\-3\\-1\end{bmatrix}$$

$$n_1\cdot n_2 = 1\cdot-1+2\cdot-3+6\cdot-1$$
$$= -1-6-6$$
$$= -13$$

$$\|n_1\| = \sqrt{1^2+2^2+6^2}$$
$$= \sqrt{1+4+36}$$
$$= \sqrt{37}$$

$$\|n_2\| = \sqrt{(-1)^2+(-3)^2+(-1)^2}$$
$$= \sqrt{1+9+1}$$
$$= \sqrt{11}$$

$$\theta = \cos^{-1}\left(\frac{-13}{\sqrt{37}\sqrt{11}}\right)$$

$$\theta = 130.1197 \text{ or } 49.8803$$

70. 1.5367

$$d_1\times d_2 = \begin{bmatrix}2\cdot1-\,-3\cdot1\\-3\cdot2-1\cdot1\\1\cdot1-2\cdot2\end{bmatrix} = \begin{bmatrix}2+3\\-6-1\\1-4\end{bmatrix} = \begin{bmatrix}5\\-7\\-3\end{bmatrix}$$

$$Q_1-Q_2 = \begin{bmatrix}2-1\\0-1\\3+1\end{bmatrix} = \begin{bmatrix}1\\-1\\4\end{bmatrix}$$

$$\frac{|(Q_1-Q_2)\cdot(d_1\times d_2)|}{\|d_1\times d_2\|}$$

$$(Q_1-Q_2)\cdot(d_1\times d_2) = 1\cdot5+-1\cdot-7+4\cdot-3$$
$$= 5-7-12$$
$$= -14$$

$$\|d_1\times d_2\| = \sqrt{5^2+(-7)^2+(-3)^2}$$
$$= \sqrt{25+49+9}$$
$$= \sqrt{83}$$

$$\frac{|-14|}{\sqrt{83}}$$

$$\frac{14}{\sqrt{83}} = \frac{14\sqrt{83}}{83} \approx 1.5367$$

74.

$$X = (3,1,0) + t\begin{bmatrix}4\\1\\-1\end{bmatrix}$$

$$\begin{bmatrix}1\\2\\6\end{bmatrix}\times\begin{bmatrix}2\\-3\\5\end{bmatrix} = \begin{bmatrix}2\cdot5-6\cdot-3\\6\cdot2-1\cdot5\\1\cdot-3-2\cdot2\end{bmatrix} = \begin{bmatrix}28\\7\\-7\end{bmatrix} = \begin{bmatrix}4\\1\\-1\end{bmatrix}$$

$x+2y=5 \qquad 2x-3y=3$
$x=5-2y \qquad 2(5-2y)-3y=3$
$x=5-2(1) \qquad 10-4y-3y=3$
$x=3 \qquad -7y=-7$
$y=1$

94. 0.5345

$$\begin{bmatrix}0\\2\\1\end{bmatrix}\times\begin{bmatrix}2\\-4\\1\end{bmatrix}$$

$$\begin{bmatrix}2\cdot1-1\cdot-4\\1\cdot2-0\cdot1\\0\cdot-4-2\cdot2\end{bmatrix}$$

$$\begin{bmatrix}2+4\\2-0\\0-4\end{bmatrix}$$

$$\begin{bmatrix}6\\2\\-4\end{bmatrix} = \begin{bmatrix}3\\1\\-2\end{bmatrix}$$

$3x+y-2z=k_1$
$3(1)+(2)-2(1)=k_1$
$3+2-2=k_1$
$3=k_1$
$3(2)+(3)-2(4)=k_2$
$6+3-8=k_2$
$1=k_2$

$$\frac{|3-1|}{\sqrt{3^2+1^2+(-2)^2}}$$

$$\frac{|2|}{\sqrt{9+1+4}}$$

$$\frac{2}{\sqrt{14}} = \frac{\sqrt{14}}{7} \approx 0.5345$$