Quantum Implementation of a Quantum-Cognitive Model of Decision-Making

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Introduction

- Quantum computers may solve certain computations much faster than classical electronic computers
- Quantum computers process information using "quantum bits" (qubits) that can be in superposition
- We are presently in the Near Term Intermediate Scale quantum era with *noisy* and *depth-limited* quantum computers → use **hybrid, variational algorithms**

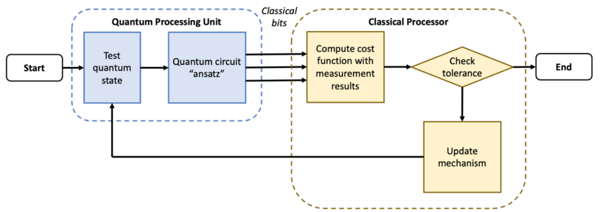


Fig. 1. The VQE uses both quantum (blue) and classical (yellow) parts connected in feedback to minimize the measured value of a quantum wave state.

Variational Quantum Eigensolvers (VQE)

- Solve for the lowest eigenenergy state by making a parameterized circuit, in Python Qiskit (compatible with IBM quantum hardware)

1 qubit

$$|0\rangle - R_x(t_1) - R_y(t_2) - |\psi\rangle \qquad |\psi\rangle = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) \\ e^{i\varphi}\sin\left(\frac{\theta}{2}\right) \end{pmatrix}$$

Fig. 2. A basic one qubit quantum circuit has just two angles to optimize over.

2 qubit

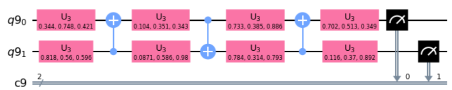


Fig. 3. At two qubits and beyond, quantum entanglement must be considered and circuit complexity increases.

$$\lambda_{min} \leq \lambda_{theta} \equiv \langle \psi(\theta)|H|\psi(\theta)\rangle$$

- Difficult to use "**universal**" ansatzes for larger circuits
- Consider heuristics-based designs
- **Trotterized** potential well; quantum Fourier transform between position and momentum basis

$$\langle j|X_{pos}|k\rangle = \sqrt{\frac{2\pi}{n}}\, l(j)\delta_{j,k} \qquad P_{pos} = F^{\dagger}X_{pos}F$$

Application to cognitive science

- Human decision making can be modelled as a multiple potential wells problem
- Probability of finding a particle in a well corresponds to likelihood of taking that decision
- Solve for lowest few energy states corresponding to decision probabilities

Finding multiple states

While VQE typically only returns the ground state, Nakanishi et al. (2019) proposed a Subspace Search variant: orthogonal input wave states (|0000⟩, |0101⟩ .etc) map to each eigenenergy state

$$H = \frac{P^2}{2} + \frac{X^2}{2}$$

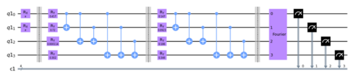


Fig. 4. An instance of "Ry-style" ansatz using two layers and four qubits, with CNOT entangling gates.

VQE algorithm design factors

Ansatz choice

Hardware Optimized vs Ry style
Method of entanglement
Subspace Search VQE

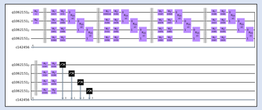


Fig. 5. An instance of "Hardware optimized" ansatz with four qubits.

Classical Optimizer

Simultaneous Perturbation Stochastic Approximation (SPSA)
Constrained Optimization by Linear Approximation (COBYLA)

Measuring the Hamiltonian

Can do measurement in position basis, and then momentum basis via a Fourier Transform
Or decompose Hamiltonian into tensor products

$$A = \sum_{i,j,k,l} h_{ijkl} \cdot \frac{1}{4}\sigma_i \otimes \sigma_j \otimes \sigma_k \otimes \sigma_l$$

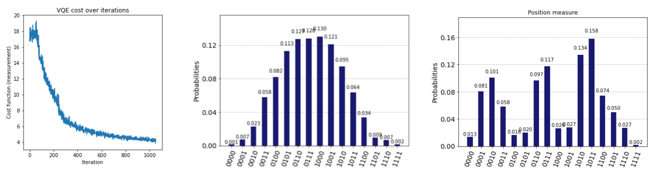
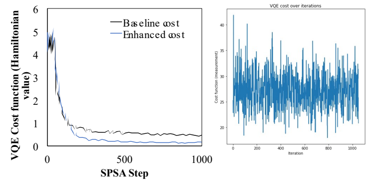


Fig. 6. (L to R) Example cost function with full ansatz, the resultant ground and third energy states found (using SSVQE).

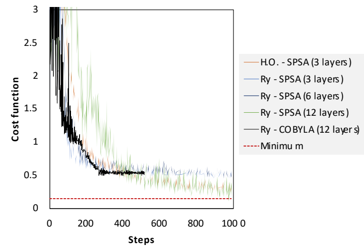
Results and discussion

- Standard VQE allows us to find ground state eigenstate and wave function in the position basis
- With more steps and complexity, the lowest four eigenstates can be found by SSVQE
- Simultaneous Perturbation Stochastic Approximation algorithm worked best; have to use stable parameters for convergence



SPSA settings could not be adjusted too aggressively

Fig. 7. A more aggressive classical optimizer tuning led to improved results for a low dimension problem but did not work as well with multiple qubits.



Convergence results of different ansatz types

Fig. 8. Cost functions for various optimizers had similar trends overall.

Conclusion

- Variational quantum eigensolver can be implemented for Hamiltonian cost estimation on near term quantum machines
- Different circuit gate arrangements possible
- Large cost of optimization steps in basic approach

References

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