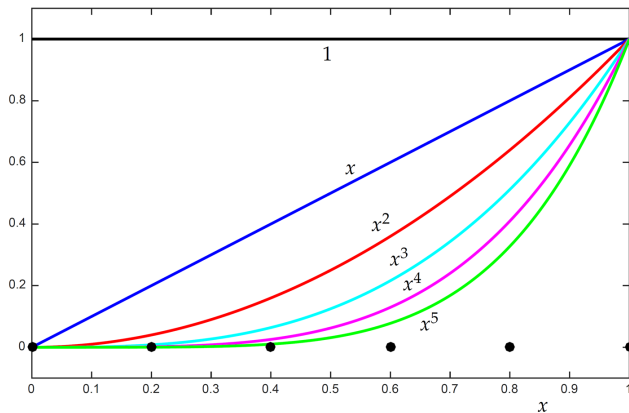


Interpolation

Marcin Kuta

Ill-conditioned base of monomials



Lagrange interpolation

$$\ell_j(t) = \frac{\prod_{k=1, k \neq j}^n (t - t_k)}{\prod_{k=1, k \neq j}^n (t_j - t_k)} \quad j = 1, \dots, n \quad (1)$$

$$\ell_j(t_i) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}, \quad i, j = 1, \dots, n \quad (2)$$

$$p_{n-1}(t) = y_1 \ell_1(t) + y_2 \ell_2(t) + \dots + y_n \ell_n(t) \quad (3)$$

Newton interpolation

$$\pi_j(t) = \prod_{k=1}^{j-1} (t - t_k) \quad j = 1, \dots, n \quad (4)$$

$$\pi_j(t_i) = 0 \quad \text{for } i < j \quad (5)$$

$$\begin{aligned} p_{n-1}(t) &= x_1 \pi_1(t) + x_2 \pi_2(t) + \dots + x_n \pi_n(t) \\ &= x_1 + x_2(t - t_1) + x_3(t - t_1)(t - t_2) + \dots \\ &\quad + x_n(t - t_1)(t - t_2) \dots (t - t_{n-1}) \end{aligned} \quad (6)$$

$$p_{n-1}(t) = f[t_1] \pi_1(t) + f[t_1, t_2] \pi_2(t) + \dots + f[t_1, t_2, \dots, t_n] \pi_n(t)$$

Evaluation of polynomial

Evaluation of polynomial $p(t)$ of degree n

Base	number of multiplications
monomials	$n(n+1)/2$ (naive approach)
	n (Horner's scheme)
Lagrange polynomials	$2n(n+1)$
Newton polynomials	$n(n+1)/2$ (naive approach)
	$2n-1$

Vandermonde matrix

Points: x_1, \dots, x_n

Vandermonde matrix:

$$\begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^{n-1} \end{bmatrix}$$

Vandermonde matrix

Data points: $(x_1, y_1), \dots, (x_n, y_n)$

In the monomial basis, coefficient a_1, \dots, a_n of polynomial interpolating points $(x_1, y_1), \dots, (x_n, y_n)$ can be found from equation:

$$\begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^{n-1} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

Vandermonde matrix

Data points: $(x_1, y_1), \dots, (x_n, y_n)$

In the basis $\phi_1(x), \dots, \phi_n(x)$, coefficient a_1, \dots, a_n of polynomial interpolating points $(x_1, y_1), \dots, (x_n, y_n)$ can be found from equation:

$$\begin{bmatrix} \phi_1(x_1) & \phi_2(x_1) & \phi_3(x_1) & \cdots & \phi_n(x_1) \\ \phi_1(x_2) & \phi_2(x_2) & \phi_3(x_2) & \cdots & \phi_n(x_2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \phi_1(x_n) & \phi_2(x_n) & \phi_3(x_n) & \cdots & \phi_n(x_n) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

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