Quadratures

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Quadratures

Quadrature	Nodes
Newton-Cotes	Equispaced nodes
Clenshaw-Curtis	Chebyshev nodes
Gauss	Legendre nodes

Quadratures

$$M(f) = (b-a)f(\frac{a+b}{2}) \tag{1}$$

$$T(f) = \frac{b-a}{2}(f(a)+f(b)) \tag{2}$$

$$S(f) = \frac{b-a}{6}(f(a)+4f(\frac{a+b}{2})+f(b)) \tag{3}$$

Simple quadratures and error bounds

Quadrature	Error
Midpoint rule	$\frac{1}{24} f''(\eta) h^3$
Trapezoid rule	$\frac{1}{12} f''(\eta) h^3$
Simpson 1/3 (parabolic)	$\frac{1}{90} f^{(4)}(\eta) h^5$
Simpson 3/8 (cubic)	$\frac{3}{80} f^{(4)}(\eta) h^5$

Quadrature	Degree of exactness	Rate of convergence
Midpoint rule	1	3
Trapezoid rule	1	3
Simpson 1/3 (parabolic)	3	5
Simpson 3/8 (cubic)	3	5

Composite quadratures

Quadrature	Rate of convergence
Midpoint rule	2
Trapezoid rule	2
Simpson 1/3 (parabolic)	4
Simpson 3/8 (cubic)	4

Gaussian quadratures and scaling

$$\int_{-1}^{1} f(\xi)\,\mathrm{d}\xi \approx \sum_{i=0}^{n} A_i f(\xi_i) \tag{4}$$

$$\int_{a}^{b} f(x)\,\mathrm{d}x \approx \frac{b-a}{2} \sum_{i=0}^{n} A_i f(x_i) \tag{5}$$

$$x_i = \frac{b+a}{2} + \frac{b-a}{2}\xi_i \tag{6}$$

Empirical order of convergence

$$E(h) \approx Ch^p \tag{7}$$

$$\log E(h) \approx \log(C) + p \log(h) \tag{8}$$

$$p \approx \frac{\log(\frac{E(h_2)}{E(h_1)})}{\log(\frac{h_2}{h_1})} \tag{9}$$

References

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