Jakub Gruś

$$\begin{cases} -\dfrac{d^2 u(x)}{d^2 x} - u = \sin x \\ u(0) = 1 \\ \dfrac{du}{dx} - u(2) = 5 \Rightarrow \dfrac{du(2)}{dx} = 5 + u(2) \\ [0,2] \ni x \to u(x) \in \mathbb{R} \end{cases}$$

$$\begin{cases} t = v & v' = u'' \\ t' = v' & w = u' \end{cases} \int = v \cdot u' - \int_0^2 v' u' dx$$

$$-u'' - u = \sin x \,/\cdot v, \int_\Omega dx \Rightarrow \int_0^2 u'' v\, dx - \int_0^2 u v\, dx = \int_0^2 \sin x\, v\, dx =$$

$$-v(2)\underbrace{u'(2)}_{5+u(2)} + \underbrace{v(0)\, u'(0)}_{\text{bo } u(0)=1,\ \text{więc } v(0)=0} + \int_0^2 u' v' dx - \int_0^2 u v\, dx = \int_0^2 \sin x\, v\, dx =$$

$$\int_0^2 u' v' dx - \int_0^2 u v\, dx - v(2)(5 + u(2)) = \int_0^2 \sin x\, v\, dx =$$

$$\underbrace{\int_0^2 u' v' dx - \int_0^2 u v\, dx - v(2)\, u(2)}_{B(u,v)} = \underbrace{\int_0^2 \sin x\, v\, dx + 5 v(2)}_{L(v)}$$

Ze względu na niezerowy warunek Dirichleta przyjmujemy rozwiązanie postaci $u = w + \bar{u}$, zatem:

$$B(w + \bar{u}, v) = L(v) \Rightarrow B(w, v) = L(v) - B(\bar{u}, v)$$