Runge phenomenon

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Runge phenomenon

Theorem

Runge (1901)
Polynomial interpolants in equispaced points may diverge exponentially, even if f is analytic

Theorem

Faber (1914)
No polynomial interpolation scheme, no matter how the points are distributed, will converge for all continuous functions

Theorem

Polynomial interpolants in Chebyshev points always converge if f is smooth (e.g. Lipschitz continuous)

Cubic splines

$$a = t_0 < t_1 < \dots t_N = b$$

$$S(x) = S_i(x) \text{ for } x \in [t_i, t_{i+1}]$$

- interpolation condition
  $S_i(t_i) = y_i \quad S_i(t_{i+1}) = y_{i+1}$ for $i = 0, \dots, N-1$
- continuity of first derivatives
  $S'_{i-1}(t_i) = S'_i(t_i)$ for $i = 1, \dots, N-1$
- continuity of second derivatives
  $S''_{i-1}(t_i) = S''_i(t_i)$ for $i = 1, \dots, N-1$

Cubic splines

- natural spline

$$S_0''(a) = S_{N-1}''(b) = 0$$

- clamped spline

$$S_0'(a) = f'(a) \quad S_{N-1}'(b) = f'(b)$$

- not-a-knot spline

$$S_0'''(t_1) = S_1'''(t_1), \; S_{N-2}'''(t_{N-1}) = S_{N-1}'''(t_{N-1})$$

- periodic spline

$$S_0'(a) = S_{N-1}'(b), \; S_0''(a) = S_{N-1}''(b)$$

- quadratic spline
  - S_0 and S_{N-1} are quadratic

References

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